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43. 1203.

43. 1203.

43. 1203.

A

THEORETICAL AND PRACTICAL SYSTEM

OF

ARITHMETIC,

COMPRISING

ALL THE USUAL RULES WITH THEIR PROOFS;

SCALES OF NOTATION;

THE CONSTRUCTION AND USE OF

LOGARITHMS;

MENSURATION OF SUPERFICIES AND SOLIDS;

FORMULA FOR CALCULATING THE HORSE-POWER OF
STEAM-ENGINES, WATER-WHEELS, &c.

TOGETHER WITH MUCH USEFUL INFORMATION NEVER BEFORE INTRODUCED
INTO TREATISES ON ARITHMETIC.

*DESIGNED FOR THE USE OF SCHOOLS AND
SELF-TAUGHT STUDENTS.*

BY

W. H. CRANK,
MATHEMATICAL MASTER, ST. MARK'S COLLEGE, CHELSEA.

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TO

THE REV. D. COLERIDGE, M.A.,

PRINCIPAL OF ST. MARK'S COLLEGE,
CHELSEA,

THIS VOLUME IS DEDICATED,

AS

A MARK OF RESPECT AND GRATITUDE,

BY

THE AUTHOR.

PREFACE.

The following Treatise, originally composed for the instruction of the Pupils at St. Mark's College, Chelsea, having been found to answer its purpose, is now submitted, with slight alterations, to public notice.

The questions, amounting to nearly four thousand, are original, and will, it is hoped, from their number and variety, be found to afford sufficient practice in each rule to students of every class.

Many rules not generally known, and many original methods, are introduced in various parts of the work; more particularly in Multiplication, Division, Rule of Three, Practice, Interest, Fractions, Decimals, Extraction of Roots, and Mental Arithmetic.

The proofs of the rules, it is hoped, will be found clearly elucidated, and adapted to the comprehension of beginners and self-taught students. Throughout the work, it has been endeavoured to remove, as far as possible, all difficulties, whether real or apparent, from the path of the student, and to explain, in familiar language, every operation. No "Royal Road" is offered for the acquisition of arithmetical knowledge, yet, perhaps, it may be remarked, that many new rules, and various improvements on the old ones, will be found. Great care has been employed in the composition of the questions, in order that the reasoning powers of the pupils may be gradually and progressively exercised.

The arrangement of the work will be best understood from a perusal of the table of Contents. In addition to the ordinary rules of Arithmetic, many of its more useful applications have been introduced; some of which are only to be found in works of much higher pretensions.

A more scientific phraseology might, of course, have been adopted, but would have been out of place in a work of this kind. Should the result prove, that the Author has been successful in rendering clear to beginners, what has before been found abstruse, and in making that easy, which has hitherto been thought difficult, his object will have been fully attained.

St. Mark's College,
February 25, 1843.

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The Author regrets to find that, in spite of great care bestowed upon the revision of this Work in its progress through the press, some error, have escaped him. Most of them, however, are errors of a single figures where the inaccuracy will be obvious, and therefore cannot embarrass the reader; and he believes the only cases where any difficulty can arise, are the following :—

Page 10, question 12, for 301799872 read 3017998727.
,, 11, ,, 13. The answer should be 669000.
,, 12, ,, 27. The date of the birth of Socrates should be B.C. 469.
,, 23, ,, 2. The answer should be 14 acres $\frac{9191}{10890}$.
,, 57, ,, 1. 8296*l*. 14*s*. 8½*d*. divided by 7 is 1185*l*. 4*s*. 11½*d*.
,, ,, ,, 3. 2847*l*. 19*s*. 11½*d*. divided by 6 is 474*l*. 13*s*. 3*d*.
,, *85*, ,, *21*. The answer should be 34384.

ARITHMETIC.

PART I.

DEFINITIONS.

1. ARITHMETIC is the science of number.

2. When it treats of *whole* numbers it is called Vulgar, or Common Arithmetic; but when of *broken* numbers, or parts of numbers, it is called *Fractions:* these being subdivided into Vulgar, Decimal, and Complex Fractions.

3. A Unit, or, as often called, Unity, represents one thing, disregarding its component part or parts. It is the chief elementary principle of all computations. Thus, a horse, a plough, &c.

4. *Number* consists of 2, 3, or more units, and of homogeneous objects. Numbers are either *concrete* or *abstract:* concrete, when they speak of specific objects, as 5 horses; and abstract, when *not* of specific objects, as 9.

5. An *Integer*, or whole number, is made up of a certain portion of units, as 1, 19, 25.

6. A *Fraction* consists of a part or parts of an integer, as $\frac{1}{5}$, $\frac{2}{5}$; which means, that the integer has been divided into 5 parts, and that 1 of them is meant by the first fraction, and 2 by the second.

7. A *Prime* number is that which has no divisor but itself or unity, and does not consist of factors, as 19: which has no other divisor than unity and itself, and consists of no other factors but 1 and 19.

8. A *Perfect* number is a number which equals the sum of all its aliquot parts; thus $28 = 14.\ 7.\ 4.\ 2.\ 1$; and $6 = 3.\ 2.\ 1$; and $496 = 248.\ 124.\ 62.\ 31.\ 16.\ 8.\ 4.\ 2.\ 1$.

9. A number is called a *Multiple* of another when it contains that *other* a certain number of times *exactly:* there being no remainder. The *number of times* contained is called a *measure*, or a *divisor*. Ex. 32 is a multiple of 8, the 8 being called a measure, or a divisor.

10. A *Common Multiple* is a number which is divisible by 2 or more numbers. Thus, 180 is a C. M. of 4, 5, and 9.

11. A *Common Measure* is a number which measures 2 or more numbers. Thus, 4 is a C. M. of 28, 40, and 160.

12. A *Composite* number is a number composed of 2 or more

factors. Thus, 48 is composed of 8 and 6, or of 2, 3, and 8, or 2, 4, and 6, &c.

13. An *Aliquot* part of any number is a measure or factor of it. Thus 4 is an aliquot part of 48.

14. A *Factor* is a number, which, with any other number or numbers, form a product equal to the given number. Thus, 7 or 8 is said to be a factor of 56, because 7 multiplied by 8=56. Therefore 7 and 8 are said to be factors of 56.

15. An *Aliquant* part of any number is any number which does not measure the given number without a remainder. Thus, is an aliquant part of 48.

SIGNS OR SYMBOLS OF ARITHMETICAL OPERATIONS.

These consist of certain characters or marks, which denote the various operations of Addition, Subtraction, Multiplication, Division, Involution, and Evolution.

$+$ is called *plus*, and indicates Addition; signifying, that any numbers between which it is placed are to be added together. Thus $5 + 6$, means that 5 is to be added to 6; or the sum of 5 and 6.

$-$ is called *minus*, and indicates Subtraction; signifying, that any number before which it is placed is to be subtracted from the number which precedes it. Thus, $9 - 4$, means that 4 is to be taken from 9.

($\times$ or $\bullet$) indicates Multiplication, and is read *into;* and signifies that the quantities between which it is placed are to be multiplied together. Thus 5×9 or $5 \bullet 9$ means that 5 is to be multiplied by 9.

$\div$ or : indicates Division, and is read *by;* and signifies that the quantity which precedes the sign is to be divided by the quantity which follows it. Thus, $56 \div 7$, or $56 : 7$, means that 56 is to be divided by 7.

: (as) :: (is to), : (so is), are the signs of proportion. Thus, $5 : 6 :: 10 : 12$, is read, 5 is to 6 as 10 is to 12.

$=$ placed between two or more quantities is read *equal*, or *equals*, or *will equal;* or it may be rendered by any tense and numbers of the verb, "to be equal." Thus, $12=12$, or $4+2=6$, or $9 + 3 + 5 = 6 + 10 + 1$, or $7. 8. 2 = 28. 4$.

$\surd$ is the sign of the Square Root, and signifies that the quantity or quantities before which it is placed is, or are, to have the square root extracted; as, $\sqrt{5}$ means that 5 is to have its square root extracted. Sometimes this sign is represented by a fractional index placed on the right of the quantity; thus, $5^{\frac{1}{2}}$ means the same as $\sqrt{5}$.

$\sqrt[3]{}$ is the sign of the Cube Root, and signifies &c., as in the last paragraph, and so of any other root.

—— is called a Vinculum, and when placed over any numbers denotes that they are to be taken together. Thus, $\overline{5+3-2} \times 9 = 54$, means that 5 is to be added to 3, and 2 to be subtracted from the sum, and the difference to be multiplied by 9: giving a product of 54. Whereas, without the vinculum the numbers would have signified that 5 is to be added to 3, and 2 multiplied by 9 (viz. 18), to be taken from the sum; or the result would have been -10.

( ) is called a Parenthesis, and means precisely the same thing as the vinculum.

NOTATION AND NUMERATION.

(1) Notation is the method of expressing Numbers by means of recognised symbols in figures, *i. e.* of writing down any number in *figures;* and Numeration is the reading of any number expressed.

(2) These symbols, ten in number, (called figures or digits,) are supposed to be of Arabic origin, and assume different values, according as other figures are placed to the right hand of them.

In this country numbers are reckoned thus:

The digits, or figures, 1. 2. 3. 4. 5. 6. 7. 8. 9. and 0. called cipher, nought, or zero.

From these all other numbers can be formed. Thus, by Art. 2, our number ten is formed by affixing zero to 1, thus 10; eleven, by affixing unity to unity, thus 11;

Twelve	by affixing	2	to unity, thus	12
Thirteen	..	3	.. ..	13
Fourteen	..	4	.. ..	14
Fifteen	..	5	.. ..	15
Sixteen	..	6	.. ..	16
Seventeen	..	7	.. ..	17
Eighteen	..	8	.. ..	18
Nineteen	..	9	.. ..	19

Twenty being 2 tens is represented by 20.

From twenty to thirty, numbers are formed by merely placing the number required, more than twenty, in the place of the 0 of the 20; thus, if we want to express twenty-seven, we merely substitute for the 0 of the 20, the 7, thus obtaining 27. We then proceed to thirty (3 tens), or 30; and any intermediate numbers to forty, or 4 tens, are found in precisely the same manner as between twenty and thirty. We then proceed from forty (40) to fifty, or 5 tens (50), introducing the intermediate numbers as before. Then from fifty (50) to sixty or 6 tens (60), introducing the intermediate numbers as before.

Then from sixty (60) to seventy or 7 tens (70), ditto.
„ seventy (70) to eighty or 8 tens (80), ditto.
„ eighty (80) to ninety or 9 tens (90), ditto.
„ ninety (90) to hundred or 10 tens (100), ditto.

On reviewing the above arrangement we perceive that *one single* number is called a unit; and of two numbers, the first, or *left* hand one, is called *ten*, or tens, and the *right* hand one the unit, or units; and that we terminated the above observations by arriving at a hundred, which consisted of 3 numbers. All intermediate numbers from 100 @ 200 are found by affixing the part above 100 to the one hundred, or displacing the 2 noughts of the 100 and substituting for them the number above one hundred.

Ex.—To express one hundred and thirty eight, we perceive that the number consists of 100, 3 tens and 8 units, and that we are told above, to express any number between 1 and 2 hundreds, we are to displace the 2 noughts and substitute the 3 tens and 8 units, thus we obtain 138.

ADDITIONAL EXAMPLES.

Put down in numbers:

Eight hundred and fifteen.
Nine thousand six hundred and fifteen.
Seventy thousand and eighty three.
Nine millions five hundred thousand and forty seven.
Eighty-five millions six hundred and nine.

The following is called the Numeration Table, in which it will be perceived that the numbers increase in value tenfold whenever a number is added to the right hand; thus,

Millions	Thousands	Hundreds
987	654	321

1 means One unit.
21 „ Two tens and a unit, and is read twenty-one.
321 „ Three hundreds, two tens and one unit; and is read three hundred and twenty-one.
4321 „ Four thousands, three hundreds, two tens and one unit; and is read four thousand, three hundred and twenty-one.
54321 „ Fifty-four thousands, three hundreds, two tens and one unit; and is read fifty-four thousand, three hundred and twenty-one.

654321 means Six hundred and fifty-four thousands, three hundreds, two tens and one unit; and is read six hundred and fifty-four thousand, three hundred and twenty-one.

7654321 „ Seven millions, six hundred and fifty-four thousands, three hundreds, two tens and one unit; and is read seven million, six hundred and fifty-four thousand, three hundred and twenty-one.

87654321 „ Eighty-seven millions, six hundred fifty-four thousands, three hundreds, two tens and one unit; and is read eighty-seven million, six hundred and fifty-four thousand, three hundred and twenty-one.

987654321 „ Nine hundred and eighty-seven millions, six hundred and fifty-four thousands, three hundreds, two tens, and one unit; and is read nine hundred and eighty-seven million, six hundred and fifty-four thousand, three hundred and twenty-one.

It will be perceived that these numbers are divided into periods; the three first on the right being composed of units, tens of units, and hundreds of units; the next three being composed of thousands, tens of thousands, and hundreds of thousands. On the left hand of these, they consist of hundreds of millions, and if three more were added they would be called thousands of millions, if three more were added they would be called hundreds of billions, if three more, thousands of billions, then trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, &c., &c.; increasing precisely in value as the millions increase.

RULE FOR ADDITION.

Addition is the first of the Rules of Arithmetic, and consists in finding the sum of two or more numbers. The method of doing which is by the following

Rule.—The numbers must be so placed that units may fall under units, tens under tens, hundreds under hundreds, thousands under thousands, and so on.

(2) A line must be drawn below the last, or bottom row of numbers, then cast up the right hand row or units column, and underneath the last unit of the bottom line of numbers place the right hand or unit figure of the sum just found, adding the left hand one, or ones, to the second column (or tens) of numbers.

Then add up the second column, and put down the right hand number, or unit figure, of this sum, carrying the left hand number or numbers to the third (or hundreds') column: and so continually proceed, always remembering to put down the *right* hand number (or unit of your sum last found) under the column last added up, and carrying the *left* hand number or numbers to the next column. Excepting after you have added the last column, when you must put down all the numbers of this last column.

Ex.—Familiar explanation of the Rule, with example.

```
 7294
 4579
 6895
 4092
 8376
------
31236
```

Here we begin by drawing a line underneath the last row of numbers, and then proceed to add up the units, or right hand column, and find the amount to be 26. We then, under the last number of the bottom line, place the right hand or unit's figure of the 26 (viz. the 6), and carry the 2 (tens) to the tens, or second column, and find the sum to be 43. We then put down the 3 of the 43 under the 7 (or second column), carrying the 4 to the third column, which, on adding up, we find amounts to 22. We then put down the right hand number (2), and carry the left hand one to the next (or thousands) column, and find on adding up this, that it amounts to 31, which we put down; there being no other column to which we can add the left hand number (3). We thus find the sum total to be 31236.

ADDITION TABLE.

The pupil should learn the following Table before attempting to perform addition:

2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and	10 and	11 and	12 and
2= 4	2= 5	2= 6	5=10	6=12	7=14	8=16	9=18	10=20	11=22	12=24
3= 5	3= 6	3= 7	6=11	7=13	8=15	9=17	10=19	11=21	12=23	
4= 6	4= 7	4= 8	7=12	8=14	9=16	10=18	11=20	12=22		
5= 7	5= 8	5= 9	8=13	9=15	10=17	11=19	12=21			
6= 8	6= 9	6=10	9=14	10=16	11=18	12=20				
7= 9	7=10	7=11	10=15	11=17	12=19					
8=10	8=11	8=12	11=16	12=18						
9=11	9=12	9=13	12=17							
10=12	10=13	10=14								
11=13	11=14	11=15								
12=14	12=15	12=16								

1. It will be perceived that we have omitted, after the 3 first columns of numbers, the numbers lower than the number to be added; as it must be evident that 7 added to 3 is the same as 3 added to 7, which appears in the column of 3.

This Table should be perfectly learnt. It will be found not only of great use in Addition, but also in Subtraction.

SIMPLE ADDITION.

(1) Add 37+29+68+47+55+63+71+32+53.
(2) „ 286+397+428+714.
(3) „ 519+967+432+568+481.
„ 298+745+326+702+255.
(4) „ 419+628+739+581+261.
„ 789+432+695+211+405.
(5) „ 809+756+937+191+63.
(6) „ 785+497+998+215+12.
(7) „ 679+385+767+321+615.
(8) „ 5849+7168+2496+3417+4151+7504+6583.
(9) „ 8174+7695+8297+7968+1826+1703+2032.
(10) „ 5698+7495+3217+7968+4302+6783+2032.
(11) „ 9157+3209+6758+7396+843+3242+2604.
(12) „ 8296+5739+7418+6982+1704+2582+3018.
(13) „ 4674+5196+3287+6573+5326+6713+3427.
(14) „ 7825+4916+3827+6952+2175+6173+3048.
(15) „ 95732 + 89674 + 92749 + 31957 + 4268 + 7251 + 68043.
(16) „ 72159+35698+74924 + 49864+27841+25076+ 50136.
(17) „ 95731 + 82973 + 65719 + 82495 + 4269+24281+ 17505.
(18) „ 49689+32174+59682+73198+ 50311 + 40318+ 49689.
(19) „ 53967 + 48674 + 59879+39247+46033+40121+ 60753.
(20) „ 96971 + 59674 + 38693 + 72794+ 3029+61307+ 27206.
(21) „ 29082+7041+3968+57321+271+380.
(22) „ 673256+49825+793298+46829+3281+751.
(23) „ 29517+496+32987+3172+559827+2098.
(24) „ 968929+49819+57321+1731+39497+4394.
(25) „ 21936+791825+4983+297+79829+1479.
(26) „ 986423+59867+4987+928+49679+3596.
(27) „ 795438+762982+43219+8731+621+47.
(28) „ 831967+49278+329+4798+65732+891.
(29) „ 598273+69749+5298+697+3849+596.
(30) „ 298654+783196+51498+3795+928+1987.
(31) „ 493741+61782+9698+3291+19681+532986.
(32) „ 409217+596827+38291+49867+53821+9867.

(1) How many days are there in the odd months of the year?
Ans. 184.

(2) How many days are there in the even months of the year? *Ans.* 181.

(3) Homer lived 907 B.C.; how many years to the present time, 1842? *Ans.* 2749.

(4) A person was born on January 1st, 1777, when will he be 86 years and 63 days old? *Ans.* 4th March, 1862.

(5) Charles V. gave Malta in 1522 to the Knights of Rhodes. How long was this to A.D. 1842? *Ans.* 320 years.

(6) The Areopagus was established at Athens in 1272 B.C.; how long to the establishment of the University of Oxford by Alfred in 886? *Ans.* 2158.

(7) Jerusalem was built 1800 B.C., and destroyed by Titus in 70 A.D. How long did it exist? *Ans.* 1870 years.

(8) The kingdom of Argos began 1586 B.C. How many years have elapsed to the establishment of the kingdom of Greece under Otho, in 1833? *Ans.* 3419 years.

(9) The kingdom of Athens began 1556 B.C.; the city of Babylon was founded by Nimrod 2640 B.C.; the Chinese Empire is said to have been founded 2100 B.C. How long would all these have existed at the foundation of the Geneva Republic in 1512, supposing they had all endured to that time?

Ans. 10832 years.

(10) Carthage was founded by the Tyrians 1259 B.C.; built by Dido 776 B.C.; supposing it had existed till 349 A.D., how long would the empire have been founded, and how long after Dido? *Ans.* 1608 founded; 1125 after Dido.

(11) Alexander the Great made conquests in India 327 B.C. How long to the time that Henry IV. conquered France in 1420? *Ans.* 1747.

RULE FOR SUBTRACTION.

Subtraction is the second of the Rules of Arithmetic, and consists in finding the difference between any two numbers. The method of doing which is by the following

RULE.—First place the numbers so that units may be under units, tens under tens, &c., as in Addition. Then subtract each number in the bottom line from the number over it, in the top line; but should it so occur (which is very frequently the case) that the bottom number is greater than the top one, we must (as it is usually termed) borrow 10 from the next number towards the left hand, and add it to the less number, then subtract the bottom number from this sum (see the proof at the end of the first four Rules). Now, as we have borrowed ten from the next, or top number, we must add ten to the bottom, or next number to the one we last subtracted, and proceed continually throughout the question in this manner. The following example will clearly illustrate the Rule.

From 78296
Take 39724
———
38572

On looking at the numbers to be subtracted, we perceive on the right hand that we have to take 4 from 6; and as we have learnt in Addition, that 6 consists of 4 added to 2, we thence perceive, that if we take away the 4 we necessarily leave 2 as remainder; we therefore put down this remainder underneath the 4 or units figure. We then proceed to the next column, and perceive that we have to take 2 from 9; and from our previous knowledge we know that 9 consists of 2 and 7, we therefore perceive, if we take away the 2, that 7 remains; we accordingly put it down under the 2 or second number. We then proceed to the 3rd column, and perceive that we have to take 7 from 2; now this we know to be an impossibility. We then, as it has been termed, "borrow 10 from the next number," which, when added to the 2 will make 12 (as in Addition 10 and 2 = 12); and as 12 (see Addition Table) consists of 7 and 5, we have to take away 7 from 12, it follows that if we take away the 7, 5 necessarily remains. We therefore put down the 5, and proceed to the next or 4th column. Here we have to take 9 from 8*; we therefore borrow (as we did in the preceding numbers) 10 from the next number; which makes the 8 an 18, and we have to take 10, *viz.*, 9 and 1 (termed borrowed) from the 18; and as we know that 18 consists of 10 and 8, we thence perceive that if we take away the 10, the 8 will remain, we then put down this 8 and proceed to the 5th column. And as we borrowed 10 we must increase the 3 by 1, we then get 4 to be taken from 7; and as 7 consists of 4 and 3, if we take away the 4, we leave as remainder 3, which we put down. This terminates the question.

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* But it is usual in such instances as this, instead of taking the 1 out of the 8, and taking the 9 from 7 to let the 8 remain an 8, and to add 1 to the 9, making it 10, and taking this from the 8 above, increased by 10 from the next number. We are justified in doing this, because it will be perceived that we have absolutely increased each number by 1, and therefore the difference will still be the same as before we so increased the numbers. Ex.—If we have to take 9 from 11, we know that 11 consists of 9 and 2, and we see that if we take away the 9, the 2 remains; but if instead of taking away the 9 from 11, we add 1 to each number, we change the 9 to 10, and the 11 to 12; and as in addition, we learnt that 12 consists of 10 and 2, it is evident if we take away the 10 that the 2 remains. It will be perceived that we obtain the same remainder as without having added 1 to each top and bottom number, which is *really* the case in the method of performing Subtraction.

SIMPLE SUBTRACTION.

393174982456493 179659832496987	29864729841983 17329586397496
99674928649573 82967498947869	7986432969874 3498749654989
573299498231698 498683295749382	7329649693298 4939824969829

(1) From 349682 take 217431.
(2) „ 6179524 take 2063113.
(3) „ 496329747 take 248116316.
(4) „ 597932148 take 249867912.
(5) „ 409963257 take 298759698.
(6) „ 932496874 take 241597329.
(7) „ 7296849528 take 784932159.
(8) „ 59493286 take 20957869.
(9) „ 69578296 take 23795968.
(10) „ 95968492743 take 28957329698.
(11) „ 896 729586 take 1596986796.
(12) „ 301799872 take 1963782095.
(13) „ 52693892471 take 17968920758.
(14) „ 5689632 49 take 5299840932.
(15) „ 983298457 take 729654932.
(16) „ 9219587432986 take 138216938475.
(17) „ 46732982968 take 2 986743299.
(18) „ 743297579 take 496324968.
(19) „ 62498743296 take 38195698732.
(20) „ 49874329941 take 18295749862.
(21) „ 52198742964 take 29817965498.
(22) „ 378294159327 take 409286732981.
(23) „ 8719324984 take 4983217849.
(24) „ 9028734968 take 2698407329.
(25) „ 9986274986 take 1695732849.
(26) „ 5273298496 take 2968749689.
(27) „ 729832964 take 615797382.
(28) „ 9582937219 take 4598632947
(29) „ 4596732989 take 2 98635825.
(30) „ 2874573510 take 1739658929.

(31) From 59864987498 take 38174829654.
(32) „ 70956849298 take 24986958329.
(33) „ 9732965749 take 3829549867.
(34) „ 3296849498 take 1782969649.

(1) The Assyrian empire began 790 B.C., and ended 538 B.C.; how long did it exist? *Ans.* 252 years.

(2) The Persian or Median empire began 538 B.C., and ended 332 B.C.; how long did it last? *Ans.* 206 years.

(3) The Grecian or Macedonian empire began 332 years B.C. under Alexander the Great; how many years elapsed after the creation of the world 4004 B.C.? *Ans.* 3672 years.

(4) The Roman empire began 753 B.C. and the kings were expelled B.C. 534; required the number of intervening years. *Ans.* 219.

(5) The Crusades began in 1096; how many years from tnat time to 1827? *Ans.* 731.

(6) America was discovered in 1492; how long was that before 1849? *Ans.* 357.

(7) The computation by Olympiads commenced 776 B.C., and terminated 480 A.D.; how many years intervened? *Ans.* 1256.

(8) Homer is said to have lived 850 B.C.; how many years elapsed to the time of Cyrus 536 B.C.? *Ans.* 314.

(9) Lycurgus lived 708 B.C.; how long before Cæsar's death 44 B.C.? *Ans.* 664.

(10) The Peloponnesian War began 431 B.C., and ended 404 B.C.; how long did it last? *Ans.* 27 years.

(11) The city of Syracuse was founded 719 B.C., by Aretais, the son of Euergetes, and was taken by the Romans 200 B.C.; how many years elapsed before this event? *Ans.* 519.

(12) The regal government at Rome lasted 218 years, the consular government till 456 B.C.; how many years did the latter government exist? *Ans.* 238.

(13) The population of Rome was said to be 1,200,000, and the population of London and suburbs, in 1831, 1,869,000; required, the difference. *Ans.* 651,000.

(14) Constantine the Great succeeded to the Roman empire 305 A.D.; how many years have elapsed to 1841? *Ans.* 1536.

(15) The Eastern empire existed till 1453 A.D.; how many years after the time of Constantine the Great 331? *Ans.* 1122.

(16) The Hegira, or flight of Mahomet, happened in 622 A.D., and the Reformation in England began in 1330; how many years elapsed between the two events? *Ans.* 708.

(17) Longinus, the Prime Minister of Zenobia, died 273

A.D.; how many years to 1842, when Sir R. Peel again resumed office? *Ans.* 1569.

(18) Mæcenas, the Prime Minister of Augustus, died 8 B.C.; how many years to the death of Mr. Pitt, 1806 A.D.? *Ans.* 1814.

(19) Dr. Johnson was born 1709 and died 1784; how old would he have been had he lived 30 years longer? *Ans.* 105.

(20) Milton was born in Bread Street, Cheapside, in 1608, and died 1674; Shakespeare was born at Stratford-upon-Avon in 1564, and died 1616; Mitford was born 1734, and died 1794; how long did these authors live? *Ans.* 178 years.

(21) Coleridge died in 1834, Socrates died 410 B.C.; how long from the latter event to the former? *Ans.* 2244.

(22) Wickliffe died 1384; how long was this before Luther was born, viz. 1483? *Ans.* 99 years.

(23) Nicholas Breakspeare (the only Englishman who ever succeeded to the Popedom) died in 1159; how many years was this event after the birth of Mahomet in 570 A.D.? *Ans.* 589.

(24) Confucius lived 500 B.C.; how many years have elapsed to 1841 A.D.? *Ans.* 2341.

(25) Alexander the Great conquered Persia 332 B.C. How many years from this event to 1841? *Ans.* 2173.

(26) The French Revolution began in 1789; how many years have elapsed since the death of Alexander the Great, 323 B.C.? *Ans.* 2112.

(27) Cuba was discovered by Columbus in 1492 A.D. What time from that event to the birth of Socrates in 496 B.C.? *Ans.* 1961.

(28) Greece was conquered by the Turks 1453 A.D.; how long was this after the Flood 2348 B.C.? *Ans.* 3801.

(29) The Flood happened 1656 A.M.; how long since is that to 1847? *Ans.* 4195.

(30) The kingdom of Macedon was subdued 168 B.C.; how many years before the partition of Poland in 1795 A.D.? *Ans.* 1963.

(31) Virgil was born at Andes, near Mantua, 70 B.C. What time has elapsed to the birth of Milton, born in Bread Street, Cheapside, in 1608? *Ans.* 1678.

(32) Wickliffe, born at Wickliffe in Yorkshire, died in 1385; how many years elapsed between the death of Mahomet, born at Medina, in 631? *Ans.* 754.

(33) Lucius, the first Christian King in Britain, lived in 183 A.D.; how many years have intervened between this date and the death of Calvin, born in Picardy, in 1654? *Ans.* 1477.

(34) St. Chrysostom, Bishop of Constantinople, died in 404; how many years would he have lived, if he had been born in the same year as Constantine the Great died, viz., 337? *Ans.* 67.

(35) Caxton, the introducer of printing, died in 1491; and Sir W. Raleigh, the introducer of the potato and tobacco to this country, was beheaded in 1618. How many years intervened? *Ans.* 127.

(36) Wickliffe opposed the Pope's supremacy in 1377; how many years elapsed before Luther's death, in 1546? *Ans.* 169.

(37) Joseph was sold to the Egyptians in 1728 B.C.; how many years was this after the Flood in 1656, A.M. *Ans.* 620.

(38) George Fox, born at Drayton, Leicestershire, the founder of the Quakers, died in 1681; how long did he live after St. Andrew was martyred, in 69? *Ans.* 1612.

(39) Xenophon died at Corinth 350, B.C., aged 90; how many years elapsed from his birth to the birth of Seneca, born at Corduba, in 12 A.D. *Ans.* 452.

MULTIPLICATION TABLE.

Twice	Three times	Four times	Five times
1 are 2	1 are 3	1 are 4	1 are 5
2 .. 4	2 .. 6	2 .. 8	2 .. 10
3 .. 6	3 .. 9	3 .. 12	3 .. 15
4 .. 8	4 .. 12	4 .. 16	4 .. 20
5 .. 10	5 .. 15	5 .. 20	5 .. 25
6 .. 12	6 .. 18	6 .. 24	6 .. 30
7 .. 14	7 .. 21	7 .. 28	7 .. 35
8 .. 16	8 .. 24	8 .. 32	8 .. 40
9 .. 18	9 .. 27	9 .. 36	9 .. 45
10 .. 20	10 .. 30	10 .. 40	10 .. 50
11 .. 22	11 .. 33	11 .. 44	11 .. 55
12 .. 24	12 .. 36	12 .. 48	12 .. 60
13 .. 26	13 .. 39	13 .. 52	13 .. 65
14 .. 28	14 .. 42	14 .. 56	14 .. 70
15 .. 30	15 .. 45	15 .. 60	15 .. 75
16 .. 32	16 .. 48	16 .. 64	16 .. 80
17 .. 34	17 .. 51	17 .. 68	17 .. 85
18 .. 36	18 .. 54	18 .. 72	18 .. 90
19 .. 38	19 .. 57	19 .. 76	19 .. 95
20 .. 40	20 .. 60	20 .. 80	20 ..100

Six times	Seven times	Eight times	Nine times
1 are 6	1 are 7	1 are 8	1 are 9
2 .. 12	2 .. 14	2 .. 16	2 .. 18
3 .. 18	3 .. 21	3 .. 24	3 .. 27
4 .. 24	4 .. 28	4 .. 32	4 .. 36
5 .. 30	5 .. 35	5 .. 40	5 .. 45
6 .. 36	6 .. 42	6 .. 48	6 .. 54
7 .. 42	7 .. 49	7 .. 56	7 .. 63
8 .. 48	8 .. 56	8 .. 64	8 .. 72
9 .. 54	9 .. 63	9 .. 72	9 .. 81
10 .. 60	10 .. 70	10 .. 80	10 .. 90
11 .. 66	11 .. 77	11 .. 88	11 .. 99
12 .. 72	12 .. 84	12 .. 96	12 .. 108
13 .. 78	13 .. 91	13 .. 104	13 .. 117
14 .. 84	14 .. 98	14 .. 112	14 .. 126
15 .. 90	15 .. 105	15 .. 120	15 .. 135
16 .. 96	16 .. 112	16 .. 128	16 .. 144
17 .. 102	17 .. 119	17 .. 136	17 .. 153
18 .. 108	18 .. 126	18 .. 144	18 .. 162
19 .. 114	19 .. 133	19 .. 152	19 .. 171
20 .. 120	20 .. 140	20 .. 160	20 .. 180

Ten times	Eleven times	Twelve times
1 are 10	1 are 11	1 are 12
2 .. 20	2 .. 22	2 .. 24
3 .. 30	3 .. 33	3 .. 36
4 .. 40	4 .. 44	4 .. 48
5 .. 50	5 .. 55	5 .. 60
6 .. 60	6 .. 66	6 .. 72
7 .. 70	7 .. 77	7 .. 84
8 .. 80	8 .. 88	8 .. 96
9 .. 90	9 .. 99	9 .. 108
10 .. 100	10 .. 110	10 .. 120
11 .. 110	11 .. 121	11 .. 132
12 .. 120	12 .. 132	12 .. 144
13 .. 130	13 .. 143	13 .. 156
14 .. 140	14 .. 154	14 .. 168
15 .. 150	15 .. 165	15 .. 180
16 .. 160	16 .. 176	16 .. 192
17 .. 170	17 .. 187	17 .. 204
18 .. 180	18 .. 198	18 .. 216
19 .. 190	19 .. 209	19 .. 228
20 .. 200	20 .. 220	20 .. 240

MULTIPLICATION.

DEFINITIONS.—1. The third of the fundamental Rules of Arithmetic is Multiplication. Its use is to find the amount of any number taken a certain number of times.

2. *The multiplicand* is the number to be multiplied: *the multiplier* is the number that shows how many times the multiplicand is to be taken; *the product* is the sum of these multiplications.

RULE.

Case 1. Place the numbers as in Addition and Subtraction, *i.e.*, units under units, tens under tens, &c. Then multiply the unit figure of the multiplicand by the unit figure of the multiplier, putting down the unit of the product and carrying the tens; then multiply the tens of the multiplicand by the unit figure of the multiplier, adding the number carried; put down the unit figure of this sum, and carry the tens; repeat this operation with the hundreds, thousands, &c., till the last number of the multiplicand is multiplied, the final product being added to the last tens carried, must be put down. Secondly, multiply the multiplicand by the tens of the multiplier, and proceed exactly as with the unit, observing to place the first number resulting from the multiplication under the number by which you multiply; then proceed to the hundreds, &c., and perform the same operation; the sum of all these multiplications is the product.

Case 2. If the multiplier has any ciphers at the end, the first numeral to the left hand of the ciphers is to be placed underneath the units figure of the multiplicand. The multiplication must then be proceeded with as in the 1st Case, affixing finally the noughts, as in Ex. 2 below.

Case 3. When the multiplier is less than 20, the multiplication can be easily effected in one line. This mode is usually called "multiplying by the back figure," and is thus performed: multiply the unit's figure of the multiplicand by the unit's figure of the multiplier, and put down the unit's figure of the product; then multiply the ten's place by the unit figure of the multiplier, adding in the number carried and the unit figure of the multiplicand; then proceed to the hundreds of the multiplicand, and multiply it by the unit of the multiplier, adding in the number carried and also the back figure (the ten's place); and so proceed till all the numbers are multiplied; taking care to add the tens carried to the last number of the multiplicand, and setting down the sum.

Case 4. It frequently happens that numbers, when numerous, admit of being divided into portions; thus the number 280|56|8, can be divided into portions, 8 . 56 and 280. It will be perceived that 56 is 7 times 8; therefore, instead of multiplying by 6 and then by 5, we multiply the 8 line by 7; so the 280 consists of 5 times the 56, we therefore multiply the 56 line by 5, thus obtaining our product by means of three lines.

EXAMPLES TO THE DIFFERENT CASES.—*Case* 1.

Mult^{y}. 7964 by 25
25
39820
15928
199100

Here it will be perceived that we first begin with the unit of the multiplicand, and multiply it by the unit figure of the multiplier; now we have learnt that 5 times 4 is 20, and we therefore put down the 0 (the unit figure of the product), and carry the 2; we then proceed to the ten's figure of the multiplicand, and multiply it by the 5, and we already know that 5 times 6 is 30, to which, if we add the 2 of the 20 before found, we obtain 32; we then put down the 2 (the unit figure), and carry the tens (the 3); we then multiply the hundreds by 5, and we already know that 5 times 9 is 45, to which we add the 3 before found, which makes 48, we then put down the 8 and carry the 4; we then proceed to the thousands, and multiply the 7 by 5; and we have learnt that 5 times 7 is 35, to which, if we add the 4 carried, we obtain 39, which we put down, there being no other number of the multiplicand to be multiplied. Secondly: we then proceed to the tens of the multiplier, multiplying every number of the multiplicand by the tens of the multiplier, placing the number resulting from the multiplication underneath the number by which we multiply, for this reason, that, in reality, we have multiplied by 20 (see proof at the end of Simple Rules); we then add the results of our multiplication together, and find we have for the sum 199100.

Case 2.

Mult^{y}. 7496 by 600
600
4497600

Here we place the 6 of the 600 underneath the unit figure of the multiplicand, and multiply as in Case 1, placing the two noughts of the 600 to the right hand of the product.

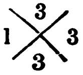
Case 3.

Mult^{y}. 2493 by 16
16
39888

Here we multiply the 3 of the multiplicand by the 6 of the multiplier, and obtain 18; we put down the 8 and carry the 1; then we say 6 times 9 is 54, and 1 is 55, and 3 (the back figure) is 58; we put down the 8 and carry the 5: we then say 6 times 4 is 24, and 5 is 29, and 9 (the back figure) is 38; we put down 8 and carry 3: we then say 6 times 2 is 12, and 3 is 15, and 4 is 19; we put down the 9 and carry 1: we then say 1 (the number carried) and 2 are 3, which we put down, making a product of 39888.

Case 4.

Multiply 24967 by 64164



64|16|4
99868 = 24967 by 4
399472 = preceding line by 4
1597888 = ———— by 4
1601982588

It will be perceived that we first multiply the multiplicand by 4; then we multiply this 4 line by 4, because 4 × 4 = 16; then we again multiply this last line by 4, because 4 times 16 is 64; taking care to put down the number resulting from the multiplication under the number we multiply by; here we put it under the 4 (thousands) of the multiplier; we then add, as in Case 1.

Case 4.

It also frequently happens that numbers are near to 100 or 1000, or 10000, &c., such as 102 or 98, or 995 or 1009, or 10007 or 9993; in all which cases an abbreviation can take place, thus:

To multiply 3824 by 98. Rule.—Add two ciphers, multiply the number 3824 by 2, and subtract this product from the original number, multiplied by 100. The reason is obvious, for we at first take the product as 100 times the multiplicand; but we should have taken 98 times only; we, therefore, subtract twice the multiplicand, leaving the product of multiplicand by 98.

382400 = 3824 × 100
Subtract 7648 = 3824 × 2
374752 = 3824 × 98

Again, multiply 4739 by 106. Here 106 is 6 more than 100; we therefore multiply the multiplicand by 100, adding 6 times the multiplicand to this product; for the same reason as above.

473900 = 4739 × 100
Add 28434 = 4739 × 6
502334 = 4739 × 106 times.

It will be perceived in all operations of this kind, that there is no necessity for multiplying by 100 or 1000, or 10000, &c., because we can easily bear in mind that we are to add 1, 2, 3, 4, or more ciphers, as the case may be; this operation can be mentally performed, remembering for every cipher to be added we are to place our first number one place to the right. Let us take the first of the above questions.

Here we perceive that we are to multiply 3824 by 100, and we therefore remove our product of 3824 by 2, two places to the right, effecting our purpose, and saving a line in the operation.

Mult^{y}. 3824
7648
374752

Simple Multiplication.

Multiply

(1) 7329864953 by 2 *Ans.*
(2) 2864738196 by 3 *Ans.*
(3) 4093769429849 by 4 *Ans.*
(4) 687494963298 by 5 *Ans.*
(5) 7496329584 by 6 *Ans.*
(6) 869490829684 by 7 *Ans.*
(7) 32968497457 by 8 *Ans.*
(8) 52794968947 by 9 *Ans.*
(9) 73295498627 by 10 *Ans.*
(10) 629578249674 by 11 *Ans.*
(11) 48317965987 by 12 *Ans.*
(12) 628974964 by 36 . 97 . 65 . 82 *Ans.* 22643098704; 61010571508; 40883372660; 51575948048
(13) 493286 by 49 . 73 . 65 . 45 . 68 . 72 *Ans.* 24171014; 36009878; 32063590; 22197870; 33543448; 35516592
(14) 939864987 by 125 . 65 . 79 . 93 *Ans.* 117483123375; 61091224155; 7424933973; 87407443791
(15) 682496864 by 329 . 796 . 473 . 84 *Ans.* 224541468256; 543267503744; 322821016672; 57329736576
(16) 7096576573 by 208 . 5009 . 7020 *Ans.* 1476087927184; 35556752054157; 49717867542460
(17) 629873245 by 596 . 789 . 2095 *Ans.* 375404454020; 496989990305; 1319584448285
(18) 3479169294 by 296 . 408 . 732 . 473 *Ans.* 10983223024; 1419501071952; 2546751923208; 1645647076062
(19) 493296872 by 539 . 682 . 3709 . 949 *Ans.* 26588693208; 336428466704; 1829638098248; 468138731528
(20) 939864987 by 125 . 65 . 79 . 93 *Ans.* 117483123375; 61091224155; 7424933973; 87407443791
(21) 682496864 by 329 . 796 . 473 . 84 *Ans.* 224541468256; 543267503744; 322821016672; 57329736576
(22) 74984329 by 144728 *Ans.* 1357066386242

23) 65298473296 by 600909 *Ans.* 39238440291027882
24) 767498698 by 84 *Ans.* 64469890632
25) 868583 by 115763 *Ans.* 102548352644
26) 1916933 by 5280 *Ans.* 10121406240
27) 10006095 by 9993908, and add 7130740 to the product. *Ans.* 100000000000000
28) 25020 by 1760 *Ans.* 44035200

DIVISION.

Definition.

Division is the fourth of the fundamental operations of Arithmetic. Its use is to find how often one number is contained in another.

The number dividing is called the *divisor.*

The number *to be divided* is called the *dividend.*

And the number resulting from the division, *i. e.*, the *number of times* the divisor is contained in the dividend, is called the *quotient.*

Case 1. When the divisor consists of a unit:—

Place the divisor on the left hand side of the dividend: taking care to separate them by a curved line. See how often the divisor is contained in the first number or numbers of the dividend, by the Multiplication Table (for the number or numbers you have to divide: it being a multiple of your divisor) placing the *contained* number under the first, or second numbers (if you divide into 2 numbers) of the dividend; proceed in the same manner to the second or third numbers, observing to carry to the following number whatever is not an aliquot part of the number divided; and so proceed to the end of the dividend, placing the remainder with the sign + before it. This rule is commonly called Short Division.

Divide 3749869 by 7.

Ex. 7)3749869

535695 + 4

Case 2. When the divisor is a composite number:—

Separate the composite number into its component factors, and divide by one of the factors, as in Rule 1; placing the remainder, if any, as directed in Rule 1. Then divide this last quotient by the next factor, placing the remainder as before. Then to find the true remainder—multiply the last remainder by the first divisor, adding in the first remainder. The sum thus found will be the true remainder.

Ex. Divide 3298495 by 32.

$$32=\left\{\begin{array}{l}4\,)\\ \times\\ 8\,)\end{array}\right.\begin{array}{r}3298495\\ \hline 824523+3\\ \hline 103077+7\\ \hline\end{array}$$

The division is performed exactly as in last example, and to find the remainder we multiply the 7 (the last remainder) into the first divisor, *viz.*, the 4, and get for a product 28: to which we add the 3, obtaining 31.

Case 3. When the divisor consists of 2 or more numbers, but is not a composite one:—

Place the divisor on the left hand side of the dividend, putting a curved line, thus), between them; also, placing on the right hand of the dividend another curve, thus (; to the right of which place the quotient. Then try the first number of the divisor into the first number or numbers of the dividend, and place the result, or number of times, in the quotient*. Then multiply the divisor by the quotient figure thus found, and place the product under the dividend on the left; subtract this product from the dividend, bringing down the next number of the dividend; try the first number of your divisor into this new dividend, and place the result in the quotient; multiply the divisor by this quotient figure, subtract the product from your *new* dividend, and bring down the next figure and proceed as before.

Case 4. When the divisor consists of numbers with ciphers at the end—

Cut off the ciphers from the divisor by a vertical or curved line. Then cut off as many numbers from the dividend as there are ciphers cut off from the divisor. Divide as in Rule 3; remembering that the true remainder will consist of the resulting remainder with the numbers cut off affixed thereto.

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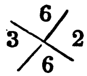
* If the number next to the trial or first number exceed 5, the trial number is to be increased by unity, and the quotient number is generally 1 more than *this test* gives. This is an empirical mode of obtaining the quotient number; but it is generally accurate. Its truth may always be tested by remembering, that if the remainder, after subtracting the product of the divisor and quotient number, be greater than the divisor, the quotient figure must be increased by unity, or if the product of the divisor and quotient figure be greater than the figure above it, that, then the quotient figure must be diminished by unity.

```
174)598647(3440
    522
    ---
     766
     696
     ---
      704
      696
      ---
       87
       --
```



Divide 598647 by 174. Here, on looking at the divisor, we perceive that the second number is above 5; we therefore increase the first or trial number by unity, we thus get 2 for the trial number. And by simple division, we find that there are 2 twos in 5; we thence obtain 3 (being 1 more than our empirical quotient) for the quotient figure: which is the correct number. We then subtract the product of the divisor and quotient figure, viz. 522, from 598; and obtain a remainder 76; and bringing down the next number 6, we obtain 766 for a new dividend. We then try 2 (our trial number) into the 7, and find it goes 3 times; but this is to be increased by unity; *i. e.*, it is to be 4. In multiplying the divisor by 4 we obtain a product of 690, which we subtract from the new dividend, and we then bring down the next number to the dividend, and get 704; we then try the trial number into the 7, and find that it goes 3, which we increase by unity, making the quotient number 4; which is correct. We then multiply the divisor by the 4, and obtain a product of 696, which we subtract from the 704, and obtain a remainder of 8, to which we affix the 7 (the next number brought down from the dividend) and find that our new dividend (87) is less than our divisor: we then perceive that 174 is not contained in 87, we therefore put a 0 in the quotient, and this concludes the question.

Useful Properties of Numbers to be learnt by the Pupil.

1. Every even number is divisible by 2.
2. Every number ending in 0 or 5 is divisible by 5.
3. If the units and tens be divisible by 4, the number is divisible by 4.
4. If the hundreds, tens, and units be divisible by 8, the number is divisible by 8.
5. If when the digits of any number be added together they be divisible by 3 or by 9, the number itself is divisible by 3 or by 9.
6. If the units place be even, and the sum of all the digits be divisible by 6, the number is divisible by 6.
7. When the sum of the 1st, 3rd, 5th, 7th, &c., digits, (*i. e.*, the sum of all the *odd* places) be equal to the sum of the 2nd, 4th, 6th, 8th, &c., places, (*i. e.*, the sum of all the *even* places,) the number is divisible by 11.

8. A number not being divisible by any other number l than the square root of the given number, is a *prime.*

9. All prime numbers, 2 and 5 being excepted, terminate 1, 3, 7, or 9. All other numbers are called *composite;* that are divisible into factors. It must be understood, however, t many numbers terminating in 1, 3, 7, or 9, are not *pri* numbers.

DIVISION (SHORT).

2)63985474968	3)4987329689
31992737484	1662443229 + 2
4)3721956824	5)789268439
930489206	155853687 + 4

Divide

(5) 3298456973 by 4 . 5 . 6
Ans. 824614243+1; 659691394+3; 549742828

(6) 1987324865 by 7 . 8 . 9
Ans. 283903552+1; 248415608+1; 2208131873

(7) 3924986719 by 10 . 11 . 12
Ans. 392498671+9; 356816974+5; 327082226

(8) 3298654 by 13 . 14 . 15 . 16
Ans. 253742+8; 23561+11; 21991; 206053

(9) 8319578 by 17 . 18 . 19 . 20
Ans. 489386+16; 462198+14; 437872+10; 41597

(10) 9173296 by 81 . 92 . 23 . 24
Ans. 1027108+36; 99709+68; 398838+22; 382220+

(11) 6873295 by 95 . 36 . 27 . 28
Ans. 72350+45; 190924+31; 254566+13; 245474

(12) 5167389 by 29 . 39 . 71 . 32
Ans. 1782185+24; 132497+6; 72780+9; 161480+

(13) 72968954 by 33 . 84 . 85 . 36
Ans. 2211180+14; 868678+2; 858458+24; 2026915+

(14) 50984732 by 97 . 85 . 42 . 37
Ans. 525615+77; 599820+32; 1213922+8; 1377965+

(15) 82749698 by 67 . 69 . 65 . 63
Ans. 1235070+8; 1199270+68; 1273072+18; 1313376+

(16) 782496587 by 86 . 93 . 51 . 17
Ans. 9098797+45; 8413941+74; 15343070+17; 46029

(17) 41769824 by 97 . 98 . 77 . 79
Ans. 430616+72; 426222+68; 542465+19; 528731+

(18) Divide 209864752 by 67 . 84 . 95 . 36 *Ans.* 3132309+49; 2490056+48; 2209102+62; 5829576+16

(19) 431792849 by 65 . 33 . 74 . 92 *Ans.* 6642966+59; 13084631+26; 5835038+37; 4693400+49

MISCELLANEOUS QUESTIONS (AFTER DIVISION).

(1) Required the product of all the odd numbers to 15 inclusive; also the product of all the even numbers below 15; also the quotient of the greater product by the less. *Ans.* Product of odd Nos. 2027025; of even Nos. 645120; quotient $3\frac{91665}{645120}$.

(2) I am desirous of placing 3599 fir trees in 12 rows, 14 ft. asunder. What space of ground shall I cover? *Ans.* 16 rods $\frac{226}{363}$.

(3) Required $7\frac{1}{2}$ the number of years, from the building of Solomon's temple, 1012 B.C. to the time of building St. Paul's cathedral in 1688. *Ans.* 20250.

(4) D is 19 years younger than B, and C, who is 32 years old, is 4 years older than D; how much older is B than C, and what is each man's age, and the sum of all their ages?

Ans. B is 15 years older than C. Their ages, B=47; C=32; D=28; and their sum 107.

(5) A gentleman died and left 7964*l.* 12*s.* between his son and daughter; the son had 1365*l.* more than the daughter; what had each? *Ans.* Son, 4664*l.* 16*s.*; daughter, 3299*l.* 16*s.*

(6) There are two numbers, the greater is 199 more than the less; and the sum of both is 677. Required the numbers. *Ans.* 239 the less, 438 the greater.

(7) The total amount of the effects of the suitors in the Court of Chancery in 1828 was 39216326*l.* 0*s.* 1*d.*; now, supposing, on an average, each suitor's property to amount to 12,490*l.*, how many causes would be down for trial? *Ans.* 3139 +.

(8) In 1827 there were 3393 convicts sent to New South Wales and Van Diemen's Land at an expense of 81682*l.* 17*s.* 8*d.* what was the average expense of one? *Ans.* 24*l.* 1*s.* $5\frac{1}{2}$*d.*+3338

(9) In 1828 there were 16564 persons in England and Wales committed for trial; how many were there to each county, there being 52 in both divisions? *Ans.* 318 $\frac{7}{13}$.

(10) In 1828 there were 35695 English settled in 18 towns of France; how many on an average to each town? *Ans.* 1871 $\frac{17}{18}$.

(11) The amount of duty paid in 1828 by the Sun Fire Office on Fire Insurances was 114205*l.* 10*s.* 10*d.* Supposing on an average each insurance to pay 7*s.* 6*d.*; how many insurances were effected? *Ans.* $304548\frac{1}{9}$.

EXPLANATION OF ADDITION.

Add 40965, 7209, 2935 together.

$$\begin{array}{lrcrcrcrcrcr}
 & 40965 & = & 40000 & + & 0000 & + & 900 & + & 60 & + & 5 \\
\smash{\left\{\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right.} & 7209 & = & & + & 7000 & + & 200 & + & 0 & + & 9 \\
 & 2935 & = & & + & 2000 & + & 900 & + & 30 & + & 5 \\
\hline
 & 51109 & = & 40000 & + & 9000 & + & 2000 & + & 90 & + & 19 \\
 & \overline{} & & 10000 & & 2000 & & 100 & & 10 & & \\
 & & & \overline{50000} & & \overline{11000} & & \overline{2100} & & \overline{100} & & \\
 & & & \overline{} & & \overline{} & & \overline{} & & \overline{} & &
\end{array}$$

It will be perceived, on reference to the number 40965 that it consists of 40 thousands, 9 hundreds, 6 tens, and 5 units, which are placed in a line with the sign + between them. Also that 7209 consists of 7 thousands, 2 hundreds, 0 tens, and 9 units. Also that 2935 consists of 2 thousands, 9 hundreds, 3 tens, and 5 units. On looking at the right hand column we perceive the sum of the units is 19, which we put down; we then proceed to the next, or tens' column, and find the sum to be 9 tens, which we put down; we then proceed to the hundreds' column, and find the sum to be 20 hundreds, which we put down; we then proceed to the thousands' column, and find the sum to be 9 thousands, which we put down; we then proceed to the tens of thousands' column, and find the sum to be 4 tens of thousands, which we put down.

On looking at the units we perceive that they consist of 1 ten and 9 units; we therefore put down the 9 units and add the 1 ten to the 9 tens, and find it equals 10 tens, or 1 hundred; we therefore add the 1 hundred, putting down 0 in the tens' place, there being no tens left when the hundred is added to the column of hundreds. The 1 hundred being added to the 20 hundreds makes 21 hundreds; but 21 hundreds are = to 2 thousand and 1 hundred; we therefore put down the 1 hundred and carry the 2 thousands to the 9 thousands, which makes the sum of this column 11 thousands; but 11 thousands consists of 10 thousands and 1 thousand; we therefore put down the 1 thousand and carry the 10 thousands to the four 10 thousands, which makes five 10 thousands: therefore the sum is 51 thousands, 1 hundred, 0 tens, and 9 units, or 51109.

In practical operations this method is shortened by adding the tens of the units to the column of tens, and the hundreds of the tens' column to the hundreds' column, and the thousands of the hundreds' column to the thousands' column, and the tens of thousands of the thousands' column to the ten thousands' column, and so on for every column.

EXPLANATION OF SUBTRACTION.

From 3287 take 1992.

$$\text{Here}\left\{\begin{array}{l}3287 = 3000 + 200 + 80 + 7\\ 1992 = 1000 + 900 + 90 + 2\end{array}\right.$$
$$\underline{\overline{1295 = 1000 + 200 + 90 + 5}}$$

Here we perceive that we are to take 2 from 7, and we learnt in Addition that 7 consists of 2 and 5; therefore if we take away the 2 the 5 remains, which we put down; we then proceed to the next column, and find that we have to take 9 tens from 8 tens, which we know to be impossible; we therefore borrow one of the next column, viz., 10 tens, or 100, and then take 9 tens from the sum of the 100 and 80, which is taking 9 tens from 18 tens; leaving a remainder of 9 tens, which we put down. We then proceed to the hundreds' column and find that we have to take 900 from 100, which we cannot; we therefore borrow one of the next column, viz., 1000, and then we find we have to take 9 hundreds from 11 hundreds, which we know leaves 2 hundreds; and which we put down. We then proceed to the thousands' column, and find that we have to take 1 thousand from 2 thousands, which leaves 1 thousand, which also we put down. We thus find our remainder to be 1 thousand, 2 hundreds, 9 tens, and 5 units, or 1295.

EXPLANATION OF MULTIPLICATION.

Multiply 3284 by 11

$$\begin{array}{r@{\;}l}
\underline{11} & \\
3284 = & 3284 \text{ taken once.}\\
\underline{32840 =} & \underline{3284 \text{ taken ten times.}}\\
\underline{36124 =} & \underline{3284 \text{ taken 11 times.}}
\end{array}$$

Hence we perceive why we put the number resulting from multiplying the multiplicand by the second or tens' number of the multiplier one place to the left after every operation.

Multiply 4296 by 453

$$\begin{array}{r@{\;}l}
4296 & \\
\underline{453} & \\
12888 = & 4296 \text{ taken 3 times.}\\
214800 = & 4296 \text{ taken 50 times.}\\
\underline{1718400 =} & \underline{4296 \text{ taken 400 times.}}\\
\underline{1946088 =} & \underline{4296 \text{ taken 453 times.}}
\end{array}$$

In this last Example we first multiply the multiplicand by the unit figure 3; then we multiply it by the tens' figure, or 50; then we multiply it by the hundreds' figure, or 400. But in c Multiplication the noughts are not used; care being ta place the numbers resulting from the multiplication one p the left at every operation.

EXPLANATION OF DIVISION.

In the division of 12 by 3 it will be seen that by takin the divisor from the dividend as often as we are able, a koning the number of times we take it away, gives the quot Division, thus,

$$\begin{array}{rl} 12 & \\ \underline{3} & = 1 \\ 9 & \\ \underline{3} & = 2 \\ 6 & \\ 3 & = 3 \\ 3 & \\ \underline{3} & = 4 \\ 0 & \end{array}$$

Hence we have taken away the divis separate times from the dividend, and fin after we have done so, we have 0 rem which shows that 3 is contained in 12 ex times.

If we multiply 436 by 217 we obtain a product of 94612; fore if we divide this product by the multiplier we evidently the multiplicand; and if we divide by the multiplicand we ev obtain the multiplier; let us take 94612 ÷ 217, we must 436 as quotient, or 400 + 30 + 6.

$$\begin{array}{r} 217)94612(400 + 30 + 6 \\ \underline{86800} \quad\quad\quad\quad\; \\ 7812 \quad\quad\quad\quad\; \\ \underline{6510} \quad\quad\quad\quad\; \\ 1302 \quad\quad\quad\quad\; \\ \underline{1302} \quad\quad\quad\quad\; \end{array}$$

Observations and Abbreviations on the preceding Rule taining Shorter Methods of Solving Particular Cas

SUBTRACTION.

There is another way of performing Subtraction. T take the 3 from 4 and 1 remains, which we put down. W take 5 from 6 and 1 remains. We then perceiv we are ordered to take 9 from 8; we cannot d but we take the 9 from 10, leaving 1, which we the 8 above and obtain 9, which we put down. then, having borrowed 10, increase the 7 by 1, makes 8, which we take from the 9 above, (which is th

$$\begin{array}{r} 329864 \\ \underline{157953} \\ \underline{171911} \end{array}$$

thing as diminishing the upper 9 by 1 and taking 7 from it,) which leaves 1, which we put down. We then proceed to the 5, and have to take it from 2, but as this is impossible, we take it from 10, leaving 5, which we add to the 2 above, and get 7, which we put down (the same remark applies here as in last parenthesis); we then, having borrowed 10, increase the 1 by unity which makes it 2, which we take from the 3, leaving 1.

SHORT RULES IN MULTIPLICATION.

RULE 1.—To multiply by 5. Add 0 to the multiplicand and ÷ by 2.

Ex.—Multiply 37299 by 5, add 0, we get 372990, which, divided by 2, we obtain 186495; but it is shorter to *conceive the* 0 *added*.

RULE 2.—To multiply by 25. Add 2 ciphers and ÷ by 4. Shorter to conceive 2 ciphers added and ÷ 4.

Ex. and Proof.—Multiply 796824 by 25. Here adding 2 noughts mentally and dividing by 4, we obtain 19920600 for product.

Proof. $$\frac{796824 \times \cancel{25} \times 100}{\underset{4}{\cancel{100}}}$$

RULE 3.—To multiply by 50. Add 2 ciphers and divide by 2.

Ex. and Proof.—Multiply 598649 by 50, dividing by 2, we obtain 29932450 for quotient.

Proof. $$\frac{598649 \times \cancel{50} \times 100}{\underset{2}{\cancel{100}}}$$

RULE 4.—To multiply by 75. Add 2 ciphers and subtract ¼.

Ex.—Multiply 37196 by 75.

$$\begin{array}{r@{\,=\,}l} 3719600 & 37196 \times 100 \\ 929900 & 37196 \times 25 \\ \hline 2989700 & 37196 \times 75 \end{array}$$

RULE 5.—To multiply by 125. Add ¼, and so on, of other numbers which are aliquot parts of 100. A slight attention to the formation of the multiplier will frequently determine a shorter process of obtaining a result than that usually pursued.

RULE 6.—To multiply by any number of 9's. Add as many ciphers as there are 9's, and subtract the given multiplicand. The remainder is the true product.

Ex.—Multiply 378314 by 9. Conceive a 0 added, and subtract the number itself. The reason is that we have multiplied

3783140
378314
3404826

the number by 10 (by adding a 0 to it,) an therefore taken it once too much; therefo subtracting the number itself, we obtain th product.

Ex.—Multiply 749682 by 99.

$$\begin{array}{rl} & 749682 \times 100 = 74968200 \\ \text{Subtract} & 749682 \times 1 = 749682 \\ \hline & 749682 \times 99 = 74218518 \end{array}$$

Here we have multipli multiplicand by 100, plain the rule; but i lute practice it is not sary.

It is sometimes necessary to discover the factors of a site number. This may easily be effected by the following Ex.—To find all the factors of the number 800. Rule.— by all the *small prime* numbers that will divide into the number. These divisions being effected, all the other fac divisors may likewise be found, as follows. Rule.—Place prime divisors in a line, preceding them by unity (wh course, is a divisor of any number); then draw a line unde and place unity as the first divisor, multiplying this b second divisor, the products are 2 and 4; we reject the 2, b we already have a divisor 2, and put down the 4; we then ply the preceding divisors, viz., 1, 2, and 4, by 2, and get 2, 4, 8; we reject the 2 and 4 (for the reason before given), and put down the 8; we then multiply all the factors already found by 5 (the next prime divisor), and we obtain 5, 10, 20, 40, which, as we have n of these factors, we put down in a vertical column. W proceed to the next divisor, 5, and multiply all the factors a found by it, and we obtain 5, 10, 20, 25, 50, 100, 200; we the 5, 10, 20, and 40, because we have them already, and down the 25, 50, 100, 200, vertically. We then multiply factors by 2, and we obtain 2, 4, 8, 16, 10, 20, 40, 80, 50, 10 400; we reject the 2, 4, 8, 10, 20, 50, 100, 200, but put the 16, 80, and 400. We then multiply by the last divi and obtain 2, 4, 8, 16, 10, 20, 40, 80, 50, 100, 20, 4 160, 800; we reject 2, 4, 8, 16, 10, 20, 40, 80, 50, 10 400 (for the reasons already given), but put down the 3 and 800. The numbers thus exhibited are the factors req

	1st
2	800
2	400
2	200
5	100
5	20
2	4
2	2
	1

2nd.

1 . 2 . 2 . 2 . 5 . 5 .

1 . 2 . 4 . 8 . 5 . 25 . 1
10 . 50 . 8
20 . 100 . 40
40 . 200

The French, in dividing, place both divisor and quotient on the right hand, the latter being placed beneath the former, a line intervening, thus

Divide 84257 by 321.

```
84257  /321
642    \262
-----
2005
1926
-----
 797
 642
 ---
 155
```

```
Thus 321\  84257
     262/  642
           -----
           2005
           1926
           -----
            797
            642
            ---
            155
```

This arrangement is advantageous in long operations, and the appearance of the work is improved; but the method we have lately introduced at St. Mark's College, of placing the divisor and quotient on the left, instead of as above, is of greater practical use.

Find all the Factors of the following numbers.

(1) 840=1. 2. 3. 6. 7. 4. 5. 8. 14. 21. 42. 12. 18. 84.
10. 15. 30. 35. 70. 105. 210. 20. 60. 140. 420.
24. 56. 168. 40. 120. 280. 840

(2) 760=1. 2. 5. 10. 4. 20. 8. 40. 19. 38. 95. 190. 76.
380. 172. 760

(3) 424=1. 2. 8. 53. 106. 212. 424

(4) 860=1. 2. 5. 10. 4. 20. 43. 86. 215. 430. 172. 860

(5) 336=1. 2. 3. 6. 7. 14. 21. 42. 4. 12. 84. 8. 24. 168.
16. 48. 336

(6) 1960=1. 2. 4. 5. 10. 20. 40

(7) 326=163. 326. 2. 1

(8) 568=71. 142. 284. 568. 8. 4. 2

(9) 200=1. 2. 5. 10. 25. 50. 4. 20. 100. 8. 40. 200

(10) 590=1. 2. 5. 10. 59. 118. 245. 590

(11) 960=1. 2. 3. 6. 5. 10. 15. 20. 30. 4. 12. 60. 8. 24.
120. 16. 48. 240. 32. 96. 480. 64. 192. 960

(12) 952=1. 2. 4. 8. 119. 238. 476. 952

(13) 480=1. 2. 5. 10. 4. 20. 3. 6. 15. 30. 12. 60. 8. 40.
24. 120. 16. 80. 48. 240. 32. 160. 96. 480

(14) 780=1. 13. 2. 26. 4. 52. 3. 39. 6. 78. 12. 156. 5.
65. 10. 130. 20. 260. 15. 195. 30. 390. 60. 780

(15) 396=11. 2. 22. 3. 33. 6. 66. 44. 12. 132. 198. 18.
36. 396

(16) 584=1. 73. 2. 146. 292. 584
(17) 735=1. 7. 49. 5. 35. 245. 3. 21. 147. 15. 105. 735
(18) 640=1. 2. 4. 8. 16. 32. 64. 128. 5. 10. 20. 40. 80. 160. 320. 640
(19) 950=1. 19. 5. 35. 475. 2. 38. 10. 190. 950
(20) 630=1. 5. 3. 15. 2. 10. 6. 30. 7. 35. 21. 105. 14. 70. 42. 210. 18. 90. 63. 315. 126. 630
(21) 208=1. 4. 8. 16. 13. 26. 52. 104. 208
(22) 196=1. 2. 4. 7. 49. 28. 14. 98. 196
(23) 240=1. 3. 5. 2. 4. 8. 16. 15. 6. 12. 24. 48. 10. 20. 40. 80. 30. 60. 120. 240
(24) 325=1. 5. 25. 13. 65. 325
(25) 564=1. 2. 3. 4. 47. 6. 12. 94. 141. 282. 188. 564
(26) 572=1. 2. 4. 11. 22. 44. 13. 26. 52. 143. 286. 572
(27) 108=1. 3. 9. 2. 6. 18. 27. 54. 4. 12. 36. 108

TABLES OF MONEY, WEIGHTS, AND MEASURES.

(1)—Money.

2 farthings	make	a halfpenny	¼	represents	a farthing
4 ———	,,	a penny	½	,,	a halfpenny
12 pence	,,	a shilling	¾	,,	3 farthings
20 shillings	,,	a pound			

Table of Coins mentioned by Old Writers.

	£	s.	d.
An angel	0	10	0
A crown	0	5	0
Half crown ..	0	2	6
A Carolus	1	3	0
A groat	0	0	4
A guinea	1	1	0
Half guinea ..	0	10	6
A Jacobus	1	5	0
A mark	0	13	0
A moidore	1	7	0
A noble	0	6	8
A pistole	0	16	10
A seven shillings' piece	0	7	0
A six and thirty ..	1	16	0
A sovereign ..	1	0	0
Half sovereign ..	0	10	0

Nominal Money is called *Stock*. *Sterling Money* is represented by any of the above denominations.

There is in all countries a standard for Gold and Silver Coin. In this country the standard for Gold "consists of 22 parts of pure gold and 2 parts of copper," called *alloy*, and this is added to *harden* the coin.

The standard for Silver consists of 37 parts of pure silver and 3 parts of copper.

24 pence are made from an avoirdupoise lb. of copper.

66 shillings are coined out of a lb. troy of silver.

TABLES OF MONEY, &c.—(*Continued.*)

6$\frac{29}{40}$ sovereigns are coined out of a lb. troy of gold.
he penny weighs 10$\frac{2}{3}$ drs. avoirdupoise, or 291$\frac{2}{3}$ grs. troy.
he shilling ,, 3 dwts. 15$\frac{3}{11}$ grs. troy.
he sovereign ,, 5 dwts. 3$\frac{171}{623}$ grs. ,,
he guinea ,, 5 dwts. 9$\frac{1}{2}$ grs. ,,
he Mint price of standard gold is £3 17*s.* 10½*d.* per oz.
,, ,, ,, silver is 5*s.* 6*d.* per oz.

THE PENCE TABLE.

	£	*s.*	*d.*	*d.*		£	*s.*	*d.*	*d.*		£	*s.*	*d.*
are	0	1	8	160	are	0	13	4	1200	are	5	0	0
..	0	2	0	168	..	0	14	0	1300	..	5	8	4
..	0	2	6	170	..	0	14	2	1400	..	5	16	8
..	0	3	0	180	..	0	15	0	1500	..	5	5	0
..	0	3	4	190	..	0	15	10	2000	..	8	6	8
..	0	4	0	192	..	0	16	0	2500	..	10	8	4
..	0	4	2	200	..	0	16	8	3000	..	12	10	0
..	0	5	0	204	..	0	17	0	3500	..	14	11	8
..	0	5	10	210	..	0	17	6	4000	..	16	13	4
..	0	6	0	216	..	0	18	0	4500	..	18	15	0
..	0	6	8	220	..	0	18	4	5000	..	20	16	8
..	0	7	0	228	..	0	19	0	5500	..	22	18	4
..	0	7	6	230	..	0	19	2	6000	..	25	0	0
..	0	8	0	240	..	0	20	0	6500	..	27	1	8
..	0	8	4	300	..	1	5	0	7000	..	29	3	4
..	0	9	0	400	..	1	13	4	7500	..	31	5	0
..	0	9	2	500	..	2	1	8	8000	..	33	6	8
..	0	10	0	600	..	2	10	0	8500	..	35	8	4
..	0	10	10	700	..	2	18	4	9000	..	37	10	0
..	0	11	0	800	..	3	6	8	9500	..	39	11	8
..	0	11	8	900	..	3	15	0	10000	..	41	16	8
..	0	12	0	1000	..	4	3	4	11000	..	45	16	8
..	0	12	6	1100	..	4	11	8	12000	..	50	0	0
..	0	13	0										

The beginner should only learn, at present, to 240 pence, rving the acquisition of the higher numbers till somewhat e advanced.

(2)—TROY WEIGHT.

24 grains make 1 pennyweight .. .. {grs. dwt.
20 pennyweights ,, 1 ounce .. .. .. oz.
12 ounces ,, 1 pound .. .. .. lb.

Gold, silver, jewels, and liquors, are weighed by this weight; and it is employed in chemical and other philosophical experiments.

There are 480 grains in an ounce.
„ 5760 ditto „ a lb.

This weight was introduced into Europe, during the Crusades, from Cairo. The city of Troyes, in France, first used it.

Avoirdupoise Weight.

16 drachms ..	1 ounce	oz.
16 ounces ..	1 pound	lb.
14 pounds ..	1 stone.	
28 ditto ..	1 quarter	qr.
4 quarters, or 112 lbs.	1 hundred weight ..	cwt*.
20 hundred weight	1 ton.	

There used also to be "the long hundred," consisting of 120 lbs. Of this long hundred, 30 lbs. made a quarter. This, with many other weights, has been discontinued since 1826.

The following weights appear to be arbitrary:—

A firkin of butter	56	lbs.
——— soap	64	do.
Barrel of anchovies	30	do.
——— gunpowder	112	do.
——— raisins	112	do.
——— potash	200	do.
——— soap	256	do.
A quintal of fish	100	do.
A puncheon of prunes	1120	do.
A faggot of steel	120	do.
A fother of lead	19½	cwt.
A seam of glass	120	lbs.
A stone of iron, shot, or horseman's weight	14	do.
——— hemp	32	do.
——— Butcher's meat	8	do.
——— glass	5	do.
A gallon of train oil	7½	do.
A truss of straw	36	do.
——— new hay	60	do.
——— old hay	56	do.
36 trusses	a load.	
Peck of flour or salt	14	lbs.
Sack of flour	280	lbs.

* From C, one hundred, and wt. the initial and final letters.

Sack of potatoes	228	lbs.
——— coals	224	do.
A bag of rice	168	do.
A chest of tea	84	do.
A tun of linseed oil	16	cwt.
A pocket of hops	112	lbs.
A bag of hops	250	do.

7560 grs. = a lb. Troy
9000 grs. = a lb. Avoirdupoise
Troy. Avoirdupoise.
Therefore, 175 lbs. = 144 lbs.
1 lb. = 13 oz. 2½ dr.
1 lb. 2 oz. 11 dwts. 16 grs. = 1 lb.

The ounce Troy is heavier than the ounce Avoirdupoise by 42½ grs.; having the ratio of 480 to 437½.

Silks used to be weighed some 24 oz. to the lb., others 16 oz. to the lb.

A grain of wheat is universally supposed to have been the element of weight.

Cheese and Butter.

The following weights are retained in this work, being useful in solving questions in old books, though a uniformity in weights and measures has been established throughout Great Britain.

A clove, or half stone	8 lbs.
A wey in Suffolk, 32 cloves, or	256 do.
——— Essex, 42 cloves, or	336 do.

Wool.

A clove	7 lbs.
A stone	14 do.
A tod	28 do.
A wey, or 6 tods and 1 stone	182 do.
A sack is 2 weys, or	364 do.
A last is 12 sacks, or	4368 do.

All mercantile sales of articles, as soap, sugar, tobacco, &c., are effected by Avoirdupoise Weight, gold and silver being the only articles sold by Troy Weight.

Apothecaries' Weight.

		Character.
20 grs.	1 scruple	℈
3 scruples	1 dram	ʒ
8 drams	1 oz.	
12 ounces	1 pound	℔

The pound Troy is precisely the same weight as the pound Apothecaries, but its sub-divisions differ.

Apothecaries buy their drugs by Avoirdupoise Weight, but retail them by this weight.

Cloth Measure.

			Character.
2¼ inches	make	1 nail	n.
4 nails	,,	1 quarter	qr.
3 qrs.	,,	1 Flemish Ell	Fl. E.
4 qrs.	,,	1 yard	yd.
5 qrs.	,,	1 English Ell	E. E.
6 qrs.	,,	1 French Ell	Fr. E.
9 inches	,,	¼ of a yard	
18 inches	,,	½ of a yard	
27 inches	,,	¾ of a yard	
36 inches	,,	1 yard	
45 inches	,,	1 English Ell	
54 inches	,,	1 French Ell..	

The French ell and Flemish ell are now disused in England.

Long Measure.

		Character.
3 barley-corns	1 inch	bar. & in.
12 inches	1 foot	ft.
3 feet	1 yard	yd.
6 feet	1 fathom	fth.
5½ yards	1 rod, pole or perch.	rod, p.
40 rods	1 furlong	fur.
8 furlongs	1 mile	m.
3 miles	1 league	l.
60 miles geographical	1 degree	deg.

Some other measures not usually columned with the above, follow:—

A barley corn	⅓ of an inch
A cubit	18 inches
A degree	69·12 miles
A hand	4 inches
A line	$\frac{1}{12}$ of an inch
A palm	3 inches
A span	9 inches

Land is measured by the chain of 4 poles, or 22 yards in length. This is sub-divided into 100 links, each containing 7$\frac{23}{25}$ inches.

Length of a Mile in different countries in English yards.

	Yards.
Denmark	7233
England	1760
Germany	5866
Hungary	8800
Italy	1467
Poland	4400
Russia	1100
Scotland and Ireland	2200

	Yards.
Spain	5028
Sweden	7233
In France they reckon by leagues:	
The small league is	2933
Mean league	3666
Great league	4400

It is generally supposed that the barley-corn was the original element of measure. The inch or ounce comes from Uncia (a twelfth part). The yard is said to have had its origin from the length of the arm of Henry I.

The following were, before 1826, the lengths of the same measures in different parts of Great Britain and Ireland:—

	Pole.
Cunningham measure	6½ yards
Forest measure	8 „
Woodland, or Burleigh measure	6 „
Perch of Ireland	7 „

11 Irish miles = 14 British ones.

"The standard yard of 1760" is the standard yard *now*. It is, however, to be observed, that this is when the temperature is at 62° Fahrenheit.

A pendulum beating seconds in lat. of London, is 39·1393 inches in length, so that should the standard yard ever be lost, it could easily be restored.

SQUARE, OR LAND MEASURE.

			Character.
144 inches	make	1 foot	in. & ft.
9 feet	„	1 square yard	sq. yd.
100 feet	„	I sq. of flooring or roofing	Sq. Fl.
30¼ yards, or 272¼ feet	„	1 rod	Rod.
40 rods, poles, or perches, or 25000 square links	„	1 rood	Rood
4 roods, or 160 rods, or 4840 yards, or 100,000 sq. links	„	1 acre of land	Acre
30 acres	„	1 yard of land	
100 acres	„	1 hide of land	H.
640 acres	„	1 square mile	Sq. M.

All superficies are measured by this measure, land, painting, glazing, plastering, thatching, flooring, plumbing, &c.

The chain of 4 poles, or 22 yards, or 66 feet long, is used in land measuring.

10 chains long by one broad make 1 acre.

SOLID, OR CUBIC MEASURE.

		Character.
1728 inches ..	.. 1 solid or cubic foot ..	Cub. ft.
27 cubic feet ..	.. 1 cubic yard ..	.. Cub. yd.
40 feet of round timber .. 50 feet of hewn timber ..	1 ton or load ..	.. T.
42 cubic feet ..	.. 1 ton of shipping	.. T. S.

A cubic inch means a figure of six sides, each being 1 inch, *i. e.* 1 inch long, 1 inch broad, and 1 inch deep. A cubic foot is of the same figure; but its sides each one foot.

A cubic yard is similarly estimated.

All solids (as timber, marble, &c.) are measured and sold by this measure.

It is also enacted, in the act before referred to, that "the Troy pound of 1758" shall be the standard unit of weight.

The weight of a cubic inch of water, (when the thermometer is at 62°, and barometer 30 inch,) is 252·458 grains; affording a means of recovering the standard should it ever be lost.

The measure of capacity for liquids is to be 10 pounds Avoirdupoise of distilled water; barometer and thermometer as before.

The contents of the gallon are 277·274 cubic inches. The weight of the bushel is 80 lbs.; its contents 2218·192 cubic inches.

WINE MEASURE.

		Character.
4 gills or noggins	.. 1 pint	.. p.
2 pints	.. 1 quart	.. qu.
4 quarts ..	.. 1 gallon	.. gal.
10 gallons ..	.. 1 anker of brandy ..	.. ank.
18 gallons ..	.. 1 rundlet	.. rund.
31½ gallons ..	.. 1 barrel, or ½ hogshead	.. bar.
42 gallons ..	.. 1 tierce	.. tier.
63 gallons ..	.. 1 hogshead	.. hhd.
84 gallons ..	.. 1 puncheon	.. pun.
2 hhds. ..	.. 1 pipe or butt	.. P.orB
2 pipes, or 4 hhds	.. 1 tun	.. tun.

The pipes and hogsheads vary in quantity according to the country they come from, or the sort of wine they contain.

	Gallons.		Gallons.
A hhd. of Cape is	92	Pipe of Teneriffe	100
——— Claret	46	Butt of Lisbon	117
——— Tent	52	——— Malaga	105
Pipe of Madeira	92	——— Port	115
——— Sicilian	93	——— Sherry	108
——— Vidonea	100		

ALE AND BEER MEASURE.

			Character.
2 pints	make	1 quart	pt. & qu.
4 quarts	„	1 gallon	gal.
8 gallons	„	1 firkin of ale	fir. A.
9 gallons	„	1 firkin of beer	fir. B.
2 firkins, or 18 gallons	„	1 kilderkin	kild.
2 kilderkins, or 36 gallons	„	1 barrel	bar.
1½ barrels, or 54 gallons	„	1 hogshead	hhd.
3 bar., or 2 hhds., or 108 gal.	„	1 butt	

DRY MEASURE.

			Character.
2 pints	make	1 quart	qu.
2 quarts	„	1 pottle	pot.
2 pottles	„	1 gallon	gal.
2 gallons	„	1 peck	pk.
4 pecks	„	1 bushel	bush.
2 bushels	„	1 strike	str.
4 bushels	„	1 coomb	c.
2 coombs, or 8 bushels	„	1 quarter	qr.
5 qrs., or 40 bushels	„	1 wey	w.
2 weys, or 10 qrs.	„	1 last	l.

Coals used formerly to be sold by the following measure, but now are sold by weight.

4 pecks	make	1 bushel	bush.
3 bushels	„	1 sack	s.
12 sacks, or 36 bushels	„	1 chaldron	ch.
21 chaldrons	„	1 score	sc.

This measure is used for measuring all kinds of corn, seeds, &c.

TIME.

			Character.
60 seconds	make	1 minute	min.
60 minutes	„	1 hour	h.
24 hours	„	1 day	d.

		Character.
7 days	make 1 week	w.
4 weeks	„ 1 month	mon.
13 months 1 day	„ 1 year	
Or 13 months 2 days ..	„ 1 leap year.	

Leap year is found by dividing the year of Our Lord by 4; if there be no remainder it is leap year; if there be a remainder, it shows how many years have past since leap year. Ex. 1840, $\div 4 = 460$ leaving no remainder, therefore it is leap year. Again 1842, $\div 4$ gives for quotient 460 and 2 remainder, which shows that the year 1842 is the second after leap year. When the *number of centuries* is not divisible by 4, it is *not* considered leap year, though the year itself may be divisible by 4, as 1800. Here 18 is not divisible by 4, therefore not leap year; but the centuries in 1600 are divisible by 4, therefore it is considered eap year.

There are different kinds of years. The length of the sidereal year is $365^d\ 6^h\ 9'\ 12''$. The solar year is $365^d\ 5^h\ 48'\ 51\frac{3}{5}''$.

It will be remembered that the months are called long and short months; *i. e.*, any one having 31 days is called a long month; all others are called short months. And that they are alternately long and short, the month of August being excepted in this order.

Names of the Calendar Months, and number of Days in each.

	Days.		Days.
January	31	July	31
February	28	August	31
(29 in Leap Year.)		September	30
March	31	October	31
April	30	November	30
May	31	December	31
June	30		

The civil year is taken at $365\frac{1}{4}$ days, making an error in 1 year of ·007736 of a day, or 1 day in 129·2657 years; and therefore, taking into account the remark above on *even* centuries, we shall find in 1752 an error of 11 days had occurred; the difference between old and new style.—See HIND's *Arithmetic*, p. 202, 3rd Edit.

TABLE OF ANGULAR MEASURE.

		Character.
60 seconds	make 1 minute..	or $1'$ & second $1''$
60 minutes	„ 1 degree..	or 1°
90 degrees	„ 1 right ∠	

In astronomical calculations, thirds ($1'''$), fourths (1^{iv}). Also the circumference of every circle is divided into 360 parts, called degrees; each degree into 60 parts, called minutes; each minute in 60 equal parts, called seconds; each second into 60 equal parts, called thirds; each third into 60 equal parts, called fourths and so on.

ASTRONOMY.

			Characters
60 fourths ..	.. make	1 third	‴
60 thirds ..	.. „	1 second	″
60 seconds ..	.. „	1 minute	′
60 minutes ..	.. „	1 degree	°
30 degrees ..	.. „	1 sign of the Zodiac	
12 signs ..	.. „	1 great circle	

DIAMETERS OF THE PLANETS.

	Miles.		Miles.
Mercury	3108	Juno	1425
Venus	7498	Ceres ..	not satisfactorily known
Earth	7964	Pallas ..	
Moon	2144	Jupiter	89069
Mars	4218	Saturn	78730
Vesta, probably ..	238	Herschel	35112

SIZE OF BOOKS.

A sheet of paper being doubled makes a	Folio	Fol. ..	4 pages
This being again doubled makes a	Quarto. . ..	4to ..	8 do.
„ „ „	Octavo.. ..	8vo. ..	16 do.
When it makes 24 pages it is called	Duodecimo ..	12mo.	
And when 36 pages, it is called	Octodecimo..	18mo.	

PARTICULAR NUMBERS OF ARTICLES.

12 articles ..	make	1 dozen	20 quires ..	make	1 ream
12 dozen ..	„	1 gross	2 reams ..	„	1 bundle
20 articles ..	„	1 score	5 bundles ..	„	1 bale
24 sheets of paper	„	1 quire			

COMPOUND ADDITION.

RULE.—1. Arrange the quantities so that the respective columns may consist only of the same denomination.

2. Begin to add up the columns of the lowest denomination,

see how many of the next higher denomination are in this sum, put down the remainder (if any), and add the quotient to the next denomination; adopt the same method in all subsequent columns.

NOTE.—It is better, to avoid trouble, where the quantities to be added are very numerous, to put them down in separate divisions, and finally add *their* totals together for the sum.

Here we place pounds under pounds, shillings under shillings, pence under pence, and farthings under farthings. We then begin to add the lowest denominations together—the farthings, and find that the sum is 8; we then see how often 4 (the number of farthings which make the next denomination) is contained in 8, and find it is contained twice, without remainder; we therefore add the 2 to the next column of pence, and find the total to be 25; we then see how often 12 (the number of pence in the next denomination) is contained in 25, and find it is twice, and remainder is 1; we put down the 1, and carry the 2 to the next column of shillings, and find the sum to be 57, we then see how often 20 (the number of shillings in the next denomination) is contained in 57, and find it to be twice, with a remainder of 17; we put down the 17, and carry the 2 to the pounds' column, and find this amount to be 227; which we put down; giving us a total of 227*l.* 17*s.* 1*d.* of all the sums added,

£	*s.*	*d.*
97	14	6½
43	9	2¼
37	16	4¾
48	16	11½
227	17	1

NOTE.—Farthings must be put under farthings, pence under pence, shillings under shillings, and pounds under pounds.

In Troy Weight, Avoirdupoise Weight, and all the other Weights and Measures the same method is to be adopted.

ADDITION OF MONEY.

£	*s.*	*d.*
761	19	8¼
482	14	9½
569	13	7½
238	0	3¾
430	6	4½

£	*s.*	*d.*
791	3	8¼
429	6	8½
374	14	7¼
208	16	3¾
625	5	4¾

£	*s.*	*d.*
973	8	10¼
496	3	11½
738	12	9¼
26	11	1¾
261	7	2¾

£	*s.*	*d.*
574	19	8¼
378	16	5½
782	19	11¼
425	0	3¾
217	0	0¾

£	*s.*	*d.*
659	7	9¼
208	13	2¾
417	6	9¼
340	12	2¾
582	13	2¾

£	*s.*	*d.*
827	13	9½
178	6	7¾
597	12	6½
172	6	2½
402	7	5½

ADDITION OF TROY WEIGHT.

Rule.—Same as in Addition of Money.

Explanation of Operation.

Here we commence with the lowest denomination (or right d) column. We find the sum of the grains to be 67, we therefore divide this by 24 (because 24 grs. make a dwt.), and obtain for quotient 2, and remainder 19, which we put down. We then add the 2 dwts. to the column of dwts., and obtain for the sum 50 dwts; we therefore divide this by 20 (because 20 dwts. make an oz.), and obtain 2 for a quotient, and 10 for a remainder, which we put down. We then carry the 2 ozs. to the column of ozs., and get for a 32; we therefore divide this by 12 (because 12 ozs. make a and obtain for a quotient 2, with a remainder of 8, which we down. We then carry the 2 lbs. to the lbs., and obtain for a 233. Our sum total being 233 lbs., 8 oz., 10 dwts., 19 grs. No further explanation of addition of the other compound tities will be given, it being deemed unnecessary after the e two explanations.

oz.	dwts.	grs.
11	19	15
8	10	19
9	11	13
0	0	9
2	8	11
8	10	19

Addition of Troy Weight.

lbs.	oz.	dwts.	grs.
72	11	19	15
33	8	10	19
42	9	11	13
27	0	0	9
57	2	8	11

lbs.	oz.	dwts.	grs.
32	11	19	23
79	8	16	14
68	3	11	12
67	0	0	1
21	8	8	12

lbs.	oz.	dwts.	grs.
58	10	18	22
49	3	17	16
69	2	14	15
41	1	1	2
30	9	5	9

lbs.	oz.	dwts.	grs.
67	8	3	14
29	8	17	19
28	5	12	13
32	3	16	10
71	6	7	11

ADDITION OF AVOIRDUPOISE WEIGHT.

tons	cwts.	qrs.	lbs.	oz.
3974	19	3	27	15
1982	11	4	14	13
2874	8	3	17	14
6025	0	0	0	1
7125	11	0	10	2

tons	cwts.	qrs.	lbs.	oz.
7498	14	2	14	11
2745	10	1	19	2
8296	7	1	18	14
2501	5	1	13	5
1703	12	2	9	2

tons	cwts.	qrs.	lbs.
4329	18	2	19
6824	18	1	27
7896	9	2	11
5670	1	1	9
2103	10	1	17

SUBTRACTION OF MONEY AND TROY WEIGHT.

RULE.—1. Arrange the numbers as if for Addition (by the Addition Rule), and commence, as in Simple Addition, at the lowest, or right hand, column.

2. If the Subtrahend (or line to be subtracted) has its terms less than the Minuend (or line from which it is to be subtracted), put down the difference below its column.

3. But if the minor terms of the Subtrahend should be partially or totally greater than the minor terms of the Minuend, then subtract the terms of the Subtrahend from one of the next greater denomination, adding the difference to the upper term, and placing the sum below the column. Proceed thus throughout the question, observing that the final, or left hand, denomination is to be subtracted, as in Simple Subtraction.

		£	s.	d.
(1)	From	4298	13	$8\frac{1}{2}$
	Take	395	8	$2\frac{1}{4}$
		2903	5	$6\frac{1}{4}$

		£	s.	d.
(2)	From	3986	14	$9\frac{1}{4}$
	Take	2198	16	$11\frac{3}{4}$
		1787	17	$9\frac{1}{2}$

Here, in the first Example, all the minor terms (viz., shillings, pence, and farthings,) are less than the corresponding upper terms; their difference is therefore put below the column. But in the second Example we are to take $\frac{3}{4}$ from $\frac{1}{4}$, we therefore, agreeably to our Rule, take the $\frac{3}{4}$ from one of the next denomination, viz., from a penny, or 4 farthings, (its value in the same denomination that we have to subtract), we obtain 1 remainder, which we add to the upper term, and the sum is $\frac{1}{2}$, which we put down. We then, having borrowed one of the next denomination, increase the bottom term by unity, and we then perceive we have to take 12 from 9, which we know we cannot; we therefore borrow one of the next denomination, viz., a shilling, or 12 pence, from which we take our bottom term and add the difference, 0, to the top; the sum is 9, which we put down. We then, having borrowed one of the next denomination, increase the next denomination of lower by unity, and we perceive we have to take 17 from 14, which :annot; we therefore borrow one, viz., a pound, or 20 shillings, n which we take our 17, leaving a remainder of 3, which we to our upper term, obtaining for sum 17, which we put down. then, having borrowed one of the next denomination, increase bottom term by unity, and we perceive we have to take 9 from The remainder of the question is performed as in Simple ıtraction.

EXAMPLE TO TROY WEIGHT.

	lbs.	oz.	dwts.	grs.
From	743	11	9	15
Take	296	10	11	19
	447	0	17	20

Here, as before, we take 19 from 24 (one of the next deno- ation), and obtain a remainder of 5, which we add to the 15, ıpper term, obtaining for sum 20, which we put down. Having rowed one of the next denomination, we increase the next :om term by unity, making it 12, which we have to take from We therefore subtract the 12 from one of the next denomina- , viz., 1 oz., or 20 dwts., and obtain a remainder of 8, which add to the top term, obtaining for sum 17, which we put down. ᴠing borrowed one of the next denomination we increase the by unity, making it 11, which we subtract from the upper 11, ing a remainder of 0, which we put down. The remaining : of the question is performed in the same manner, as in ımon Subtraction.

SUBTRACTION OF MONEY.

Borrowed.			Paid.			
£	s.	d.	£	s.	d.	
968	14	11¾	209	10	8¼	*Ans.*
729	16	9½	428	13	7¼	*Ans.*
12096	18	8¼	3982	12	11½	*Ans.*
74287	19	6½	2438	16	11¼	*Ans.*
40986	11	9¾	39698	19	10½	*Ans.*
29847	12	6½	5697	19	8¾	*Ans.*
79328	14	7¼	32968	19	6¾	*Ans.*
596896	18	7¾	249896	13	7¾	*Ans.*
72968	11	7¼	69748	19	10½	*Ans.*
48295	18	8¾	25869	18	9¼	*Ans.*
46987	19	10½	37896	12	9¼	*Ans.*
342968	14	8¼	96987	19	10½	*Ans.*
47983	19	6½	23896	12	8½	*Ans.*
64967	18	9¼	49698	12	11½	*Ans.*

Borrowed.			Paid.			
£	s.	d.	£	s.	d.	
42958	12	11¼	31874	15	9¾	*Ans*
37929	17	4¼	19865	12	7½	*Ans.*
69572	10	8¼	58269	15	11½	*Ans.*
20957	13	7½	18699	8	9¾	*Ans.*
85672	11	9¼	39896	13	10¾	*Ans.*
69379	11	4¼	51967	18	11½	*Ans.*
79845	19	6¼	31784	1	8¾	*Ans.*
54986	14	7¼	24967	19	10½	*Ans.*
40867	12	11½	29698	18	7¾	*Ans.*
25968	14	9¼	14869	15	10½	*Ans.*
56987	10	9¼	29596	14	11¼	*Ans.*
67824	18	10½	59098	12	7¾	*Ans.*

SUBTRACTION OF LONG MEASURE.

	deg.	m.	fur.	p.		deg.	m.	fur.	p.	
From	39824	14	3	19	Take	17396	48	6	12	*Ans.*
„	95732	48	5	22	„	68259	55	7	35	*Ans.*
„	67425	13	2	17	„	48657	47	6	32	*Ans.*
„	82916	22	1	13	„	49573	19	7	31	*Ans.*
„	29174	16	0	15	„	7968	32	6	30	*Ans.*
„	91738	17	2	19	„	47396	51	5	27	*Ans.*
„	86957	31	5	29	„	79864	57	3	35	*Ans.*
„	59327	45	3	16	„	48974	49	6	36	*Ans.*
	yds.	ft.	in.	b.		yds.	ft.	in.	b.	
„	74967	1	3	1	„	29732	2	10	2	*Ans.*
„	49832	0	7	0	„	24739	2	9	1	*Ans.*
„	29754	1	2	1	„	18295	2	8	2	*Ans.*
„	18924	0	3	1	„	2746	2	7	2	*Ans.*
„	79234	2	1	2	„	28629	1	11	1	*Ans.*
„	28311	1	0	0	„	17965	2	7	2	*Ans.*
„	68592	0	10	1	„	24979	2	3	2	*Ans.*
„	25957	1	3	0	„	18259	2	10	2	*Ans.*

SUBTRACTION OF WINE MEASURE.

	tuns	hhds.	gal.	qu.		tuns	hhds.	gal.	qu.	
From	7914	3	51	1	Take	2089	1	48	2	*Ans.*
„	5969	1	32	1	„	2974	3	35	2	*Ans.*
„	9582	0	33	0	„	4979	1	29	3	*Ans.*
„	1698	0	37	2	„	1299	0	38	3	*Ans.*
„	8274	0	48	0	„	7498	1	35	1	*Ans.*
„	5695	1	32	1	„	4957	0	49	2	*Ans.*
„	9821	2	31	0	„	5897	1	35	1	*Ans.*
„	7498	0	16	1	„	6989	1	35	0	*Ans.*

	hhds.	B.	gal.	qu.		hhds.	B.	gal.	qu.	
From	3957	1	31	3	Take	2895	2	34	1	*Ans.*
„	4986	0	11	1	„	2578	1	32	0	*Ans.*
„	6748	1	34	0	„	5921	1	35	2	*Ans.*
„	6973	0	29	1	„	5169	1	20	2	*Ans.*
„	6573	1	21	1	„	2995	0	27	2	*Ans.*
„	4973	2	25	1	„	2869	1	17	2	*Ans.*
„	6719	2	27	0	„	4973	1	32	1	*Ans.*
„	5967	1	25	1	„	2983	2	33	1	*Ans.*

SUBTRACTION OF ALE AND BEER MEASURE.

	hhds.	gal.	qu.	p.		hhds.	gal.	qu.	p.	
From	7924	37	1	0	Take	2847	51	2	1	*Ans.*
„	5968	32	1	1	„	4170	48	2	1	*Ans.*
„	6297	45	0	0	„	2919	49	3	1	*Ans.*
„	7396	21	1	1	„	4129	41	2	1	*Ans.*
„	6298	31	1	0	„	4732	48	2	1	*Ans.*
„	5982	43	0	1	„	3178	51	3	1	*Ans.*
	butts	hhds.	gal.	q.		butts	hhds.	gal.	q.	
„	3829	1	50	0	„	2198	2	43	1	*Ans.*
„	7863	2	17	2	„	3796	1	45	0	*Ans.*
„	5849	1	19	0	„	4983	0	36	2	*Ans.*
„	2743	0	17	2	„	1989	1	39	0	*Ans.*
„	9849	1	13	2	„	4658	1	44	3	*Ans.*
„	2578	0	24	1	„	1796	1	41	1	*Ans.*

COMPOUND MULTIPLICATION.

						Answers.		
			s.	*d.*		*£.*	*s.*	*d.*
47 yards	..	at	3	9½	=	10	7	5
54 ells	..	at	2	7¼	=	7	0	7½
95 pecks	..	at	1	2½	=	5	14	9½
37 pecks	..	at	3	8¼	=	6	16	5¼
92 hndkfs.	..	at	4	2½	=	19	7	2
35 lbs.	..	at	2	9¾	=	4	16	4
45 ditto	..	at	3	1½	=	7	0	7½
72 ditto	..	at	7	9¼	=	27	19	6
48 ditto	..	at	3	11¾	=	9	11	0
49 ditto	..	at	4	9½	=	11	14	9½
95 gallons	..	at	7	8¼	=	36	10	3¾
67 pecks	..	at	3	2¾	=	10	15	4¼
96 cwt.	..	at	14	8½	=	70	12	0
79 oz.	..	at	3	9¼	=	14	17	10¾

							Answers.		
				s.	*d.*		£.	*s.*	*d.*
59	lbs.	..	at	2	3¾	=	6	16	7¾
45	ditto	..	at	1	9½	=	4	0	7½
77	ditto	..	at	3	4¼	=	12	18	3¾
53	ditto	..	at	10	6½	=	28	14	8½
87	yards	..	at	2	2¼	=	9	10	3¾
95	ditto	..	at	1	11½	=	9	6	0½
34	ditto	..	at	6	2¾	=	10	11	9½
59	ditto	..	at	7	9½	=	22	19	8½
89	ditto	..	at	3	2¼	=	14	3	8¼
53	ditto	...	at	1	10¾	=	5	0	5¾
92	ditto	..	at	1	8½	=	7	17	2
75	ditto	...	at	3	6¾	=	13	7	2¼
95	ditto	...	at	9	3½	=	44	2	8½
78	ditto	..	at	5	9¼	=	22	10	1½
47	ditto	...	at	3	2½	=	7	10	9½
69	ditto	...	at	4	11¼	=	17	0	8¼
85	bushels	..	at	4	7¼	=	19	11	4¼
63	ditto	..	at	2	9½	=	8	15	10½
91	ditto	..	at	3	7¼	=	16	7	11¾
41	ditto	..	at	2	11¾	=	6	2	1¾
67	ditto	..	at	1	9½	=	6	0	0½
85	ditto	..	at	6	11¾	=	29	13	2¾
43	ditto	..	at	14	9¼	=	31	15	1¾
62	ditto	..	at	3	10½	=	11	19	3
98	ditto	..	at	15	8¼	=	76	17	4½
29	ditto	..	at	19	7½	=	28	9	1½
65	ditto	..	at	14	11¾	=	48	13	7¾
71	ditto	..	at	12	9¼	=	45	6	8¾
59	ditto	..	at	17	8½	=	52	4	9½
31	ditto	..	at	12	10½	=	19	19	1½
67	ditto	..	at	14	2½	=	47	11	11½
39	ells	..	at	2	1¾	=	4	3	5
47	ditto	..	at	1	8½	=	4	0	3½
64	pieces	..	at	2	11½	=	9	9	7
98	ditto	..	at	3	9¾	=	19	9	11½
46	ditto	..	at	17	8¼	=	40	13	7½
79	yards	..	at	2	7¼	=	10	5	8¾
41	ditto	...	at	3	8½	=	7	12	0½
69	ditto	..	at	2	8¼	=	9	5	5¼
97	ditto	..	at	3	2¾	=	15	13	2¾
45	ditto	..	at	11	8½	=	26	6	10½
67	ditto	..	at	2	4¾	=	8	0	6¼

			s.	d.		*Answers.* £	s.	d.
54 yards	..	at	7	9½	=	21	2	9
48 ditto	..	at	3	4¼	=	8	1	0
92 ditto	..	at	1	11½	=	9	0	2
65 ditto	..	at	3	7¾	=	11	16	11¾

		£	s.	d.			£	s.	d.
Multiply		829	14	7¼	by 2 . 9 . 5 . 6				
Ans.	(2)	1659	9	2½		(9)	7467	11	5¼
	(5)	4148	13	0¼		(6)	4978	7	5½
Multiply		734	19	11½	by 3 . 4 . 7 . 9				
Ans.	(3)	2204	19	10½		(4)	2939	19	6
	(7)	5144	19	8½		(9)	6614	19	7½
Multiply		969	12	10½	by 4 . 6 . 9 . 7				
Ans.	(4)	3878	11	6		(6)	5817	17	3
	(9)	8726	15	10¼		(7)	6787	10	1½
Multiply		8396	19	8¼	by 5 . 8 . 2 . 9				
Ans.	(5)	41984	18	5¼		(8)	67175	17	6
	(2)	793	19	4½		(9)	3572	17	2¼
Multiply		9689	17	4½	by 6 . 3 . 8 . 5				
Ans.	(6)	58139	4	3		(3)	29069	12	1½
	(8)	77518	18	8		(5)	48449	6	10½
Multiply		2908	19	6½	by 7 . 9 . 5 . 12				
Ans.	(7)	20362	16	9½		(9)	26180	15	10½
	(5)	14544	17	8½		(12)	34907	14	6
Multiply		9832	14	11¼	by 8 . 9 . 11 . 12				
Ans.	(8)	78661	19	6		(9)	88494	14	5¼
	(11)	108160	2	5¾		(12)	117992	19	3
Multiply		29684	15	8½	by 6 . 9 . 4 . 7				
Ans.	(6)	267163	1	4½		(9)	178108	14	3
	(4)	118739	2	10		(7)	207793	10	9½
Multiply		40989	19	6½	by 8 . 10 . 2 . 4				
Ans.	(8)	409899	15	5		(10)	327919	16	4
	(2)	81979	19	1		(4)	163949	18	2
Multiply		29684	12	7¾	by 7 . 11 . 6 . 5				
Ans.	(7)	322530	19	1¼		(11)	207792	8	6¼
	(6)	178107	15	10½		(5)	148423	3	2¾
Multiply		69598	18	9½	by 8 . 9 . 12 . 4				
Ans.	(8)	835187	5	6		(9)	556791	10	4
	(12)	628390	9	1½		(4)	270395	15	2

hen the number is above 12 to 144.

ase 1.—If it be a composite number, multiply the price by factors, of which the number is composed; ex. 48 consists of the 2 factors, 4 × 12 or 8 × 6, or 16 × 3, or 24 × 2: the 2 former to be used.

Case 2.—If it be not a composite number, find the greatest number in it which is a composite number, and multiply as in Case 1; 2ndly, then multiply the price by the remaining units, the sum of the products will be the answer: ex. in 57, the greatest composite number is 56, which consists of 7 × 8 and 57=7 × 8+1, we multiply the price by 7, and that product by 8, to which we add the price of one for the value of the unit 1.

		£	s.	d.				£	s.	d.
(12)	Multiply	925	18	11¼	by	24	=	23222	14	6
(13)	"	629	11	2¾	by	36	=	22664	4	3
(14)	"	754	18	9½	by	45	=	33971	4	8¼
(15)	"	837	12	11¼	by	56	=	46908	4	6
(16)	"	1239	19	8½	by	64	=	79359	1	4
(17)	"	3682	17	4¼	by	72	=	235709	10	8
(18)	"	6984	12	5½	by	84	=	586708	6	6
(19)	"	7829	9	4¼	by	90	=	704652	1	10½
(20)	"	8294	12	6¾	by	110	=	912409	1	10½
(21)	"	3596	17	2½	by	121	=	435220	2	2½
(22)	"	6298	14	5¼	by	132	=	821431	5	9
(23)	"	7473	11	2½	by	288	=	2152385	18	0
(24)	"	5	19	10½	by	296	=	1775	3	0
(25)	"	4	17	6½	by	29	=	141	8	8½
(26)	"	26	18	6½	by	498	=	13400	13	9
(27)	"	3	19	9½	by	69	=	275	5	7½
(28)	"	78	13	6½	by	796	=	31156	3	0
(29)	"	137474	18	10½	by	97	=	13335069	10	10½
(30)	"	8	19	6½	by	296	=	1775	11	10

Case 3.—When the quantity consists of a ¼, ½, ¾, besides integers, multiply by the integers, as in Case 1 or 2 (according as it is or is not a composite number), and if there is ¼, divide the price by 4; if ½, ÷ by 2; if ¾, ÷ by 2; and that also by 2, and add to the product of the integer with the price: the sum is the answer required.

		£	s.	d.				£	s.	d.
(31)	Multiply	36	19	10½	by	179¼	=	6541	10	1
(32)	"	38	14	6½	by	368½	=	11270	18	7¼
(33)	"	0	15	6½	by	272¾	=	211	18	11¾
(34)	"	1	5	9¼	by	276½	=	356	5	7½
(35)	"	2	10	8½	by	596¾	=	1513	0	2¼

		£.	s.	d.				£	s.	d.
)	„	1	8	7¾	by	892¼	=	1277	19	2¾
)	„	2	7	4¼	by	568¾	=	1346	12	8
)	„	0	19	8¾	by	127¼	=	125	10	6¼
)	„	1	18	9½	by	86½	=	124	10	5¾
)	„	2	18	11¼	by	75¾	=	223	4	6
)	„	1	17	6½	by	968½	=	1817	19	1¼
)	„	2	19	3¾	by	96½	=	286	11	7¾
)	„	3	19	9½	by	795¾	=	3274	14	2½

Multiply £	s.	d.			Answers. £	s.	d.
4926847296	14	11¾	by	6492	=3198509265049	4	9
32968479684	19	7½	by	6843	=4331403306484326	13	10½
8906	3	7½	by	4296894	=38268916776	0	9
45	18	6½	by	1593686	=73193349	14	7
26718	10	10½	by	4296894	=114806750328	2	3
321	9	9½	by	1593686	=512353448	2	1
29684	3	8½	by	4329432	=128515662236	17	0
1607	8	11½	by	3187372	=512353448	0	10
9623968491298	14	9½	by	9468	=46357597336756164 66	7	6
47	16	8¼	by	46924	=2244580	4	3
45	18	6½	by	796843	=36596674	17	3½
0	17	6½	by	496892432	=435816070	11	4
490000000180757	17	8½	by	113616	=27824558402053698 7982	10	0
1898905439054	18	10½	by	13686	=25988419838905960	3	3

		£	s.	d.	
)	Multiply	32	19	8½	by 2
)	„	85	19	6¼	by 3
)	„	78	12	11½	by 5
)	„	93	19	6¼	by 6
)	„	35	11	7¾	by 8
)	„	65	18	9½	by 9

) Reqd. the value of 35 boxes of indigo at £ 38 s. 16 d. 9½ per box
Ans. £1359 7 8½

) „ 64 ditto cochineal at 69 11 8¼ each
Ans. £4453 8 0

) „ 47 pckts. of money at 88 12 11¾ ditto
Ans. £4166 10 0¼

		£	s.	d.	
(10)	Reqd. the value of 99 lbs. of gold	at 35	17	6½	each
	Ans. £3551 16 7½				
(11)	„ 57 boxes of tea	at 29	18	7¼	ditto
	Ans. £1706 0 5¼				
(12)	„ 235 chronometers	at 73	15	6¼	ditto
	Ans. £17337 7 4¾				

(13) Multiply £73 15 6¼ by 235
Ans. £17337 7 4¾
(14) „ £69 18 11 by 33¼
Ans. £2325 13 11¾
(15) „ £820 15 6½ by 44½
Ans. £36524 11 7¼
(16) „ £3698 15 6¼ by 352
Ans. £1301969 3 4

TROY WEIGHT.

lbs.	*ozs.*	*dwts.*	*grs.*	*lbs.*	*ozs.*	*dwts.*	*grs.*
5196	11	19	13	2940	10	15	19
			2				3
10393	11	9	2	8822	8	7	9
9174	10	13	15	4824	7	14	16
			4				5
36699	6	14	12	24123	2	13	8
6274	9	15	19	5829	5	13	15
			6				7
37648	10	14	18	40806	3	15	9
5296	7	19	23	7168	3	15	21
			8				9
42373	3	19	16	64514	10	2	1

AVOIRDUPOISE WEIGHT.

	tons.	*cwt.*	*qrs.*	*lbs.*	*ozs.*	*drs.*
(1)	7968	19	1	25	14	13
						2
(2)	48296	11	2	24	15	11
						3

	tons.	cwt.	qrs.	lbs.	ozs.	drs.
(3)	78219	4	1	17	13	2
						4
(4)	46731	13	2	19	12	15
						5
(5)	28173	13	1	15	13	11
						6
(6)	19862	14	3	19	10	12
						7

Apothecaries' Weight.

	lbs.	ozs.	drs.	scr.	grs.		lbs.	ozs.	drs.	scr.	grs.
(1)	5978	11	7	2	19	(2)	6738	10	5	1	13
					2						3
(3)	9731	9	4	1	14	(4)	7825	8	6	2	15
					4						5
(5)	6958	7	3	0	13	(6)	8296	4	5	1	18
					6						7

Cloth Measure.

	Fl. ell.	qrs.	n.		yds.	qrs.	n.
(1)	3954	1	1	(2)	7982	3	2
			2				3
	7908	2	2		15965	3	2

	Fr. ell.	qrs.	n.		Eng. ell.	qrs.	n.
(3)	4968	3	3	(4)	8215	2	3
			4				5
	19875	3	0		41078	1	3

	Eng. ells.	qrs.	n.		yds.	qrs.	n.
(5)	6958	3	3	(6)	2917	1	3
			6				7
	41752	2	2		20422	0	1

LONG MEASURE.

d.	m.	yds.	ft.	in.	d.	m.	yds.	ft.	in.
7196	59	1720	2	1	8296	53	1698	2	1
				2					3
6521	48	697	1	0	5579	45	1398	2	1
				4					5
4759	39	1597	1	1	3675	44	1678	1	1
				6					7
2497	46	1538	1	1	8297	49	1639	2	1
				8					9

ALE AND BEER MEASURE.

butts.	hhds.	galls.	qu.	hhds.	b.	k.	g.	qu.
7492	1	49	1	4537	1	2	32	1
			2					3
6197	0	43	2	6572	1	1	31	1
			3					8
9582	1	41	3	7329	1	0	33	1
			4					9
2894	0	39	2	8629	1	2	29	1
			5					10

butts.	*hhds.*	*galls.*	*qn.*
6732	2	19	3
			6

hhds.	*b.*	*k.*	*g.*	*qn.*
7965	1	1	35	1
				11

Long Measure.

The circumference of the earth is 25020 miles; how many times would a wheel, 9 feet in circumference, turn in going round the earth? *Ans.* 14678400 times round

How many minutes would a cannon ball be in reaching the sun, 95173117 miles distant, going at the rate of 2000 feet in a second? *Ans.* 4187617$\frac{37}{250}$ minutes

Light travels at the rate of 200000 miles in a second; how long would it be in going round the earth's orbit 596902654 miles in extent? also, how long across the diameter 190000000?
Ans. 2984$\frac{51327}{100000}$ seconds in going round the earth's orbit; and 950 seconds in going along its diameter.

A man took 5 paces in 4 yards; how many paces would he go in 849 days, supposing he travelled 10 hours a day, and went 92 yards per minute? *Ans.* 58581000 paces.

The diameter of the orbit of Saturn is 1814178064 miles how long would a carriage-wheel be in traversing it, supposing the wheel turned 122000 times in a day, and was 16$\frac{1}{2}$ feet in circumference? *Ans.* 4758499$\frac{1281}{1525}$ hours.

The orbit of the planet Mercury is 74216324 miles in diameter; how long would two men be (starting from opposite points in the orbit) before they met, supposing one travelled 45 miles, and the other 57 miles per day; and how far would the latter man have travelled when they met? *Ans.* 727611$\frac{1}{51}$ days before meeting; 41473828$\frac{2}{17}$ miles the latter travels.

The Earth travels in its orbit at the rate of 68092 miles per hour; how far will it have gone in 34 days, 7$\frac{1}{2}$ hours? *Ans.* 56073762 miles.

If the circumference of the orbit of Saturn be 5699408962 miles; and if the circumference of Saturn be 275500 miles; how many times would Saturn roll round (supposing such a thing could be) whilst making one entire circuit of its orbit? *Ans.* 20687$\frac{70231}{137750}$.

WINE MEASURE.

tuns.	*p.*	*g.*	*qu.*	*tuns.*	*hhds.*	*g.*	*qu.*
4319	3	61	1	7329	3	57	1
			2				3
9396	2	59	1	4289	1	49	1
			4				5
8291	1	57	1	3679	1	55	1
			6				7

DRY MEASURE.

w.	*q.*	*b.*	*p.*	*g.*	*w.*	*q.*	*b.*	*p.*	*g.*
7498	4	7	3	1	6291	3	6	6	1
				2					3
5473	3	6	2	1	7985	3	4	1	0
				4					5
3296	1	5	1	1	4869	1	3	0	1
				6					7

TIME.

m.	*w.*	*d.*	*h.*	*m.*	*s.*	*m.*	*w.*	*d.*	*h.*	*m.*	*s.*
7319	3	6	23	59	59	4829	3	5	22	48	47
					2						3
6582	2	5	19	48	51	5167	2	4	21	45	13
					4						5
8698	1	4	13	52	19	2095	1	3	22	44	17
					6						7

COMPOUND ADDITION

(1) A gentleman bought spoons, valued at 48*l.* 16*s.* 4*d.*, forks, 16*l.* 18*s.* 6*d.*, knives, 26*l.* 12*s.* 8*d.*, teapot, 28*l.* 16*s.* 6*d.*, and snuffers at 12*l.* 8*s.* 4*d*; carriage came to 4*s.* 8½*d.*; how much did he lay out, including the carriage? *Ans.* 133*l.* 12*s.* 4*d.*

(2) A butcher bought oxen, which cost 44*l.* 16*s.* 8*d.*, calves, 16*l.* 12*s.* 4½*d.*, pigs, 37*l.* 16*s.* 8¼*d.*; and sheep, 19*l.* 12*s.* 8½*d.*; what did he lay out? *Ans.* 118*l.* 18*s.* 5¼*d.*

(3) A gentleman left his widow 14,968*l.* 12*s.*, his eldest son 4964*l.* 16*s.* 9*d.*, his second son 3298*l.* 19*s.* 10*d.*, his youngest son 1969*l.* 18*s.* 9*d*, and his two daughters 3000*l.* each: what did he die worth? *Ans.* 31202*l.* 7*s.* 4*d.*

(4) A father's income is 6984*l.* 13*s.* 9*d.*, and his eldest son's 9649*l.*, his youngest son's 4932*l.* 18*s.* 6*d.*; what was the income of them all? *Ans.* 21566*l.* 12*s.* 3*d.*

(5) A. owes to B. 295*l.* 13*s.* 11½*d.*, to C. 84*l.* 9*s.* 2¼*d.*, to D. 489*l.* 10*s.*, to E 687*l.* 14*s.* 10½*d.*; what does A. owe? *Ans.* 1557*l.* 8*s.* 0¼*d.*

(6) If I owe to A. 493*l.* 12*s.* 9¼*d.*, to B. 698*l.* 14*s.* 9½*d.*, to C. 87*l.* 12*s.* 9½*d.*, to D. 475*l.* 16*s.* 8½*d.*; how much do I owe? *Ans.* 1855*l.* 17*s.* 0¾*d.*

(7) A gentleman bought a horse for 95*l.*, a chaise for 82*l.* 10*s.* 6*d.*, harness for 55*l.* 18*s.* 6*d.*; what did they all cost him? *Ans.* 233*l.* 9*s.*

(8) A person owes his butcher 48*l.* 19*s.* 6½*d.*, his baker 74*l.* 12*s.*, his fishmonger 32*l.* 10*s.* 6¼*d.*, his hatter 7*l.* 12*s.* 4*d.*, his bootmaker 9*l.* 14*s.* 8*d.*, his tailor 37*l.* 19*s.* 6*d.*; how much does he owe in all? *Ans.* 211*l.* 8*s.* 6¾*d.*

(9) A gentleman left by will to different charities, after certain charges had been paid on each legacy, to one 500*l.*, to another 708*l.* 12*s.* 6*d.*, to another 6724*l.* 3*s.* 8½*d.*, to another 3695*l.* 19*s.* 11¼*d.*, to another 509*l.* 16*s.* 8½*d.*; how much did he leave? *Ans.* 12138*l.* 12*s.* 10¼*d.*

(10) A farmer sold a field for 16*l.* 10*s.* 6*d.*, a cow for 12*l.* 13*s.* 4*d.*, a ram for 2*l.* 12*s.* 6*d.*, 5 sheep for 12*l.* 8*s.*, 3 pigs for 7*l.* 9*s.* 10*d.*; what did he get for all? *Ans.* 57*l.* 14*s.* 2*d.*

(11) A farmer's debts were as follows: — his saddler's 36*l.* 18*s.* 10¼*d.*, smith's 17*l.* 6*s.* 3¾*d.*, nurseryman 56*l.* 17*s.* 8½*d.*, veterinary surgeon 26*l.* 18*s.* 9½*d.*, wheelwright, 49*l.* 15*s.* 3¾*d.*; what did he owe? *Ans.* 187*l.* 16*s.* 11¼*d.*

MULTIPLICATION.

(1) If there are 7 leap years in 26 years, how many days are there in all the years, reckoning 365 days for a year, except in the leap years which are of the usual length? *Ans.* 9497.

(2) A ship's company took a ship and cargo; the ship was sold for 3268*l.* 12*s.* 6*d.*; the cargo, which consisted of the following articles, viz. indigo, worth 2496*l.* 14*s.* 9*d.*, sugar, 1793*l.* 16*s.* 6½*d.*, cochineal, 2096*l.* 15*s.* 9*d.*, and Brazil wood, worth 596*l.* 18*s.* 6*d.*; what would have been the value of 9½ such ships and cargoes? *Ans.* 97402*l.* 11*s.* 4¾*d.*

(3) If a man were to take 500 steps an hour for 3 hours every day, how many yards would he have stepped in the year (omitting Sundays), supposing he stepped 3 feet at each step? *Ans.* 469,500.

(4) A gentleman had a cabinet, containing 40 drawers, each drawer contained 5 purses, and each purse 20*l.*, 5 guineas, 13 crowns, and 44 half-crowns; required the total amount of the cash? *Ans.* 6800*l.*

(5) Two men set out to walk: A., from London, at the rate of 26 miles per day; and B., from Edinburgh, at the rate of 31 miles per day. They travel for 2½ days, when B. fell sick, and remained at an inn; A. pursues his journey for 3 days longer. Required the distance they are apart at this time; the distance from London to Edinburgh being 409 miles? *Ans.* 188½ miles.

(6) A horse and an ass set out from two towns, distant 256 miles; the horse travels 3 times as fast as the ass; and the ass goes 25 miles in 2½ days. Required their distance, after 5 days' travelling? *Ans.* 56 miles.

(7) There are two cart-wheels, A. and B., each in circumference 10 feet. The cart is sent to a town 22 miles off; one wheel is constantly turning round, except for the last 300 yards, when the carter, by mistake, put the drag on it, instead of on the other. It happened that there were 7 hills on the road: the first was 200 yards in its slope; the 2nd, 1500; the 3rd, 169 yards; the 4th, 1133 yards; the 5th, 49 yards; the 6th, 128 yards; and the 7th, of 300 yards. Now the carter puts the drag on the wheel A. for the first 6 hills precisely at the commencement of what is called the hill, and takes it off on arriving at the base, and also puts the drag on the wheel B. on descending the last hill. What distance will each wheel have gone at the end of the journey, and how much further will B. have gone than A.? *Ans.* A., 35,541 yards; B., 38,420; and B. will have gone 2879 yards farther than A.

(8) A hare sets off to run 500 miles; it goes 44 miles every day, but returns the 1st night 9 miles, the 2nd 4 miles, the 3rd 6 miles, and the 4th 10 miles; how far will it have travelled in 6½ days? *Ans.* 257 miles.

(9) Suppose a man to walk 27 miles on an average every day for 39 years, omitting Sundays; what distance will he have travelled at the end of this time? *Ans.* 329589 miles.

(10) A gentleman has a cabinet, containing 56 drawers, each drawer contains 42 purses, and each purse 14 guineas, 20 seven-shilling pieces, 14 crowns, and 92 sixpences; what is the value of the money in the cabinet? *Ans.* 64680*l.*

(11) If the earth be on the equator 25020 miles in circumference, and in lat. of London 15360 miles in circumference; what would be 13½ times the difference of these two numbers? *Ans.* 130410.

(12) Three men, A. B. and C. found 4 parcels of money; A got 75*l.* 13*s.* 6*d.*, and B 5 times as much as A. and C. double of what they both got; what was the amount of all their sums? *Ans.* 1362*l.* 3*s.* 0*d.*

COMPOUND SHORT DIVISION OF MONEY.

Divide £ *s.* *d.* £ *s.* *d.*

(1) 8296 14 8½ by 2 . 7 . 9 . 6 *Ans.* 4148 7 4¼;
1186 13 6¼+3; 921 17 2¼+1; 1382 15 9¾

(2) 4659 17 9¾ by 4 . 3 . 5 . 8 *Ans.* 1164 19 5¼+3;
1553 5 11¼; 931 19 6¾; 582 9 8½+7

(3) 2847 19 11½ by 7 . 4 . 6 . 9 *Ans.* 406 17 1½+4;
711 19 11¾+2; 474 19 11½+2; 316 13 3½+8

(4) 6987 12 3¼ by 6 . 5 . 7 . 8 *Ans.* 1164 12 0½+1;
1397 10 5¼+4; 998 4 7¼+2; 873 14 0¼+5

(5) 7298 13 10½ by 9 . 6 . 8 . 4 *Ans.* 810 18 2½;
1216 8 11¾; 912 6 8¾+2; 1829 13 5½+2

(6) 2869 18 4¼ by 2 . 7 . 5 . 4 *Ans.* 1434 19 2+1;
409 19 9+5; 573 19 10+1; 717 9 7+1

(7) 6987 14 11¾ by 3 . 8 . 2 . 9 2329 4 11¾+2;
873 14 4¼+7; 3493 17 5¾+1; 776 8 3¾+8

(8) 10968 15 8½ by 7 . 9 . 5 . 6 *Ans.* 1568 7 11½;
1218 15 0¾+7, 2193 15 1½+4; 1828 2 7¼+4

(9) 29867 17 3¾ by 10 . 11 . 12 . 4 *Ans.* 2986 15 9¾+7;
2715 5 2½+1; 2488 19 9¼+3; 7466 19 3¾+3

(10) 98674 12 9½ by 11 . 9 . 7 . 3 *Ans.* 8970 8 5+7;
10963 16 11½+6; 14094 18 11½+4; 32891 10 11+2

(11) 49638 19 7¼ by 12 . 6 . 8 . 4 *Ans.* 4136 11 7½;
8273 3 3+5; 6204 17 5¼+5; 12409 14 10¾+1.

COMPOUND LONG DIVISION OF MONEY.

Divide	£	s.	d.				£	s.	d.
(1)	69732	19	4¼	by	294	*Ans.*	237	3	8¾+63
(2)	48749	18	9½	by	692	*Ans.*	70	8	11¼+674
(3)	29684	15	9½	by	27	*Ans.*	1099	8	8¾+5
(4)	4368969	19	10½	by	3674	*Ans.*	1189	3	2 +218
(5)	79682	14	11½	by	676	*Ans.*	117	17	5½+630
(6)	576898	16	8½	by	3964	*Ans.*	145	10	8¼+550
(7)	79684	15	9½	by	1796	*Ans.*	44	7	4¼+370
(8)	829167	18	9	by	3782	*Ans.*	219	4	9½+368
(9)	49685	18	10½	by	5238	*Ans.*	9	9	8½+127
(10)	23222	14	6	by	24	*Ans.*	925	18	11¼
(11)	22664	4	3	by	36	*Ans.*	629	11	2¾
(12)	33971	4	8¼	by	45	*Ans.*	754	18	9½
(13)	46908	4	6	by	56	*Ans.*	837	12	11¼
(14)	79359	1	4	by	64	*Ans.*	1239	19	8½
(15)	235709	10	8	by	72	*Ans.*	3682	17	4¼
(16)	880464	14	0¾	by	15	*Ans.*	58697	12	11¼
(17)	1113175	2	4	by	16	*Ans.*	69573	8	10¾
(18)	1409431	16	1¼	by	17	*Ans.*	82907	19	9¼
(19)	892437	5	7½	by	18	*Ans.*	49579	16	11¾
(20)	1196706	0	5½	by	19	*Ans.*	68247	13	8½
(21)	1591659	18	9	by	20	*Ans.*	79582	19	11¼
(22)	851	6	10½	by	1009	*Ans.*	0	16	10½
(23)	1547	18	1½	by	2090	*Ans.*	0	14	9¾
(24)	5087	18	0	by	7968	*Ans.*	0	12	9¼
(25)	272	16	1½	by	369	*Ans.*	0	14	9½
(26)	468	6	9	by	724	*Ans.*	0	12	11¼
(27)	242	16	10½	by	409	*Ans.*	0	11	10½
(28)	104	15	2¾	by	617	*Ans.*	0	3	4¾
(29)	111	9	6¾	by	759	*Ans.*	0	2	11¼
(30)	49	1	9	by	126	*Ans.*	0	7	9½
(31)	135	5	11¼	by	429	*Ans.*	0	6	4¼
(32)	183	11	5½	by	973	*Ans.*	0	3	11½
(33)	42	7	11	by	298	*Ans.*	0	4	9½
(34)	463	19	6	by	536	*Ans.*	0	17	3¾

COMPOUND DIVISION.

Divide	£	s.	d.		
(1)	31985092650494	4	9	by	6492
	Ans. 4926847296	14	11¾		
(2)	4331403306484326	13	10½	by	6843
	Ans. 632968479684	19	7½		

Divide	£	*s.*	*d.*		
	4635759733675616466	7	6	by	9468
Ans.	489623968491298	14	9½		
	259884198338905960	3	3	by	13686
Ans.	18989054390054	18	10½		
	27824558402053698798 2	10	0	by	113616
Ans.	244900000000180757	17	8½		
	76537833552			by	8593788
Ans.	8906	3	7½		
	229613500656	4	6	by	2148447
Ans.	106874	3	6		
	2786445589639559	8	6	by	796846
Ans.	3496843291	14	6½		
	153982727	9	3½	by	469846
Ans.	27	14	7¼		

APOTHECARIES' WEIGHT.

		lbs.	oz.	drs.	scr.	grs.			
(1)	Divide	3196	11	7	2	19	by	2	*Ans.*
(2)	,,	7732	8	4	1	18	by	3	*Ans.*
(3)	,,	5196	7	3	2	17	by	4	*Ans.*
(4)	,,	6712	9	1	1	19	by	5	*Ans.*
(5)	,,	9326	10	7	2	19	by	6	*Ans.*
(6)	,,	4932	11	7	1	11	by	7	*Ans.*
(7)	,,	6731	10	6	2	19	by	8	*Ans.*
(8)	,,	8296	11	4	1	15	by	9	*Ans.*
(9)	,,	7293	10	7	1	14	by	10	*Ans.*
(10)	,,	2968	11	6	2	15	by	11	*Ans.*
(11)	,,	5897	9	5	1	19	by	12	*Ans.*

TROY WEIGHT.

		lbs.	oz.	dwts.	grs.			
(1)	Divide	7195	11	17	15	by	2	*Ans.*
(2)	,,	6217	10	18	19	by	3	*Ans.*
(3)	,,	5168	10	15	13	by	4	*Ans.*
(4)	,,	9782	9	14	13	by	5	*Ans.*
(5)	,,	2987	11	12	19	by	6	*Ans.*
(6)	,,	4928	5	19	22	by	7	*Ans.*
(7)	,,	6597	8	13	16	by	8	*Ans.*
(8)	,,	5967	4	19	23	by	9	*Ans.*
(9)	,,	4739	11	18	17	by	10	*Ans.*
(10)	,,	9298	6	13	19	by	11	*Ans.*
(11)	,,	7169	10	14	19	by	12	*Ans.*

Avoirdupoise Weight.

		tons.	cwt.	qrs.	lbs.	oz.	drs.			
(1)	Divide	3719	19	3	26	14	15	by	2	*Ans.*
(2)	„	6582	14	1	19	13	14	by	3	*Ans.*
(3)	„	2987	12	0	15	14	15	by	4	*Ans.*
(4)	„	9698	13	1	19	13	11	by	5	*Ans.*
(5)	„	8296	19	3	17	14	15	by	6	*Ans.*
(6)	„	5219	15	2	26	13	11	by	7	*Ans.*
(7)	„	2715	17	1	27	11	14	by	8	*Ans.*
(8)	„	6573	18	3	22	12	15	by	9	*Ans.*
(9)	„	8776	15	1	17	15	15	by	10	*Ans.*
(10)	„	9152	13	3	19	11	10	by	11	*Ans.*
(11)	„	2917	15	1	26	10	14	by	12	*Ans.*

Cloth Measure.

		yds.	qrs.	n.			
(1)	Divide	4916	3	2	by	2	*Ans.*
		E. E.					
(2)	„	7829	3	1	by	3	*Ans.*
		yards.					
(3)	„	9874	3	3	by	4	*Ans.*
		E. E.					
(4)	„	8296	1	3	by	5	*Ans.*
		yards.					
(5)	„	2917	3	1	by	6	*Ans.*
		F. E.					
(6)	„	7219	4	3	by	7	*Ans.*
		yards.					
(7)	„	6517	3	3	by	8	*Ans.*
(8)	„	2827	3	3	by	9	*Ans.*
(9)	„	5869	3	3	by	10	*Ans.*
(10)	„	6957	3	1	by	11	*Ans.*
		Fr. E.					
(11)	„	5731	4	3	by	12	*Ans.*

Ale and Beer Measure.

		hhds.	B.	G.	qu.			
(1)	Divide	4934	2	29	1	by	2	*Ans.*
(2)	„	2698	2	35	1	by	3	*Ans.*
(3)	„	5192	1	35	3	by	4	*Ans.*
(4)	„	6982	1	27	3	by	5	*Ans.*
(5)	„	8695	2	32	2	by	6	*Ans.*
(6)	„	5493	2	31	3	by	7	*Ans.*
(7)	„	7329	1	35	1	by	8	*Ans.*
(8)	„	6249	2	33	1	by	9	*Ans.*
(9)	„	8967	2	33	1	by	10	*Ans.*
(10)	„	4729	1	32	2	by	11	*Ans.*
(11)	„	6292	2	34	2	by	12	*Ans.*

WINE MEASURE.

		tuns	hhds.	gals.	qrs.			
(1)	Divide	3892	3	59	1	by	2	*Ans.*
(2)	„	7968	2	47	1	by	3	*Ans.*
(3)	„	6579	2	48	3	by	4	*Ans.*
(4)	„	6974	3	42	3	by	5	*Ans.*
(5)	„	2909	3	55	1	by	6	*Ans.*
(6)	„	8296	3	55	1	by	7	*Ans.*
(7)	„	9299	3	57	3	by	8	*Ans.*
(8)	„	6284	2	59	3	by	9	*Ans.*
(9)	„	5739	3	62	1	by	10	*Ans.*
(10)	„	4732	1	62	1	by	11	*Ans.*
(11)	„	3986	3	61	2	by	12	*Ans.*

TIME.

		yrs.	wks.	d.	h.	m.	sec.			
(1)	Divide	3974	51	6	23	58	51	by	2	*Ans.*
(2)	„	6297	45	6	21	48	57	by	3	*Ans.*
(3)	„	2864	50	3	20	49	49	by	4	*Ans.*
(4)	„	7319	17	6	20	17	48	by	5	*Ans.*
(5)	„	9174	48	3	19	51	47	by	6	*Ans.*
(6)	„	6986	27	2	19	48	57	by	7	*Ans.*
(7)	„	2796	51	1	21	49	53	by	8	*Ans.*
(8)	„	2974	45	1	17	13	19	by	9	*Ans.*
(9)	„	8296	19	1	17	51	47	by	10	*Ans.*
(10)	„	7398	51	2	13	15	18	by	11	*Ans.*
(11)	„	7298	14	1	19	48	51	by	12	*Ans.*

LONG MEASURE.

		deg.	m.	fur.	p.			
(1)	Divide	2791	51	7	39	by	2	*Ans.*
(2)	„	7968	48	6	31	by	3	*Ans.*
(3)	„	5196	19	4	33	by	4	*Ans.*
(4)	„	6274	19	3	29	by	5	*Ans.*
(5)	„	6987	22	4	19	by	6	*Ans.*
(6)	„	8296	25	4	17	by	7	*Ans.*
(7)	„	3965	21	1	18	by	8	*Ans.*
(8)	„	8095	17	3	19	by	9	*Ans.*
(9)	„	7395	37	6	19	by	10	*Ans.*
(10)	„	8196	24	5	39	by	11	*Ans.*
(11)	„	6294	45	6	37	by	12	*Ans.*

SQUARE MEASURE.

		m.	sq. ac.	roods	rods			
(1)	Divide	3986	591	3	37	by	2	*Ans.*
(2)	„	7965	517	2	33	by	3	*Ans.*
(3)	„	2874	389	1	33	by	4	*Ans.*
(4)	„	6987	419	2	35	by	5	*Ans.*
(5)	„	3982	199	3	31	by	6	*Ans.*
(6)	„	8546	379	3	29	by	7	*Ans.*
(7)	„	5967	379	2	35	by	8	*Ans.*
(8)	„	6597	582	1	31	by	9	*Ans.*
(9)	„	7916	495	1	37	by	10	*Ans.*
(10)	„	2874	613	3	37	by	11	*Ans.*
(11)	„	9679	387	2	31	by	12	*Ans.*

MISCELLANEOUS QUESTIONS.

(1) If a cubic foot of water weighs 1000 ounces, how many portions, each of $17\frac{1}{2}$ ozs., are there in it?
Ans. 280 oz. 11 dwts. 10 grs.

(2) If 1 person in 34 die annually, how many die in half-a-year, supposing the population of the globe to be 890000000?
Ans. $13088235\frac{10}{34}$.

(3) If the circumference of the earth be 24,873 miles, at what rate are the inhabitants of the equator carried in $\frac{1}{4}$ hour?
Ans. $259\frac{1}{12}$miles.

(4) A cubic foot of air is said to weigh $1\frac{1}{4}$ oz.; what weight does a room contain whose dimensions are 79 ft. long, 53 wide, and $19\frac{1}{4}$ high? *Ans.* $6379\frac{107}{128}$.

(5) A field contains 4728 yards, and its length is 53 yards; what is the breadth? *Ans.* $89\frac{11}{53}$.

(6) Take 5 times 396847 from 3795968. *Ans.* 1811733.

(7) Remitted a bill of exchange, valued at 796*l.* 14*s.* The person receiving it purchases the following articles:—sugar, which costs 49*l.* 12*s.*, tea 219*l.* 12*s.* 7*d.*, indigo 133*l.* 9*s.* 7*d.*, and charges 3*l.* 9*s.* 10*d.* for commission; how much has he to remit to me? *Ans.* 390*l.* 10*s.*

(8) I have 375 guineas, and I give to a charity 48*l.* 12*s.*, to 500 poor men 1*s.* 6*d.* each, to 50 poor women 2*s.* 6*d.* each, to 20 boys 9*d.* each, and to 53 girls 1*s.* each; what money have I left?
Ans. 298*l.*

(9) What is the sum of 370 guineas, 48*l.*, and 150 pence?
Ans. 437*l.* 2*s.* 6*d.*

(10) Add 7 times 24321, 45 times 260, 71 times 45, together, and subtract 13 times 840 and 9 dozens, from the sum.
Ans. 174114.

(11) What number subtracted from 58 times 473 will leave 9 times 54? *Ans.* 23708.

(12) From 7960 pence take 22*l.* 19*s.* 8¼. *Ans.* 10*l.* 3*s.* 7¾*d.*

(13) A merchant has in cash 24791*l.* 10*s.* 11½*d.*; 3 ships, ach valued at 2948*l.* 15*s.* 10*d.*; indigo, 4981*l.* 19*s.* 4*d.*; cochineal, 371*l.* 14*s.* 9*d.*; tea, 48 boxes, valued at 1273*l.* 19*s* 8*d.*; 5 ouses, each 872*l.* 12*s.*; miscellanies, valued at 7963*l.* 4*s.* 9*d.* What is he worth? *Ans.* 52591*l.* 16*s.* 11½*d.*

(14) A man has 5 granaries, A, B, C, D, and E; in granary A, he has 700 quarters of wheat, and 50 quarters of oats; in granary B, 709 quarters of wheat and 480 of oats; in C, D, and E, 470 quarters of wheat, and 320 of oats, each: how many uarters in all the granaries? *Ans.* 2729 qrs.

(15) The poor-rate in this country was, in 1828, 7391528*l.* How much, on an average, is paid to the poor daily?

Ans., 20250*l.* 15*s.* 2¾*d.* + $\frac{15}{73}$

(15) In 1828 there were 13456 British vessels entered inwards with a tonnage of 2094357 tons. What was the average tonnage of each ship? *Ans.* 155 tons + 11777 rem.

(16) In 1828 there were 4405 foreign vessels cleared outwards with a tonnage of 608,118. What was the average tonnage of each vessel. *Ans.* 1380 + 2218 rem.

(17) Pope received for his Iliad 5300*l.* Sir J. Macintosh for his Fragments of English History 5000*l.* Byron for his poetry 23000*l.* Scott for his works 210000*l.* Lingard for his History of England 4600*l.* The Wilberforces for the Life of their Father 4600*l.* Moore for his Life of Byron 4200*l.* Moore for his Lalla Rookh 3500*l.* Southey for his Cowper 1050*l.* What would be the average each author would receive if their sums were equally divided amongst them? *Ans.* 32631*l.* 5*s.* 0*d.*

(18) In 1828, there were 21709 persons buried in the 153 parishes within and without the walls of the cities of London and Westminster. What proportion to each parish?

Ans. 135 persons + 54 rem.

(19) In 1828, 8475 persons deposited money in the Savings' Bank to the amount of 251138*l.* 18*s.* 7*d.* What did each deposit on an average? *Ans.* 29*l.* 12*s.* 7½*d.* + 3522.

(20) In 28 years there were 32580515 chaldrons of coals imported into the port of London. What was the average for each year? *Ans.* 1163589 chaldrons + 21 rem.

(21) A merchant gained 49864*l.* 19*s.* 6*d.* in 8½ years. What was that per year? *Ans.* 5866*l.* 9*s.* 4*d.* + 4 rem.

(22) I spend 1769*l.* 18*s.* 4*d.* per annum. How much is this per month, week, and day? *Ans.* 137*l.* 6*s.* 2*d.* + 120 rem. per month; 4*l.* 18*s.* 0¾*d.* + 265 per day.

(23) What is the 729th part of 127849 moidores?
Ans. 236*l.* 15*s.* $1\frac{3}{4}$*d.*+$\frac{1}{9}$.

(24) 73 men and 19 boys are to receive 176*l.* 9*s.* 10*d.*; the boys are to receive 16*s.* 6*d.* each. What will each man receive?
Ans. 2*l.* 4*s.* $0\frac{1}{2}$*d.*+$\frac{61}{73}$.

(25) If 795 oxen are worth 14722*l.*, what is the value of $2\frac{1}{2}$?
Ans. 46*l.* 5*s.* $10\frac{3}{4}$*d.*+$\frac{41}{53}$.

(26) $373\frac{3}{4}$ cwts. of sugar cost 842*l.* 12*s.* 6*d.*, what is that per cwt.? *Ans.* 2*l.* 5*s.* 1*d.* $\frac{29}{299}$.

(27) Required the amount each person will receive, supposing 137,596*l.* are divided amongst 473 men? *Ans.* 290*l.* 18*s.* 0*d.* $\frac{6}{473}$.

(28) If a gentleman receives annually 500*l.*, and pays for taxes and other things 137*l.* 12*s.*, what can he spend per day supposing he saves 4*s.* 9*d.* per day? *Ans.* 0*l.* 15*s.* 1*d.* $\frac{7}{365}$.

(29) In January, 1840, 748396 people deposited 22425812*l.* in the English Savings' Banks. What was the average of each person? *Ans.* 29*l.* 19*s.* $3\frac{1}{2}$*d.*+420184 rem.

(30) The Royal Library at Paris contains 626000 vols. and 80,000 MSS. Supposing there are 3596 shelves, how many vols. are on each shelf? *Ans.* 196+1184 rem.

(31) There are 12000 vols. in the Public Library, Baltimore, United States; and there are 520000 vols. in the Royal Library at Munich; the New York Mercantile Library contains 11400 vols.; and the St. Petersburgh Imperial Library contains 432000 vols. How many libraries, containing the same number of vols. as Baltimore and New York Mercantile Library could be formed out of the two European libraries mentioned above?
Ans. 40 libraries and 16,000 vols.

(32) The population of Virginia is 1239797, and the square acres 64000. How many people are there to a square acre?
Ans. 19 + 23797 rem.

(33) The returns handed in of slaves in America in 1840 were 2369953, and the population of the United States in 1841 was about 17000000. How many whites are there to a slave?
Ans. 7+410329 rem.

(34) There are 2000000 of acres in the United States planted with cotton, and the permanent capital invested in its cultivation is estimated at 740000000 dollars. What is that per acre? *Ans.* 370.

(35) England imported, in 1838, 1124180 bales of cotton from America. A bale weighs 400 lbs. How many half-pounds were imported? *Ans.* 899344000.

(36) In 1839, Great Britain manufactured 1512000 tons of iron. Suppose this was to be turned into bars, and each ton to

produce 1 mile, how many furlongs would the iron produce, and how many times would it reach round the globe: the circumference of the earth being 25020 miles?

Ans. 12096000 fur. 60 times round the earth; 1080 rem.

(37) The coal-trade of Great Britain is said to produce 21000000 tons, and the colliers employed 150000. What does each man produce? *Ans*. 140.

(38) The Mining Journal gives for the production of gold for 40 years, viz., from 1790 to 1830, Mexico, 286006569 in American dollars-worth. If each dollar be worth 4*s*. 6*d*. what amount of gold was produced from the mines of Mexico?

Ans. 64351478*l*. 0*s*. 6*d*.

REDUCTION

Is a reducing of quantities from a high denomination to a *lower;* or it is a converting of quantities of a low denomination into a higher. The former process is called *reduction descending*, and the latter *reduction ascending*. The method of operation is by the two following rules:—

Case 1.—To reduce quantities of a high denomination to a lower.

Rule.—Consider how many of the next less denomination make one of the quantities you are going to reduce, and multiply by it; adding any part that belongs to the term or terms to be reduced of the same name as your multiplier. Again, consider how many of the next denomination to the reduced one make one of the reduced quantities, and multiply by it; adding in as before. Proceed in this manner till you have brought your highest denomination to the one required.

N. B.—This rule may be perhaps more explicitly enunciated, Thus, consider what makes the thing to be reduced, and multiply by it.

Case 2.—To convert quantities of a low denomination to quantities of a higher.

Rule.—Consider how many of the *things to be converted* make the next higher denomination, and divide by the number, and so proceed after every division until you have obtained the name required.

Ex. *Case* 1.—Reduce 26*l*. 14*s*. 7½*d*. to farthings.

£. s. d.
26 14 7½
20
534
12
6415
4
25662

Here, as we have to reduce a *high* denomination to a *lower*, we perceive that the question belongs to Case 1. We therefore first consider what makes a pound, and from what we have before learned, find it is 20 shillings; we then multiply by 20, adding in the 14 shillings, because it is of the same name as the multiplier; we thence obtain 534 shillings. We then consider what makes a shilling, and we find 12 pence; we therefore multiply by 12, and add in the 7*d.* of the given quantity, for the reason before given; we thence obtain 6415 pence. We then consider what makes a penny; and, as before learned, we find 4 farthings; we therefore multiply by 4, and add in the ½*d.*, of the given quantity making 25662 farthings.

Example to Case 2.—Convert 396824 farthings into pounds.

4)396824
12)99206
20)8267+2
£413 7 2

We see that farthings are the given term; we therefore consider what is the next greater denomination to farthings, and we find pence is the next, we therefore divide by as many farthings as make a penny; which gives us a quotient of pence. We then consider what is the next greater denomination to pence, and find shillings; we therefore consider how many pence make a shilling, and divide by the number, which gives us a quotient of shillings. We then consider what is the next denomination to shillings, and find it is pounds; we therefore divide by as many shillings as make a pound, and thus we get a quotient of pounds, which is what the question required.

MISCELLANEOUS QUESTIONS.

(1) A spider spins a web of the following dimensions: there are 84 concentric rings; the outer 6 are ½ of the length of the next inner 12, and these 12 are ¼ the length of the next inner 48, which are 5 times as long as the remaining 16; whose circumferences are equal to 4 yards; all the radii from its centre measure 8 yards in length. Required the length of all the web?

Ans. 39½ yards.

(2) 75 men have to divide 48 ozs. 10 dwts. 14 grs. equally amongst themselves; how much had each?

Ans. 12 dwts. 22 grs.+44.

(3) A ship's crew of 280 men and 60 boys took a vessel, which, with its cargo, amounted to the value of 7969*l.* 14*s.* 8*d.* Every boy was reckoned equal to half a man. How much had

each man, and each boy? *Ans*. Each man had 25*l*. 14*s*. 2+$\frac{144}{310}$*d*. each boy 12*l*. 17*s*. 1+$\frac{72}{310}$*d*.

(4) 4 fields are to be divided amongst 157 labourers; the 1st consists of 47 acres, 3 roods, and 2 poles; the 2nd of 14 acres, 1 rood, and 36 poles; the 3rd of 16 acres, and 17 poles; the 4th the same size as the 2nd. How much had each labourer?
Ans. 2 roods, 15 poles, 18$\frac{78}{157}$ yards.

(5) 76 silk-weavers, each weave 26 yards of silk in 10 days; and 45 inferior workmen make 18 yards in the same time; supposing all the men had worked 95 days, how much would they all have made; and how much, on an average, would each man have made during that time? *Ans*. 26467 yards altogether; and, on an average, 218 yards, 2$\frac{114}{121}$qrs.

(6) 45 coal-whippers bring up 4963 tons of coals in 3 days; and 69 boys bring up 4196 tons in the same time; now, supposing that three boys are equal to a man, how many tons were brought up daily by all of them, and what weight by each man and each boy? *Ans*. 3053 tons daily: 44 tons, 17$\frac{63}{68}$ cwt. by each man, and 14 tons, 19$\frac{31}{68}$ cwt. by each boy.

(7) There are two fly-wheels in motion; one turns round 584700 times in 4 hours, and the other 5986742 times in three hours; required how many times each would turn round in 12 minutes; also the difference in the number of turns between the 2 wheels during this time? *Ans*. The 1st, 29735; the 2nd, 399116$\frac{2}{15}$, and the difference 369381$\frac{2}{15}$.

(8) There are 196649494 square miles on the surface of the earth; how many times larger is the earth's superficies than England, which contains 57960 square miles?
Ans. 3392+rem. 49174.

(9) Suppose sound to travel from the earth to the sun at the rate of 1142 feet in a second; how long would it be reaching the planet Mercury, supposing the latter to be in a right line with the earth and sun? *Ans*. 170211669 sec.+882 rem.

(10) How many times larger is the earth than the moon, the diameter of the former being 7964 miles, and that of the latter 2142 miles? The rule for finding it being expressed in the tables.
Ans. 51+3898845656 rem.

(11) There are 19117440 acres in Scotland; how many parcels of land, each containing 157 acres?
Ans. 121767+21rem.

(12) There are 19441920 acres in Ireland; how many estates, each 253 acres, could be formed out of them?
Ans. 76845, rem. 135.

(13) The planet Jupiter is 494499107 miles from the sun;

how many semi-diameters of the orbit of Venus (distant 68791752 miles) would it require to measure Jupiter's distance?
Ans. 14+12956843 rem.

(14) The circumference of the orbit of Saturn is 5699408962 miles; and the circumference of the earth's orbit is 596902654 miles; how many times less is the earth's orbit than that of Saturn? *Ans.* 9+327285076 rem.

(15) Venus revolves round the sun in 224d. 16h. 49m. 10sec., and the year of Saturn is 29 years, 174 days, 1 hour, 51 minutes, 11 seconds; how many revolutions in its orbit will Venus make whilst Saturn is going once round its orbit, supposing one of our years to be 365¼ days long? *Ans.* 47+17745621.

(16) Mercury moves at the rate of 109561 miles per hour in its orbit, and the earth at the rate of 68092 miles per hour; required the difference in miles they will have gone in 33 days of 24 hours long? *Ans.* 32843448.

(17) If a sidereal year is 365 d. 6 h. 9 min. 12 sec.; how many periods of time, each 12 h. 16 min. are there in one of these years?
Ans. 714+27912 rem.

(18) 59 men discovered, in various parts of a common, the following articles: 14 brazen helmets, valued at 34*l.* 19*s.* 6*d.*; 257 swords, valued at 12*l.* 18*s.* 4*d.*; 16 silver spurs, valued at 13*l.* 5*s.* 9*d.* Required each man's share?
Ans. 1*l.* 0*s.* 8¼*d.*+27 rem.

(19) 55 oxen, each weighing 1372 lbs.; hides, each, 29 lbs.; offal, 173 lbs. each, are to be divided amongst 64 men. How much meat will each man have? *Ans.* 1005+30 rem.

REDUCTION OF MONEY.

(1) In 28*l.*, how many farthings? *Ans.* 26880 farthings.

(2) Find how many halfpence there are in 36 guineas?
Ans. 18144 halfpence.

(3) In 496 guineas, how many crowns and farthings?
Ans. 2083 crowns, 1*s.*, 499968 farthings.

(4) In 79684 moidores, how many farthings?
Ans. 103270464 farthings.

(5) In 8692 moidores, how many half-guineas? *Ans.* 22350+6.

(6) In 5376 sovereigns, how many farthings and half-crowns?
Ans. 560960 farthings, 43008 half-crowns.

(7) In 94379 half-crowns, how many pieces, each of 3¾*d.*?
Ans. 755032.

(8) In 2173859 farthings, how many eighteen-penny pieces?
Ans. 30192+8¾*d.*

(9) In 73562 half moidores, how many half-pence?
Ans. 23834088 half-pence.

(10) In 9219 guineas, how many groats and half-crowns ? *Ans*. 77439+18.

(11) In 36247964 groats, how many pence, shillings, crowns, and guineas ? *Ans*. 144991856 farthings, 12082654 shillings, 2416530 crowns, 575364+10 guineas.

(12) In 279 moidores, how many angels ? *Ans*. 753+3*s*.

(13) In 78 guineas, how many two-penny pieces ? *Ans*. 9828.

(14) In 84 boxes, each containing 5 drawers, each drawer 4 purses, and in each purse a moidore, an angel, a guinea, and 8½*d*., how many four-penny pieces ? *Ans*. 399840.

(15) In 79 half-moidores, how many quarter farthings ? *Ans*. 2047682.

(16) In 68 drawers, each containing 15 purses, each containing a guinea, a sovereign, 3 crowns, and 8 pence, how many pence ? *Ans*. 693600.

(17) In 79 purses, each containing 58 packets, each containing 48 guineas, 57 half-sovereigns, and 98 crowns, how many two-pences? *Ans*. 56853456 two-pences.

(18) Reduce 8296 French crowns, each 5*s*. 4¼*d*., to farthings ? *Ans*. 2132072 farthings.

(19) In 95284 farthings, how many pieces, each containing 2½*d*.? *Ans*. 9528+1.

(20) Divide 47½ guineas amongst 27½ men, and find how many farthings each half man will have ? *Ans*. 870 farthings + 30 rem.

(21) In 10*l*. 18*s*. 9¼*d*., how many farthings and two-penny pieces ? *Ans*. 10501 farthings, 1312+5 two-penny pieces.

(22) In 1009 half-guineas, as many sovereigns, shillings, and sixpences, how many eighteen-penny pieces ? *Ans*. 21525+6*d*.

(23) Find what each man's share will be, if 59234 guineas, 40 of which are counterfeits, and 3955 only worth 19*s*. 3½*d*. each, and the remainder of full value, are divided amongst 733 men ? *Ans*. 89*l*. 19*s*. 11½*d*.+148.

(24) Find how many farthings in 274 moidores. *Ans*. 355104 farthings.

(25) In 6249*l*., how many crowns and half-crowns ? *Ans*. 24996 crowns, and 49992 half-crowns.

(26) In 48 guineas, how many pence ? *Ans*. 12096.

(27) In 732 moidores, how many four-penny pieces ? *Ans*. 59292 four-penny pieces.

(28) In 396 angels, how many farthings ? *Ans*. 190080.

(29) In 2874 half-guineas, how many half-pence ? *Ans*. 724248 half-pence.

(30) In 382974 farthings, how many pounds and guineas?
Ans. 388*l*. 18*s*. 7½*d*., and 379 guineas + 19*s*. 7½*d*. rem.

(31) How many three-pences are there in 549 Spanish doubloons, each 48*s*.? *Ans*. 105408.

(32) In 49 crowns, how many two-penny pieces? *Ans*. 1470.

(33) In 5986*l*. 14*s*., how many half-pence and guineas?
Ans. 2873616 half-pence, 5701 guineas, 13*s*. over.

(34) How many half-sovereigns are there in 596 moidores, 48 guineas, and 496 crowns? *Ans*. 1958.

(35) In 791494 farthings, how many moidores?
Ans. 610 moidores + 19*s*. 5½*d*.

(36) In 396,874 farthings, how many half-crowns and nobles?
Ans. 3307 + 8½ half-crowns, 1240 + 58 nobles.

(37) In 296*l*. 18*s*. 6½*d*., how many half-farthings?
Ans. 5701000 half-farthings.

(38) In 396 guineas, how many angels and nobles?
Ans. 8316 shillings, 1247 + 32*d*. nobles, 831 angels + 6*s*.

(39) In 284796 crowns, how many guineas and marks?
Ans. 67808 guineas + 12*s*., 213597 nobles.

(40) In 3172 moidores, how many half-guineas and crowns?
Ans. 8156 + 6*s*., half-guineas; 17128 + 4*s*. crowns.

TROY WEIGHT.

(1) In 48 lbs., how many half ounces?
Ans. 5760 half ounces.

(2) In 3968 dwts., how many lbs. and grs.?
Ans. 16 lbs., 6 ozs., 8 dwts., and 1904640 grs.

(3) In 496994 grs., how many half lbs.?
Ans. 172 half lbs. + 3 8 2.

(4) In 7984½ grains, how many quarter lbs.?
Ans. 4 quarter lbs., 2 dwts. + 16½ grs.

(5) In 72989 grains, how many half lbs.?
Ans. 25 half lbs + 2 1 5 rem.

(6) In 39684 pounds, how many half grs. and dwts.?
Ans. 457159680 half grs., 9524160 dwts.

(7) In 73 silver chains, each weighing 3 ozs. 10 dwts. 9 grs., how many grs.? *Ans*. 123297 grs.

(8) In a silver mug, weight 2 lbs. 5 ozs. 18 dwts., how many spoons, each weighing 2 ozs. 8 dwts. 5 grs.?
Ans 12 + 486 grs. rem.

(9) In 642 lbs., 11 ozs., 19 dwts., how many dwts and grs.?
Ans. 154319 dwts., 3703656 grs.

(10) In 7294382 grs., how many parcels are there of 4 lbs., 10 ozs., 2 dwts. each? *Ans*. 261 + 15814 grs.

(11) In 32 lbs. 4 ozs. 11 dwts., 13 grs., how many ozs. dwts. and grs.? *Ans.* 388 oz., 7771 dwts., 186517 grs.

(12) In 8 packages, each 2 lbs. 9 oz. 17 dwts. 14 grs., how many ozs. dwts. and grs.? *Ans.* 271 oz., 5420 dwts., 130096 grs.

(13) In 24986 grs., how many spoons, each 2½ oz.? *Ans.* 20+986 grs.

(14) In 84 table spoons, each weighing 4½ oz., 59 tea spoons, each 2 oz. 19 dwts., how many salt spoons can be made, each weighing 1 oz. 17 dwts.? *Ans.* 238 spoons, 15 dwts. over.

(15) A gentleman has 3 silver salvers, weighing 6 lbs. 10 oz. 9 dwts. 4 grs., and wants to have them made into tea spoons, of 2 oz. 3 dwts. each, and salt spoons, each 1 oz. 11 dwts.; how many will there be, and of each an equal number.? *Ans.* $56\frac{7}{148}$.

(16) In 64 ozs., how many grs.? *Ans.* 30720 grs.

(17) In 8294 lbs., how many half grs. and ozs.? *Ans.* 95546880 grs., 99528 oz.

(18) In 37 lbs. 2 ozs. 15 dwts., how many grs. and dwts.? *Ans.* 214440 grs., 8935 dwts.

(19) In 82 lbs. 11 oz. 19 dwts. 12 grs., how many grs. ozs. and dwts.? *Ans.* 478068 grs., 19919 dwts.

(20) In 24835 grs., how many quarters of lbs.? *Ans.* 17 qrs., 13 dwts., 23 grs.

(21) In 32 lbs., how many quarter grs.? *Ans.* 737280 quarter grs.

(22) How many grs. in 14 lbs. 10 ozs. 15 dwts. 14 grs.? *Ans.* 85814 grs.

(23) In 732984 half grs., how many quarters of lbs.? *Ans.* 19 qr. lbs. + 7 dwts. 12 grs.

(24) In 729 lbs. 11 oz., how many ingots, each weighing 7 lbs. 9 oz. 3 dwts. 11 grs.? *Ans.* 94 ingots + 358 grs.

(25) In 29684 grs., how many half lbs. and quarter dwts.? *Ans.* 30 half lbs. + 16 and 4947 qr. dwts. + 2.

(26) In 427968 grs., how many lbs.? *Ans.* 74 lbs., 3 oz., 12 dwts.

(27) In 249684 quarter grs., how many half lbs.? *Ans.* 21 half pounds + 4.

(28) In 82967 dwts., how many lbs. and grs.? *Ans.* 2991208 grs., 345 lbs., 8 ozs., 7 dwts.

(29) In 96984 grs., how many dwts. and lbs.? *Ans.* 16 lbs., 10 ozs., 1 dwt.

(30) Reduce 7968 lbs. to ozs. and half dwts.? *Ans.* 95616 ozs., 3824640 half dwts.

(31) In 48¼ dwts., how many grs.? *Ans.* 1158 grs.

(32) Reduce 46 lbs. to ozs. and half grs.? *Ans.* 552 ozs., 529920 half grs.

(33) In 48 lbs., how many ozs. and grs.?
Ans. 576 ozs., 276480 grs.

(34) In 67½ lbs., how many ozs. and grs.?
Ans. 810o zs., 388800 grs.

(35) In 489¾ lbs., how many ozs. dwts. and grs.?
Ans. 3877 ozs., 117540 dwts., 2820960 grs.

(36) Reduce 84 lbs. to half grs. *Ans.* 967680 half grs.

(37) Reduce 93¾ lbs. to ozs. dwts. and quarter grs.
Ans. 1125 oz., 22500 dwts., 2160000 qr. grs.

(38) In 84 lbs. 10 oz. 11 dwts. 4 grs., how many parcels, each containing 15 grs.? *Ans.* 32593+13.

Avoirdupoise Weight.

(1) 791549 quarter ozs., how many half cwts.?
Ans. 220 half cwts., 47 lbs.+15¼ ozs. rem.

(2) In 3892 lbs. 1968 quarters of cwt. 36 cwt. 3 qrs. 27½ lbs.; how many parcels, supposing 2 divisions, the first containing 5½ lbs. in each parcel, the second 6¾ lbs., and of each an equal number? *Ans.* 5154 parcels+3 lbs.

(3) Of 294 cwt. of tobacco, how many cigars can be made, each half an oz.? *Ans.* 1053696.

(4) Reduce 14 cwt. to lbs. and ozs. *Ans.* 1568 lbs. 25088 ozs.

(5) Reduce 95 tons to half lbs. *Ans.* 425600.

(6) In 796 cwt., how many lbs. and half ozs.?
Ans. 89152 lbs., 2852864 half ozs.

(7) In 298641 ozs., how many lbs. and cwts.?
Ans. 1865 lbs.+1 oz., or 16 cwt., 2 qrs., 17 lbs., 1 dr.

(8) Find how many qrs. lbs. and drs., there are in 96 cwt. 2 qrs. 18 lbs.?
Ans. 386 qrs., 10826 lbs., 173216 ozs., 2771456 drs.

(9) In 48 tons, how many qrs. lbs. and half ozs.?
Ans. 3840 qrs., 107520 lbs., 3440640 ozs.

(10) How many parcels, each weighing 16¼ lbs., are there in cwt.? *Ans.* 48 parcels+16.

(11) In 749889 drs., how many stones and cwts.?
Ans. 209 st., 3 lbs., 4 oz., 1 dr., 26 cwt., 1 qr.

(12) In 26796 stones, how many lbs. and tons?
Ans. 375144 lbs., 167 tons, 9 cwts., 2 qrs.

(13) In 95867 half drs., how many cwts.?
Ans. 1 cwt., 2 qrs., 19 lbs., 3 ozs., 13½ drs.

(14) In 567456 qrs., how many half stones and half tons?
Ans. 1134912 half stones, 7093 2.

(15) In 48 cwt. 2 qrs. 22 lbs., how many lbs. and half ozs.?
Ans. 5454 lbs., 174528 half ozs.

(16) Reduce 45 tons to ozs. and half drs.
Ans. 1612800 ozs., 51609600 drs.

(17) In 895 cwt. 2 qrs. 18 lbs., how many lbs. and half ozs.?
Ans. 100314 lbs., 3210048 half ozs.

(18) In 4296 stones, how many half cwts. and quarter tons?
Ans. 1074 h. cwt., 107+4 qr. tons.

(19) In 796 lbs. 11 ozs. 2 dwts., how many drs.? *Ans.* 204112

(20) In 439 cwts., how many parcels, each 1¾ lbs.?
Ans. 14047+3 rem.

(21) Reduce 296 tons to lbs. and quarter ozs.
Ans. 663040 lbs., 42434560 qr. ozs.

(22) Find how many cwts. in 49082947 drs.
Ans. 1732 cwt., 3 qrs., 6 lbs., 0 ozs., 4 drs.

(23) In 17949 drs. how many half lbs. and quarters? *Ans.*

(24) How many parcels, each of 3 lbs. 10 oz. 6 drs., are there in 32 cwt. 3 qrs. 19 lbs.? *Ans.* 1010 parcels+532 drs.

(25) In 569874 half lbs. how many quarter oz. and half drs.: also cwts.?
Ans. 18235968 qr. oz., 9117984 h. drs., 2544 cwt.+9.

(26) In 382 tons how many oz. and half quarters of cwts.?
Ans. 13690880 oz., 61120 half qrs. of cwts.

(27) In 289698 drs. how many half lbs. and quarters of cwts.? *Ans.* 2263 h. lbs., 40 qr. cwts.+11 rem.

(28) How many parcels are there of 3 lbs. 11 oz. each in 796 cwt. 2 qrs.? *Ans.* 24192.

(29) Reduce 49684 qrs. to lbs. oz. and tons.
Ans. 22258432 oz.

(30) In 49 cwt. 3 qrs. 19 lbs. 7 oz., how many qrs. lbs. oz. and drs.? *Ans* 5591 lbs., 89463 oz.

(31) In 8 packages, each 4 cwt. 2 qrs. 17 lbs., how many qrs. lbs. oz. and drs.? *Ans.* 148 qrs.

(32) In 9329684 drs., how many qrs. and cwts.?
Ans. 1301 qrs. 16 lbs. 1 oz. 4 drs., 325 cwts.

(33) In 382 tons, how many half oz.? *Ans.* 27389760 h. oz.

(34) In 84 tons, how many parcels, each containing 13 lbs. 9 oz.? *Ans.* 13873+119.

(35) In 32 cwt., how many half oz. and quarter stones?
Ans. 1024 qr. stones, 114688 h. oz.

(36) In 73 cwt. 2 qrs. 19 lbs., how many packages, each containing 28 lbs. 2 oz.? *Ans.* 293 parcels+166.

(37) In 396 cwt., how many lbs. oz. and drs.?
Ans. 44352 lbs., 709632 oz., 11354112 drs.

(38) In 249849 drams, how many cwts.?
Ans. 8 cwt. 2 qrs. 23 lbs. 15 oz. 9 drs.

(39) In 3769847 drams, how many tons?
Ans. 6 tons, 11 cwt., 1 qr., 25 lbs., 15 oz., 7 drs.

(40) Reduce 749 tons, 13 cwt., to qrs. lbs. and oz.
Ans. 59972 qrs., 1679216 lbs., 26867456 oz.

(41) In 3865692 tons, how many half ounces?
Ans. 277092802560 half oz.

(42) In 82968 hhds., each weighing 15 cwt., how many lbs.?
Ans. 139386240 lbs.

(43) In 149 cwt. 3 qrs. 16 lbs., how many half ounces?
Ans. 537216.

(44) How many lbs. are there in 8964 cwt. 2 qrs. 19 lbs.?
Ans. 1004043.

(45) How many parcels, each weighing 34 lbs. 12 oz., are there in 567 cwt. 0 qrs. 14 lbs.? *Ans.* 1827+476.

(46) In 68296 tons, how many half lbs. and stones?
Ans. 305966080 half lbs., 10927360 stones.

(47) In 682 cwt., how many parcels of 16 lbs. 14 oz. Avoirdupois? *Ans.* 4526 parcels (7 lbs. 12 oz. over.)

(48) In 7968 barrels of fish, each weighing 7½ cwt., how many parcels, each weighing 20½ lbs.?
Ans. 234846+18 remainder.

(49) In 175964 hhds. of tobacco, each weighing 14 cwt., how many ounces and parcels containing 74 lbs. 6 oz.?
Ans. 3709735+182.

(50) In 76 cwt., how many parcels, each weighing 2 lbs. 11 oz.? *Ans.* 3167 parcels+11 remainder.

(51) In 4938 tons, how many quarters of stones and half cwts.? *Ans.* 3160320 qr. stones, 197520 h. cwt.

APOTHECARIES WEIGHT.

(1) How many lbs. in 4909689 grs.? *Ans.* 852 . 2 . 4.

(2) In 7349 lbs., how many scruples and half grains?
Ans. 2116512 scr., 84660480 h. grs.

(3) Find how many ½ oz. and grs. in 49657 lbs.?
Ans. 1191768 h. oz., 286024320 grs.

(4) In 2986 lbs. 10 oz. 6 dr. 3 scr. 15 gr., how many quarter grains and pills, each 3½ grs.? *Ans.* 4915598+4 pills.

(5) Reduce 4 lbs. 3 oz. 2 dr. and 2 scr. to grains and parcels, each containing 5 scr. 2 gr.? *Ans.* 24640 gr., 241 par.+58.

(6) In 4587 dozens of lbs., how many drams and grains?
Ans. 5284224 drs., 317053440 grs.

(7) In 694 lbs., how many drams, scruples, and grains?
Ans. 199872 scr., 3997440 grs.

(8) In 4098 half lbs., how many scruples and grains?
Ans. 590112 scr., 11802280 gr.

(9) In 73295 scr., how many lbs.?
Ans. 254 lbs. 5 oz. 7 dr. 2 scr.

(10) In 6395 scr., how many lbs.?
Ans. 22 lbs. 2 oz. 3 drs. 2 scr.

(11) In 99845 lbs., how many drams and grains?
Ans. 9585120 drs., 575107200 grs.

(12) In 43829 qu. grs., how many scruples and lbs.?
Ans. 1 lb. 10 oz. 6 drs. 1 scr. 17 grs. 1 qu. gr.

(13) In 78495 lbs., how many pills, each 3½ grs.?
Ans. 129180342 + 6.

(14) In 249 lbs. 3 oz. 7 drs. 4 grs., how many drams and grains?
Ans. 23935 drs., 1436104 grs.

(15) In 48 lbs., how many scruples and grains?
Ans. 13824 scr., 276480 grs.

(16) Reduce 3 lbs. 7 oz. 2 drs. 1 scr. to grains.
Ans. 1339 scr., 26780 grs.

(17) Reduce 49689 oz. to scruples and half grains.
Ans. 1192538 scr., 47701440 h. grs.

(18) In 249 lbs., how many drams and half scruples?
Ans. 23904 drs., 143424 h. scr.

(19) In 74 oz., how many pills of 4 grs. each?
Ans. 8880 pills.

(20) Find how many half ounces there are in 379 lbs.
Ans. 7098 h. oz.

(21) How many quarter drams and scruples in 69 lbs. 4 oz.?
Ans. 6656 qu. drs., 79872 qu. scr.

(22) In 27498 scr., how many drams, ounces, and lbs.?
Ans. 9166 drs., 1145 oz., 95 lbs. 6 oz.

(23) In 346825 grs., how many ounces and lbs.?
Ans. 60 lbs. 0 oz. 4 drs. 1 scr. 5 grs.

(24) Reduce 24689 lbs. to grains, and parcels each containing 3 scr. 5 grs.
Ans. 142208640 grs., 2187825 + 15 parcels.

(25) In 24 lbs., how many ½ oz. and ½ grs.?
Ans. 576 h. oz., 276480 h. grs.

(26) How many boxes of pills can be made from 48 lbs. of calomel, each pill containing 4 grs., and each box 3 dozen pills?
Ans. 69120 pills, 1920 boxes.

(27) In 496892 h. grs., how many ½ lbs.? *Ans.* 43 . 1 . 4 . 0 . 6.

(28) In 294 lbs., how many quarter scruples?
Ans. 338680 qr. scr.

(29) In 429 lbs., how many ounces and grains?
Ans. 41184 oz., 2471040 grs.

(30) In 34 lbs. 7 oz. 1 scr. 3 grs., how many parcels, each containing 4 scr. 13 grs.? *Ans.* 2146+62.

(31) In 2449 oz., how many lbs. and grains?
Ans. 1175520 grs., 204 lbs.+1

(32) In 8 parcels, each 8 lbs. 1 oz. 3 drs. 1 scr. 19 grs., how many ounces, drams, and grains?
Ans. 779 oz., 6237 drs., 18711 scr., 374232 grs.

(33) In 79 lbs. 3 oz. 1 dr. 1 scr. 19 gr., how many grains?
Ans. 456579 gr.

(34) In 796849 grs., how many half scruples and lbs.?
Ans. 79684 h. scr., 138 lbs.

(35) In 82914 oz., how many lbs. and grains?
Ans. 6909+6 lbs., 39798720 grs.

(36) How many drams and grains in 829 lbs.?
Ans. 79584 drs., 4775040 grs.

(37) In 38297 scr., how many grains and lbs.?
Ans. 765940 grs., 132 lbs. 11 . 5 . 2.

(38) How many scruples and half grains in 9674 lbs.?
Ans. 2786112 scr., 111444480 h. grs.

Cloth Measure.

(1) In 7298 yards, how many French ells and nails?
Ans. 116768 nails, 5838+2 French ells.

(2) In 382 English ells, how many half nails? *Ans.* 15280.

(3) In 6954 French ells, how many quarter inches?
Ans. 1502064.

(4) How many half yards and half inches are there in 7849 Flemish ells? *Ans.* 11773 half yards, 423846 half inches.

(5) In 793824 inches, how many yards? *Ans.* 22050+24.

(6) In 37284 Flemish ells, how many English ells?
Ans. 22370+2.

(7) How many yards are there in 799842 nails?
Ans. 49990.

(8) How many English ells are there in 78495 Flemish ells?
Ans. 47097.

(9) How many French ells are there in 39694 English ells?
Ans. 66156+2.

(10) How many Flemish ells are there in 29456 English ells?
Ans. 49093+1.

(11) How many yards are there in 72957 Flemish ells?
Ans. 54717+3.

(12) How many nails are there in 579826 English ells?
Ans. 11596520.

(13) How many quarters and yards are there in 74968 French ells? *Ans.* 112452.

(14) How many Flemish ells are there in 381762 yards?
Ans. 508989+1.

(15) In 48 yards, how many nails? *Ans.* 768.

(16) In 6724 yards, how many nails and inches?
Ans. 107584 nails, 242064 inches.

(17) In 7596 English ells, how many quarters and nails?
Ans. 379809 quarters, 151920 nails.

(18) In 5796 French ells, how many quarters and nails?
Ans. 347769 quarters, 139104 nails.

(19) Find how many yards in 493,298 quarters.
Ans. 123324+2 yards.

(20) How many English ells in 7958¼ yards?
Ans. 6366+⅗.

(21) In 795968 French ells, how many Flemish?
Ans. 17591936.

(22) In 428475 English ells, how many French ells?
Ans. 357062+3.

(23) How many yards in 594372 Flemish ells?
Ans. 445777.

(24) In 79483 French ells, how many half Flemish ells?
Ans. 317932.

(25) In 382475 English ells, how many French ells?
Ans. 318729+1.

(26) How many quarter English ells are there in 79243 French ells? *Ans.* 380366+4½ inches.

(27) How many half quarters are there in 79684 Flemish ells?
Ans. 1912416.

(28) In 7924 yards, how many nails and inches?
Ans. 116704 nails, 262584 inches.

(29) In 7498 French ells, how many nails and half inches?
Ans. 179952 nails, 809784 half inches.

(30) In 4521 English ells, how many half nails?
Ans. 180840.

(31) In 682497 quarters, how many English ells, French ells, and Flemish ells? *Ans.* 136499+2 English ells, 113749½ French ells, 227499 Flemish ells.

(32) In 7151 Flemish ells, how many nails and inches?
Ans. 85815 nails, 193077 inches.

(33) In 2095 yards, how many Flemish and English ells?
Ans. 2793 Flemish ells, 1674 English ells.

(34) In 42157 Flemish ells, how many half yards and inches?
Ans. 6323 half yards, 1136299 inches.

(35) In 14725 quarters, how many English and French ells?
Ans. 2945 English ells, 2454+1 French ells.

(36) In 2917417 half inches, how many yards and French ells? *Ans.* 40519 . 2 . 0 . 1 yards, 32415+3 French ells.

(37) In 6259 Flemish ells, how many quarter inches and quarter yards? *Ans.* 18777 quarter yards, 675972 quarter inches.

LONG MEASURE.

(1) How many German miles, each 5866 English yards, in 95891672 English yards? *Ans.* 16347 miles + 170 yards.

(2) How many Swedish miles, each 7233 English yards, in 29673916 English yards? *Ans.* 4102+4550.

(3) How many Italian miles, each 1467 English yards, in 65829671 English yards? *Ans.* 44873 miles +980 yards.

(4) How many Spanish miles, each 5028 English yards, in 43829679 English yards? *Ans.* 8717 miles + 603.

(5) How many Danish miles, each 7233 English yards, in 54986925 English yards? *Ans.* 7602 miles + 1659 yards.

(6) How many Hungarian miles, each 8800 English yards, in 82967984 English yards? *Ans.* 9428 miles + 1584.

(7) In 729 leagues, how many half furlongs and yards?
Ans. 34092 half furlongs, 3849120 yards.

(8) In 62984 miles, how many degrees (each 69½ miles) and yards? *Ans.* 906+34 degrees, 110851840 yards.

(9) In 72 degrees, how many half inches and barleycorns?
Ans. 634106880 half inches, 951160320 barleycorns.

(10) In 72 miles, how many half yards?
Ans. 253440 half yards.

(11) In 398649 barleycorns, how many half miles?
Ans. 4 half miles, 171 yards, 7 inches.

(12) In 37692 furlongs, how many inches? *Ans.* 298520640.

(13) In 7896 miles, how many half barleycorns?
Ans. 3001743360.

(14) In 1738964 leagues, how many inches?
Ans. 330542277120.

(15) Paris is 982 miles from a certain place, how many inches to the same? *Ans.* 62219520.

(16) How many degrees are there in 782968214 barleycorns?
Ans. 68 degrees + 39 miles, 265 yards, 2 feet.

(17) In 44 leagues, how many yards and rods?
Ans. 232320 yards, 42240 rods.

(18) Reduce 67 miles to feet and half inches.
Ans. 350760 feet, 8418240 half inches.

(19) How many feet and barleycorns are there in 56 half miles?
Ans. 147840 feet, 5322240 barleycorns.

(20) In 78 half furlongs, how many half yards and quarter inches? *Ans.* 3120 half yards, 224640 quarter inches.

(21) In 396 degrees, how many miles and half yards?
Ans. 27522 miles, 26877440 half yards.

(22) Reduce 79 furlongs to measures, each of 7 barleycorns.
Ans. 268148+4.

(23) How many half rods are there in 6 furlongs?
Ans. 480 half rods.

(24) In 496 perches, how many half inches?
Ans. 49104 half inches.

(25) In 629 leagues, how many quarter yards?
Ans. 13284480 quarter yards.

(26) Reduce 48 miles to half rods and inches?
Ans. 30720 half rods, 3041280 inches.

(27) In 82½ furlongs, how many perches and quarter yards?
Ans. 3300 perches, 72600 quarter yards.

(28) In 796 degrees, how many yards and half perches?
Ans. 86057600 yards, 30566400 half perches.

(29) How many Polish miles, each 4400 English yards, are there in 7968492 English yards? *Ans.* 1811 miles + 92.

(30) How many Irish miles, each 2200 English yards, in 32159879? *Ans.* 14618+278.

(31) In 9496882471 barleycorns, how many miles?
Ans. 2614 miles, 0 fur. 216 yds. 0 ft. 7 in. 2 bc.

(32) In 59864 degrees, how many barleycorns?
Ans. 17068423680.

(33) How many perches are there in 27649 miles?
Ans. 8847680.

(34) In 86 leagues, how many half yards? *Ans.* 908160.

(35) In 7 leagues, how many yards and inches?
Ans. 1330560.

(36) In 14 miles, how many half inches? *Ans.* 1774080.

ALE AND BEER MEASURE.

(1) How many firkins and quarts in 793892 gallons?
Ans. 882102 firkins, 3175568 quarts.

(2) In 9847 bottles, each containing 1¾ of a pint, how many hogsheads? *Ans.* 39 hhds. + 48¼.

(3) In 6287396 half pints, how many hogsheads?
Ans. 72770 hhds.+42 gallons.

(4) In 48 barrels, how many pints? *Ans.* 13824 pints.

(5) In 876 hogsheads, how many kilderkins and pints?
Ans. 2628 kilderkins, 378,432 pints.

(6) In 3979 butts, how many gallons and half pints?
Ans. 419732 gallons, 6715712 half pints.

(7) In 982964 half pints, how many hogsheads?
Ans. 11377 hhds. + 4 gallons.

(8) In 3649 butts of beer, how many gallons and pints?
Ans. 394092 gallons, 3152736 pints.

(9) In 3869 hogsheads of beer, how many half pints and kilderkins? *Ans.* 3342816 half pints, 11607 kilderkins.

(10) In 758 barrels of beer, how many firkins and half pints?
Ans. 3032 firkins, 436608 half pints.

(11) Reduce 4969 hogsheads to kilderkins and gallons.
Ans. 14907 kilderkins, 268326 gallons.

(12) In 294 barrels, how many half kilderkins and pints?
Ans. 1176 half kilderkins, 84672 pints.

(13) In a vat, containing 48 barrels, 72 kilderkins, and 3968 gallons, how many pints? *Ans.* 66304 pints.

(14) In 796824 gallons, how many barrels and butts?
Ans. 22134 barrels, 7391 . 1 butts.

(15) Reduce 496 butts to gallons and pints.
Ans. 53568 gallons, 428544 pints.

(16) Reduce 28496 hogsheads to firkins and quarts.
Ans. 170967 firkins, 6155136 quarts.

(17) In 79 butts of beer, how many gallons and half pints?
Ans. 8532 gallons, 68256 half pints.

(18) In 49 hogsheads, how many gallons and pints?
Ans. 2646 gallons, 21168 pints.

(19) In 673 kilderkins, how many gallons and quarts?
Ans. 12114 gallons, 96912 quarts.

(20) In 398249 gallons, how many butts and hogsheads?
Ans. 49160 butts, 73740 hogsheads.

(21) In 63947 firkins, how many gallons and pints?
Ans. 575523 gallons, 4604184 pints.

(22) In 7294 hogsheads, how many bottles, each 1½ pint?
Ans. 2100672 bottles.

(23) In 3742 butts, how many half gallons?
Ans. 808272 half gallons.

(24) In 5964 hogsheads, how many firkins and half pints?
Ans. 35784 firkins, 5152896 half pints.

(25) In 249 butts, how many kilderkins and quarts?
Ans. 1494 kilderkins, 107568 quarts.

(26) In 79684 gallons, how many firkins and kilderkins, and of each an equal number? *Ans.* 2951 each + 7 gallons.

(27) In 3749 kilderkins, how many hogsheads and half gallons? *Ans.* 1249 hhds + 2 kilderkins, 134964 half gallons.

(28) In 31769 hogsheads, how many pints? *Ans.* 13724208 pints.

(29) In 398495 barrels of beer, how many gallons and pints? *Ans.* 114766560 pints, 14345820 gallons.

(30) In 783294 barrels of beer, how many hogsheads and pints? *Ans.* 225588672 pints, 522172 + 1 hhd.

WINE MEASURE.

(1) In 829684 pipes, how many quarts? *Ans.* 418160736 quarts.

(2) In 62174 pipes, how many gills and gallons? *Ans.* 250685568 gills, 7833924 gallons.

(3) In 65 hogsheads, how many quarts? *Ans.* 16380 quarts.

(4) In 7968249 quarts, how many tuns? *Ans.* 7905 tuns + 2 gall. + 1 quart.

(5) In 78296 pipes, how many half pints? *Ans.* 157844736 half pints.

(6) In 38692 gallons of brandy, how many rundlets and pints? *Ans.* 2149 + 10 rundlets, 309536 pints.

(7) In 4279 tuns, how many quarts? *Ans.* 4313232.

(8) In 32 tuns, how many half gallons? *Ans.* 16128 half gallons.

(9) In 44 rundlets, how many pints? *Ans.* 6336.

(10) In 29 pipes, how many gallons and pints? *Ans.* 29232 pints, 3654 gallons.

(11) In 29684 gallons, how many half pints? *Ans.* 474944 half pints.

(12) In 37864 tuns of wine, how many half pints and rundlets? *Ans.* 152667648 half pints, 530096 rundlets.

(13) In 48 hogsheads, how many rundlets and gallons? *Ans.* 3024 gallons, 168 rundlets.

(14) In 3682914 hogsheads, how many tuns and gallons? *Ans.* 232023582 gallons, 920728 tuns, 2 hhds.

(15) In 82964 pipes, how many gallons and rundlets? *Ans.* 10453464 gallons, 580748 rundlets.

(16) In 294987 half pints, how many dozens of wine, supposing each bottle to contain 1¼ pint? *Ans.* 9832 dozen + 10 over.

(17) In 2968 rundlets, how many tuns, gallons, and hogsheads? *Ans.* 212 tuns, 53424 gallons, 848 hogsheads.

(18) In 3284 pints, how many gills and rundlets? *Ans.* 13136 gills, 22 r. + 14 galls. + 4 pints.

(19) In 493817 gallons, how many tuns? *Ans.* 1959 + 149.

(20) In 28 puncheons, how many tuns and gallons? *Ans.* 9 tuns + 1, and 2352 gallons.

(21) In 49329449 half pints, how many hogsheads and rundlets? *Ans.* 48937 hhds., 59 gallons, 4½ pints.

(22) In 294987 half pints, how many dozens of wine, supposing each bottle to contain 1¼ pint? *Ans.* 9832 dozen + 10

(23) In 2968 rundlets, how many tuns, gallons, and hogsheads? *Ans.* 212 tuns, 53424 gallons, 848 hogsheads.

(24) In 79649284 quarts, how many tuns and pipes? *Ans.* 158034 tuns + 74 gallons, 316068 pipes.

(25) In 3298496 pints, how many hogsheads? *Ans.* 6544 hhds. + 320 pints.

(26) In 965982 pipes of wine, how many half pints? *Ans.* 2947419712 half pints.

(27) In 829734 hogsheads, how many gallons and rundlets? *Ans.* 2903155 + 9.

(28) In 82916478 gallons, how many hogsheads, pipes, and tuns? *Ans.* 1316134 + 36 hhds., 658067 pipes, 329033 + 1 tuns.

(29) In 4498 tuns, how many hogsheads and rundlets? *Ans.* 17992 hhds., 62972 rundlets.

(30) In 738 puncheons, how many gallons and half pints? *Ans.* 61992 gallons, 991872 half pints.

(31) In 34984 gallons, how many hogsheads and pipes? *Ans.* 555 hhds. + 19 gallons, 277 pipes + 1 hhd.

(32) In 79316928 pints, how many tuns? *Ans.* 39343 tuns + 144.

DRY MEASURE.

(1) In 682947 pecks of corn, how many quarters? *Ans.* 21342 quarters, 0 bushels, 3 pecks.

(2) In 82457 chaldrons of coals, how many bushels and pecks? *Ans.* 2969100 bushels, 11876400 pecks.

(3) How many half pints are there in 59684974 weys? *Ans.* 305587066880 half pints

(4) In 72964 strikes, how many half pints? *Ans.* 18678784 half pints.

(5) In 7968 lasts, how many pints? *Ans.* 40796160.

(6) How many quarters of corn are there in 92178249 lasts, and how much would 29 men receive; the quantity to be equally divided amongst them? *Ans.* 31785603 + 3 rem.

(7) In 2968497 half pints, how many quarters and weys? *Ans.* 4547 . 4 . 0 . 3 . 1 . 1.

(8) In 82974 quarts, how many gallons and quarters?
Ans. 324 0 3b. 1 2q.

(9) In 248649 pecks, how many lasts and weys?
Ans. 777 lasts, 0 w. 2 q. 1 .

(10) In 38294 lasts, how many gallons and half pints?
Ans. 24508160 galls., 392130560 half pints.

(11) In 6824 quarters, how many pecks, lasts, and weys?
Ans. 315368 p., 682 l. + 128 p., 1364 weys + 4 qrs.

(12) In 42 lasts, how many quarts? *Ans*. 21504 quarts.

(13) In 82 weys, how many bushels and pecks?
Ans. 3280 bushels, 13120 pecks.

(14) In 4769 quarters, how many quarters and half pints?
Ans. 12208649 quarters, 4883456 half pints.

(15) In 63 bushels, 2 pecks, how many pints? *Ans*. 4064 pints.

(16) In 796842 pints, how many pecks, bushels, and lasts?
Ans. 49802 pecks + 10 pints, 12450 b., 2 pecks, 10 pints,
155 lasts, 5 bushels, 2 pecks, 10 pints.

(17) In 49379678 quarters of wheat, how many gallons?
Ans. 5160299392 gallons.

(18) In 6739 lasts of corn, how many pints?
Ans. 34503680 pints.

(19) In 37968 quarters of corn, how many half pints?
Ans. 38879232 half pints.

(20) In 7493947 pints, how many lasts?
Ans. 1463 l. 6 q. 4 bu. 3 p. 1 g. 1 p. 1 p.

(21) In 5939 lasts, how many pecks, gallons, and half pints?
Ans. 1900480 pecks, 3800960 gallons, 60975360 half pints.

(22) Reduce 2 weys, 4 quarters, 7 bushels, to quarts and gallons. *Ans*. 952 gallons, 3808 quarts.

(23) In 17829645782 bushels, how many half pecks?
Ans. 142637066256 gallons, 445741144 + 176 half pecks.

(24) How many pecks, bushels, and pints, are there in 7493 weys, 2 quarters, 1 bushel?
Ans. 299737 bushels, 1198948 pecks, 19183168 pints.

(25) How many quarter gallons are there in 75 lasts, 45 quarters, 18 bushels, and 1 peck? *Ans*. 2041042 gallons.

(26) In 7 lasts, how many gallons and half pints?
Ans. 4480 gallons, 71680 half pints.

(27) Reduce 498 quarters to pottles. *Ans*. 63744 pottles.

(28) In 48 weys, how many pecks and quarts?
Ans. 7680 pecks, 61440 quarts.

(29) How many lasts are there in 36578294 pints?
Ans. 71440 lasts.

(30) In 379498 pecks, how many bushels, quarters, and weys?
Ans. 94874 + 2 bushels, 11859 + 2 quarters, 2371 + 4 weys.

Time.

(1) In 79 years, how many days and half hours (365$\frac{1}{4}$ days to a year)? *Ans.* 28854$\frac{3}{4}$ days, 1385028 half hours.

(2) In 489680 hours, how many days of 12 hours each?
Ans. 40806 + 8.

(3) In 729 years, how many weeks, minutes, and seconds?
Ans. 37908 weeks, 3275251200 seconds,
54587520 minutes.

(4) In 56 years, how many months and half weeks?
Ans. 728 months, 5824 half weeks.

(5) Reduce 73 years to days, and quarters of hours, reckoning 365$\frac{1}{4}$ days to a year.
Ans. 26663$\frac{1}{4}$ days, 2559672 quarters of hours.

(6) In 496849 hours, how many years? *Ans.* 5 years, 243 days.

(7) In 46$\frac{1}{2}$ years, how many days and half minutes (365$\frac{1}{4}$ days to a year)? *Ans.* 16984$\frac{1}{8}$ days, 48394080 half minntes.

(8) In 45 centuries, how many hours and half secouds?
Ans. 3944700 hours, 284018400000 half seconds.

(9) In 20 centuries, how many periods of time, each containing 2 hours 20 minutes? *Ans.* 7513714 periods, 40 min. rem.

(10) Find how many days and hours it is from August 10th, 1807, to February 29th, 1844.
Ans. 13343$\frac{1}{4}$ days, 320238 hours.

(11) In 42 years, how many days, hours, and seconds?
Ans. 15351 days, 368424 hours, 1326326400 seconds.

(12) Reduce 49 years, 3 months, 2 weeks, 13 days, 4 hours, to hours and half minutes (13 months to a year).
Ans. 1721980 hours, 206637600 half minutes.

(13) In 29784 months, how many weeks and years, reckoning 360 days to a year, and 30 days to a month?
Ans. 119136 months, 893520 days, 2482 years.

(14) In 457329 days, how many years, reckoning 365$\frac{1}{4}$ days to a year? *Ans.* 1252 years + 36 days.

(15) In 7 years, how many days, hours, and minutes?
Ans. 2556$\frac{3}{4}$ days, 61362 hours, 3681720 minutes.

(16) In 49 years, how many hours and seconds?
Ans. 429574 hours, 1546466400 seconds.

(17) In 697 years, how many half hours and half seconds?
Ans. 254579$\frac{1}{4}$ days, 12219804 half hours,
43991294400 half seconds.

(18) Reduce 497315 hours to years of 365¼ days each.
Ans. 56 years, 9 months, 2 weeks, 1 day, 11 hours.

(19) Reduce 247981 minutes to half hours and months of 30 days each. *Ans.* 5 months, 22 days, 5 hours, 1 minute.

(20) In 78 sidereal days, how many civil days?
Ans. 77+1128.

(21) In 4298 hours, how many periods of time each 7½ minutes? *Ans.* 3105+10.

(22) In 11973 weeks, how many quarter days and years of 366 days each?
Ans. 33524 quarter days, 228 years + 363 hours.

(23) In 19 years, 176½ days, how many half hours (years 365¼ days each)? *Ans.* 341580 half hours.

(24) How many years are there in 721496 days?
Ans. 1976+256 years.

(25) In 209573 months, 28 days each, how many days, hours, and quarter hours? *Ans.* 5868044 days, 140833056 hours, 563332224 quarter hours.

(26) In 209 years, how many days, hours, and half seconds?
Ans. 76337¼ days, 1832094 hours, 13191076800 half secs.

(27) How many minutes to July 19th, 1841, from the birth of our Saviour (365¼ days to a year)? *Ans.* 969012360 minutes.

(28) In 734 years, how many days? *Ans.* 268093½ days.

(29) How many equal spaces, each containing 15 days, 4 hours, are there in 7 years, 144 days? *Ans.* 178+$\frac{13}{182}$.

(30) In 684937 hours, how many half years (the year 365¼ days)? *Ans.* 78+$\frac{1189}{8766}$.

(31) In 56 years, how many spaces, each consisting of 196¼ days? *Asn.* 104+$\frac{176}{785}$.

(32) In 38576495 minutes, how many half weeks?
Ans. 7654+395 minutes rem.

(33) In 1376498316 seconds, how many ¾ weeks?
Ans. 3034 $\frac{442517}{453600}$.

(34) How many years are there in 7246517 half minutes (365¼ days to a year)? *Ans.* 68 yrs., 22 days, 10 hrs., 50½ rem.

MISCELLANEOUS QUESTIONS.

(1) The great bell of St. Paul's weighs 5 tons, 2 cwt., 1 qr., 22 lbs.; how many bells of 4 lbs., 7 ozs., could be formed out of this? *Ans.* 2608$\frac{16}{71}$.

(2) A man walked 4 miles, 3 furlongs, 2 poles; rode 7 miles, 4 furlongs, 29 poles; his son rode 10 miles, 3 furlongs, 17 poles,

and walked 2 miles, 1 furlong, 8 poles. How much farther did the son go than the father? *Ans.* 4 furlongs, 34 poles.

(3) A gentleman bought a parcel of ground, consisting of 14 acres, 2 roods, 19 poles; his house covered 2492 square yards; his coach-house and stables, 846; his farm-yard, 7292 square yards; his green-house, 420 square yards; his kitchen-garden, 2 acres, 0 roods, 16 poles; 2496 yards in shrubberies; 7984 in flower-beds; and the rest in lawns. How much remained for this purpose? *Ans.* $39054\frac{3}{4}$ square yards.

(4) How many crown pieces would reach round the world, supposing each to measure $2\frac{1}{4}$ inches in diameter, and the earth to be 25020 miles in circumference? *Ans.* 704563200.

(5) A gentleman travels per steamer; goes from London to Malta in 14 days; and takes 4 days to reach Alexandria; 3 days to reach Cairo; 2 to Suez; 10 to go down the Red Sea; 13 in passage to Bombay. How many minutes is he going from London to Bombay? *Ans.* 66240.

(6) A ship's crew, of 593 men, and 60 boys, consume 248 quarters of corn in 3 months; now, reckoning 2 boys to a man, how many pecks had each man and boy?

Ans. Man $12\frac{260}{623}$ pecks, and boys $6\frac{130}{623}$ pecks.

(7) 95 paupers dig up a common, whose superficial content is 338 acres, 3 roods; now, supposing each man to dig 603 square yards per day, how long would they be in digging up the common; and how much would each man dig? *Ans.* $2643\frac{107}{121}$ days, and each dug $16837\frac{7}{19}$ square yards.

(8) If a gentleman's income is 496*l.* 19*s.* 10*d.* per annum, and he pays in rent 48*l.* 10*s.*; taxes 23*l.* 14*s.*; gives to the poor quarterly 5*l.* 12*s.*; and at Christmas 14*l.* 8*s.* 6*d.*; how much per day can he spend on himself and family? *Ans.* 1*l.* 1*s.* $3\frac{37}{365}$*d.*

(9) What is that number, which, if divided by 732, with 492 remainder, will make a quotient of 463116? *Ans.* 339001404.

(10) If I multiply 2409685 by 329, and divide by 4319, then subtract 500091, and divide the remainder by 3; what quotient do I obtain? *Ans.* $44517\frac{625}{12957}$.

(11) A man intended to give to A, 48*l.* 12*s.* $9\frac{1}{2}$*d.*; to B, 50*l.* 0*s.* 1*d.*; to C, 101*l.* 18*s.* $9\frac{1}{2}$*d.*; but he lost 36*l.* 19*s.* 6*d.* before distribution; he then determines to divide the remainder equally amongst them. How much would each have, and how much did he originally intend to distribute?

Ans. To each 54*l.* 10*s.* $8\frac{1}{2}$*d.* + $\frac{2}{3}$; his first sum 200*l.* 11*s.* 8*d.*

(12) 54 soldiers take some baggage, in value 79*l.* 12*s.* 8*d.*, what will each man have, supposing 13 to be killed whilst taking it to quarters, and each dead man's friends to have half of what a survivor had, and the other half to be divided amongst the survivors? *Ans.* 1*l.* 13*s.* $6\frac{1}{4}$*d.* $+\frac{5}{19}$.

(13) The great bell of Moscow weighs 432000 lbs., and the bell of St. Paul's, London, 11470 lbs., the Vienna bell (cast from Turkish cannon) 40,200 lbs.; how many St. Paul's and Vienna bells can be cast out of the Moscow bell?

Ans. $37\frac{761}{1147}$ St. Paul's, and $10\frac{50}{67}$ Vienna bells.

(14) Notre Dame, Paris, is 204 feet high; how many churches of the height of Notre Dame must be piled on each other to make a height 17 times that of the great pyramid of Egypt, whose height is 520 feet? *Ans.* $43\frac{1}{3}$.

(15) The ancient population of Thebes, A.M. 3470, is said to have been 3600000, and the population of Baltimore, in A.D. 1840, was 101378; how many times was the population of Baltimore contained in that of Thebes? *Ans.* $35\frac{25885}{50689}$.

(16) Malte Brun says, that Europe contains 213000000 inhabitants; now, if the population of Great Britain is 27000000, of Prussia 15000000, and of Russia 42000000; how many nations, each with a population equal to Portugal, containing 3800000, would be contained in the remainder? *Ans.* $33\frac{18}{19}$.

(17) There are 3 pipes of wine, Sherry, Port, and Madeira; each pipe contains as in the tables. It is required to know how many of these pipes, and of each an equal number, would be contained in a vessel containing 7968 gallons? *Ans.* $25\frac{31}{105}$.

(18) A gentleman gave 24*l.* 9*s.* 2*d.* to be distributed to some poor families: to those families consisting of 6 children, 3*l.* 10*s.* were given; to those consisting of 4 children 1*l.* 19*s.* was given; there were 3 families, consisting of 6 children each, and 2 containing 4 children; the remainder of the money being given to the distributor. It is required to know which families obtained the greatest bounty, supposing the father and mother of each family to receive as much as 4 children; and what the distributor had for his trouble? *Ans.* The families consisting of 6 children had the greatest bounty; the distributor had 6*s.* 2*d.* for his trouble.

CONTINUATION OF MISCELLANEOUS QUESTIONS.

(1) In 78 barrels of figs, each $68\frac{1}{2}$ lbs., how many half ounces? *Ans.* 170976.

(2) Five drawers, each containing 72 purses, each purse con-

taining 5 guineas, 7 half crowns, and 96 half moidores; how many seven shilling pieces in all? *Ans.* 7295 + 2*s.* over.

(3) In 84 barrels of pimento, 30 were damaged, and about 15 of these were saleable at half the weight; they all originally weighed 56 lbs. each. What was the weight they all sold for? *Ans.* 9594.

(4) In 83465 lbs. avoirdupois, how many quarter ounces? *Ans.* 5341760.

(5) In 3847 cwts., how many parcels, each $52\frac{1}{2}$ lbs.? *Ans.* $8206\frac{98}{105}$.

(6) In 3749 yards, how many quarters and Flemish ells? *Ans.* $4998\frac{2}{3}$.

(7) How many half barleycorns are there in 745 miles? *Ans.* 283219200.

(8) How many half pints are there in 7496 quarters of oats? *Ans.* 479744.

(9) How many square yards in 673 acres? *Ans.* 3257320.

(10) In 3296*l.* 19*s.*, how many halfpence? *Ans.* 1582536.

(11) In 7345*l.* 16*s.* 8*d.*, how many fourpenny pieces? *Ans.* 440750.

(12) In 785*l.* 16*s.* 9*d.*, how many farthings? *Ans.* 754404.

(13) In 73569 crowns, how many half farthings? *Ans.* 35313120.

(14) How many half guineas can I obtain for 31698 moidores? *Ans.* 81509 + 3 rem.

(15) How many half days are there in 73296 quarter years? *Ans.* 13385682.

(16) In 7 lbs., 8 ounces, 15 drams, how many parcels, each containing 12 ounces 2 drams? *Ans.* 9 parcels, and $15\frac{57}{87}$ drs.

(17) How many times will a coach-wheel turn round, whose circumference is $13\frac{3}{4}$ feet, in going to York, 180 miles distant? *Ans.* 69120.

(18) How many $\frac{1}{4}$ drams are there in 245 ounces, 11 drams? *Ans.* 15724.

(19) In 34 lbs., 7 ounces, 3 drams, how many half drams? *Ans.* 17638.

(20) How many times would a hoop turn round in going to the Cape of Good Hope; the hoop being 4 feet 7 inches in circumference, and distance 4840 miles? *Ans.* 5575680.

(21) How many hogsheads, pipes, and tuns are thee in 76953 half gallons, and of each an equal number? *Ans.* $8\frac{73}{294}$.

(22) How many butts and hogsheads are there in 72 barrels of beer, and of each an equal number? *Ans.* 16!

(23) In 686*l.* 12*s.*, how many milrees, at 6*s.* 7½*d.* each?

Ans. $2072\frac{40}{53}$.

(24) A grocer mixes 3 different kinds of teas together, in equal quantities, valued at 4*s.* 6*d.*, 5*s.*, and 6*s.* 6*d.* per lb.; in making a mixture, to be sold for 96*l.*, what quantity will he sell?

Ans. 120 of each, or 360 in all.

(25) A man wishing to gain 3*s.* per dozen on some shoes that cost 4*s.* 6*d.* per pair, wishes to know what he will obtain, and what he shall gain on the sale of 588 pairs?

Ans. Obtained 139*l.* 13*s.*, and gained 8*l.* 12*s.*

PART II.

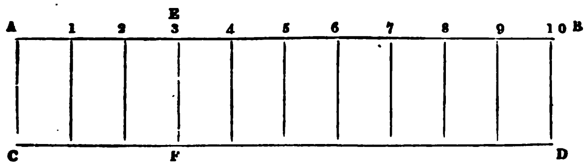
VULGAR FRACTIONS.

DEFINITIONS.

All integers, or whole numbers, consist of a certain number of units; one or more of these units, when divided into parts, is called a fraction: which literally means, a *broken piece*.

According to the number of parts into which the unit is divided, so the value of the fraction varies. Thus, if the unit be divided into ten parts, and we take three of these parts, the fraction resulting from this process is represented by the character $\frac{3}{10}$. The bottom number, showing the parts into which the unit is divided, and the top one the number of the parts to be taken for the fraction. A more familiar example will probably tend to put this explanation in a clearer light.

Ex.—Suppose we have a plot of ground, and we divide it into 10 equal parts, and suppose we give to you 3 of these parts, your share of the field would be represented by the fraction $\frac{3}{10}$.



Let A B C D be the field, and suppose it is divided into ten equal parts, as in the figure; if we wish to give you 3 of these parts (viz. the fig. A E F C), we shall give 3 parts out of the 10; or, as it is expressed in fractions, $\frac{3}{10}$. The upper number (the 3) is called the *numerator*; and the lower one (the 10) is called the *denominator*. The two numbers themselves (the 3 and the 10) being called the *terms* of the fraction.

When the numerator is less than the denominator, the fraction is *less* than unity, as $\frac{2}{5}$.

If the numerator be equal to the denominator, the fraction *represents* unity, as $\frac{5}{5}=1$.

If the numerator be *greater* than the denominator, the fraction is greater than unity, as $\frac{7}{5}=1\frac{2}{5}$.

The fraction $\frac{3}{10}$, an example of the first kind, is called a *proper* fraction.

The fraction $\frac{10}{10}$ represents unity, and is also called a *proper* fraction.

The fraction $\frac{12}{7}$ represents a fraction greater than unity, and is called an *improper* fraction.

The *reciprocal* of a fraction is found by inverting it; thus, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$; the reciprocal of 3 is $\frac{1}{3}$, because unity is the denominator.

A *mixed fraction*, or *mixed number*, consists of a whole number and a fraction, as $5\frac{1}{2}$.

A *compound fraction* consists of two or more fractions, connected by the word *of*, as $\frac{1}{8}$ of $\frac{7}{12}$ of $\frac{3}{5}$.

A *complex fraction* is a fraction whose numerator and denominator consist of other fractions:

$$\text{As } \frac{\frac{1}{2}}{\frac{7}{8}}, \text{ or } \frac{\frac{2\frac{1}{2}}{9}}{5\frac{1}{4}}$$

CASE 1.

To find the least Common Multiple of two or more Numbers.

DEFINITION.

The least common multiple of two or more numbers is the least number into which all the numbers will divide.

1. Place the numbers, of which you are to find the least common multiple, in succession; then *any* of the numbers that are factors of the others may be cancelled.

2. Divide the remaining numbers by any of the low *prime* numbers, as 2, 3, 5, 7, 11, 13, 17; (the numbers 2, 3, 5, and 7 are generally the most useful for the purpose.)

3. Then divide the remaining numbers by any prime number, and the next remaining numbers by a prime number; and so on till you obtain *primes* only in the remainders.

4. Multiply all the divisors by all the remaining prime numbers, the product is the least common multiple.

Ex.—Find the least common multiple of 2, 3, 4, 9, 15, 28, 36, 140.

3	2.3.4.9.15.28.36.140
5	5. 12. 140
2	1. 12. 28
2	1. 6. 14
	1. 3. 7

Here we perceive that 2, 3, 4, 9, will each divide into 36, and 28 into 140; we therefore cancel them. We then divide by the prime 3, and obtain for quotients the numbers 5, 12, and 140. We then divide by the prime 5, and obtain for quotients 1, 12, 28. We then divide by the prime 2, and obtain for quotients 6 and 14. We then again divide by 2, and obtain for quotients the primes 1, 3, 7. Lastly, we multiply 3, 5, 2, 2 (the divisors) by 3, 7 (the remainders), and obtain as a product 1620, the least common multiple.

Note.—One division might have been avoided in this, by dividing by 4 instead of 2, 2; but this should only occur at *the end* of the process.

Common Multiple.

Find the least common multiple of the following sets of numbers. Of

(1)	2.4.8.6.32	*Ans.* 96
(2)	4.9.8.6.14.26	*Ans.* 6552
(3)	3.8.9.6.4.28	*Ans.* 504
(4)	4.8.12.16.32	*Ans.* 96
(5)	14.8.6.10.16	*Ans.* 1680
(6)	12.8.9.15.7.6	*Ans.* 2520
(7)	8.12.16.24.32	*Ans.* 96
(8)	7.9.8.4.12	*Ans.* 504
(9)	4.3.9.4.12	*Ans.* 36
(10)	2.8.12.28.64	*Ans.* 1344
(11)	12.18.36.24	*Ans.* 72
(12)	4.9.13.29.68.37.7. and 42	*Ans.* 59757516
(13)	3.8.2.16.48.39.13.28. and 14	*Ans.* 4368
(14)	5.13.22.8.108.72.57.3. and 19	*Ans.* 2934360
(15)	7.25.35.96.100.3.49.8. and 2	*Ans.* 117600
(16)	6.4.9.8.12.24.98. and 16	*Ans.* 7056
(17)	9.12.17.4.29.48.68. and 87	*Ans.* 70992

COMMON MULTIPLE—(*continued*).

(18) 10. 14. 19. 16. 22. 54. 78. and 16 *Ans.* 41080040
(19) 3. 8. 9. 12. 36. 49. 72. and 6 *Ans.* 3528
(20) 4. 7. 14. 9. 38. 36. 49. and 58 *Ans.* 971964
(21) 5. 9. 38. 47. 79. 84. and 68 *Ans.* 1511116740
(22) 8. 12. 39. 48. 84. 117. and 64 *Ans.* 52416
(23) 36. 7. 8. 44. 93. 49. and 132 *Ans.* 1203048
(24) 8. 14. 9. 36. 49. 98. and 7 *Ans.* 3528
(25) 9. 16. 3. 6. 18. 73. and 81 *Ans.* 94608
(26) 7. 2. 9. 16. 48. 54. and 192 *Ans.* 12096
(27) 3. 8. 14. 24. 56. 96. 336. and 768 *Ans.* 5376
(28) 8. 4. 16. 68. 54. 76 *Ans.* 139536
(29) 3. 32. 18. 15. 28. 36. 48. 54 *Ans.* 30240
(30) 4. 19. 38. 63. 9. 8. 57 *Ans.* 9576
(31) 2. 6. 8. 18. 15. 14. 9. 18 *Ans.* 2520
(32) 12. 8. 4. 9. 6. 5. 36. 49 *Ans.* 17640
(33) 5. 17. 35. 68. 15. 35 *Ans.* 804900
(34) 4. 2. 16. 19. 54. 68. 75 *Ans.* 3488400
(35) 8. 6. 18. 14. 19. 38. 64 *Ans.* 76608
(36) 5. 7. 65. 63. 540. and 451 *Ans.* 22162140
(37) 2. 4. 10. 35. 48. 64. and 144 *Ans.* 20160
(38) 7. 2. 9. 84. 81. 45. 63. and 30 *Ans.* 52920
(39) 2. 4. 8. 12. 36. 96. 84. and 34 *Ans.* 34272
(40) 3. 2. 9. 8. 4. 7. 6. 42. 64. 63. and 40 *Ans.* 15120

CASE 2.

To reduce Fractions to a Common Denominator.

RULE 1.—If the *least common* denominator be *not* required (as it generally now is in questions), but only a common denominator, it may be found as follows :—

Multiply each numerator into all the denominators of the fractions, except the one below it; the several products will give the new numerators, then multiply all the denominators together for the common denominator.

RULE 2.—But if the fractions are required to be in their lowest terms (which the preceding rule in its application does not effect), the following rule must be employed :—

Find the least common multiple of the denominators by Case 1, and divide each denominator into the least common multiple, then multiply the numerator into the quotient for a numerator, underneath which place the least common multiple, and the fractions will be reduced to the least common denominator.

Ex.—Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$ to a common denominator.

$$\begin{array}{rl} \text{Here} & 2\times6\times8\times12=1152 \\ & 5\times3\times8\times12=1440 \\ & 7\times3\times6\times12=1512 \\ & \underline{5\times3\times6\times\ \ 8=\ \ 720} \\ & \underline{3\times6\times8\times12=1728} \text{ common denominator.} \end{array}$$

$\therefore$ the fractions are, $\frac{1152}{1728}$, $\frac{1440}{1728}$, $\frac{1512}{1728}$, $\frac{720}{1728}$.

Here it will be perceived that we first multiply the numerator of the first fraction, viz. the 2 by all the denominators, except the one under it, viz. by 6, 8, 12.

Secondly, Multiply the next number (5) into all the denominators except the one under it, viz. by the numbers 3, 8, 12.

Thirdly, We multiply the third number (7) by all the denominators except the one under it, viz. by 3, 6, and 12.

Fourthly, We multiply the fourth number (5) by all the denominators except the one under it, viz. by 3, 6, and 8; the product being placed after the sign =, gives the new numerator to each fraction.

Finally, We multiply all the denominators together for a common denominator, placing it under each new numerator.

But should we want the fractions to be reduced to their *lowest terms* at the same time that we find a common denominator, then we employ the second rule.

Ex.—$\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{12}$.

$$\left.\begin{array}{l} 2\,|\,\underline{3.6.8.12} \\ 2\,|\,\underline{\quad\ \ 4.\ 6} \\ \quad\ \ \underline{\quad\ \ 2.\ 3} \\ \therefore 24 \text{ is common multiple} \end{array}\right\} 3.2.2.2=24 \text{ least common multiple.}$$

$\therefore$ The fractions are, $\frac{16}{24}$, $\frac{20}{24}$, $\frac{21}{24}$, $\frac{10}{24}$.

Here, it will be perceived, we first find the least common multiple of all the denominators, which is 24. We then divide the denominator (3) of the first fraction into this least common multiple, and obtain for a quotient 8, which we multiply by the numerator 2, and obtain for the product 16, underneath which we place the least common multiple; thereby obtaining the first reduced fraction. We proceed precisely in the same manner with all the other fractions, and obtain respectively 20, 21, and 10, for the numerators and the fractions as above.

Reduce the following fractions to common denominators:—

(1) $\frac{5}{9}$, $\frac{6}{7}$, $\frac{3}{28}$, $5\frac{2}{3}$, $2\frac{1}{7}$. *Ans.* $\frac{140}{252}$, $\frac{216}{252}$, $\frac{27}{252}$, $\frac{1848}{252}$, $\frac{560}{252}$.

(2) $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{12}$, and $\frac{6}{15}$. *Ans.* $\frac{240}{60}$, $\frac{50}{60}$, $\frac{40}{60}$, $\frac{24}{60}$.

(3) $\frac{2}{5}$, $\frac{5}{7}$, $\frac{8}{9}$, $\frac{3}{5}$, and $\frac{5}{18}$. *Ans.* $\frac{252}{630}$, $\frac{450}{630}$, $\frac{560}{630}$, $\frac{378}{630}$, $\frac{175}{630}$.

(4) $\frac{7}{15}$, $\frac{3}{4}$, $\frac{6}{7}$, $\frac{5}{21}$, and $\frac{7}{16}$. *Ans.* $\frac{784}{1680}$, $\frac{1260}{1680}$, $\frac{1440}{1680}$, $\frac{400}{1680}$, $\frac{735}{1680}$.

(5) $\frac{2}{7}$, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{2}{35}$, and $\frac{13}{54}$. *Ans.* $\frac{540}{1890}$, $\frac{1680}{1890}$, $\frac{1575}{1890}$, $\frac{108}{1890}$, $\frac{455}{1890}$.

(6) $\frac{2}{3}$, $\frac{4}{9}$, $\frac{9}{11}$, $\frac{12}{17}$, $\frac{15}{18}$, $\frac{1}{9}$, $\frac{3}{4}$, $\frac{5}{7}$ *Ans.* $\frac{31416}{47124}$, $\frac{20944}{47124}$, $\frac{38556}{47124}$, $\frac{33264}{47124}$, $\frac{39270}{47124}$, $\frac{5236}{47124}$, $\frac{35343}{47124}$, $\frac{33660}{47124}$.

(7) $\frac{5}{6}$, $\frac{3}{7}$, $\frac{9}{12}$, $\frac{8}{9}$, $\frac{9}{13}$, $\frac{6}{7}$, $\frac{4}{5}$ *Ans.* $\frac{13650}{16380}$, $\frac{7020}{16380}$, $\frac{12285}{16380}$, $\frac{14560}{16380}$, $\frac{11340}{16380}$, $\frac{14040}{16380}$, $\frac{13104}{16380}$.

(8) $\frac{4}{7}$, $\frac{3}{28}$, $\frac{13}{15}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{9}{14}$, $\frac{8}{11}$, $\frac{5}{7}$ *Ans.* $\frac{2640}{4620}$, $\frac{495}{4620}$, $\frac{4004}{4620}$, $\frac{3696}{4620}$, $\frac{3960}{4620}$, $\frac{2970}{4620}$, $\frac{3360}{4620}$, $\frac{3300}{4620}$.

(9) $\frac{3}{5}$, $\frac{5}{9}$, $\frac{6}{13}$, $\frac{8}{9}$, $\frac{7}{10}$, $\frac{4}{13}$, $\frac{6}{15}$, $\frac{7}{9}$, $\frac{8}{17}$ *Ans.* $\frac{35802}{59670}$, $\frac{33150}{59670}$, $\frac{27540}{59670}$, $\frac{53040}{59670}$, $\frac{41769}{59670}$, $\frac{18360}{59670}$, $\frac{23868}{59670}$, $\frac{46410}{59670}$, $\frac{28080}{59670}$.

(10) $\frac{2}{3}$, $\frac{1}{9}$, $\frac{6}{7}$, $\frac{8}{15}$, $\frac{3}{11}$, $\frac{2}{5}$, $\frac{1}{13}$, $\frac{5}{18}$, $\frac{5}{9}$ *Ans.* $\frac{60060}{90090}$, $\frac{10010}{90090}$, $\frac{77220}{90090}$, $\frac{48048}{90090}$, $\frac{24570}{90090}$, $\frac{36036}{90090}$, $\frac{69030}{90090}$, $\frac{25025}{90090}$, $\frac{50050}{90090}$.

(11) $\frac{8}{13}$, $\frac{6}{7}$, $\frac{5}{13}$, $\frac{2}{5}$, $\frac{4}{9}$, $\frac{7}{8}$, $\frac{10}{19}$, $\frac{6}{8}$ *Ans.* $\frac{383040}{622440}$, $\frac{533520}{622440}$, $\frac{239400}{622440}$, $\frac{248976}{622440}$, $\frac{276640}{622440}$, $\frac{544635}{622440}$, $\frac{327600}{622440}$, $\frac{466830}{622440}$.

(12) $\frac{5}{8}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{7}{10}$, $\frac{3}{14}$, $\frac{5}{9}$, $\frac{13}{15}$ *Ans.* $\frac{1575}{2520}$, $\frac{1890}{2520}$, $\frac{1680}{2520}$, $\frac{280}{2520}$, $\frac{1764}{2520}$, $\frac{540}{2520}$, $\frac{1400}{2520}$, $\frac{2184}{2520}$.

(13) $\frac{2}{9}$, $\frac{4}{7}$, $\frac{3}{8}$, $\frac{7}{10}$, $\frac{5}{20}$, $\frac{6}{14}$, $\frac{5}{7}$, $\frac{14}{19}$ *Ans.* $\frac{10640}{47880}$, $\frac{27360}{47880}$, $\frac{17955}{47880}$, $\frac{33516}{47880}$, $\frac{11970}{47880}$, $\frac{20520}{47880}$, $\frac{34200}{47880}$, $\frac{35280}{47880}$.

(14) $\frac{13}{12}$, $\frac{7}{9}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{12}{19}$, $\frac{11}{13}$, $\frac{17}{19}$ *Ans.* $\frac{48165}{44460}$, $\frac{34580}{44460}$, $\frac{35568}{44460}$, $\frac{13338}{44460}$, $\frac{28080}{44460}$, $\frac{37620}{44460}$, $\frac{39780}{44460}$.

(15) $\frac{4}{7}$, $\frac{9}{13}$, $\frac{6}{7}$, $\frac{4}{15}$, $\frac{8}{14}$, $\frac{2}{5}$, $\frac{1}{17}$ *Ans.* $\frac{1560}{2730}$, $\frac{1890}{2730}$, $\frac{2340}{2730}$, $\frac{728}{2730}$, $\frac{1560}{2730}$, $\frac{1092}{2730}$, $\frac{390}{2730}$.

(16) $\frac{3}{14}$, $\frac{1}{9}$, $\frac{7}{10}$, $\frac{5}{14}$, $\frac{6}{7}$, $\frac{}{9}$, $\frac{7}{18}$ *Ans.* $\frac{135}{630}$, $\frac{70}{630}$, $\frac{441}{630}$, $\frac{225}{630}$, $\frac{540}{630}$, $\frac{350}{630}$, $\frac{245}{630}$.

CASE 3.

To reduce a mixed Number to an Improper Fraction.

ULE.—Multiply the denominator of the fractional part into tegral part, adding in the numerator of the fractional part; neath which sum place the denominator of the fractional

—Reduce $354\frac{5}{12}$ to an improper fraction.

Here we multiply 354 (the integral part) by 12, (the denominator of the fractional part,) adding the 5 (the numerator of the fractional part), obtaining for sum 4253; underneath which we place the 12 (the denominator of the fractional part),

To reduce a mixed Fraction to its proper Terms.

uce the following mixed numbers to improper fractions:—

$26\frac{11}{13}$	*Ans.* $\frac{349}{13}$	(16)	$3829\frac{2}{43}$	*Ans.* $\frac{164649}{43}$
$384\frac{5}{9}$	*Ans.* $\frac{3461}{9}$	(17)	$4792\frac{1}{25}$	*Ans.* $\frac{119801}{25}$
$298\frac{6}{19}$	*Ans.* $\frac{5668}{19}$	(18)	$8294\frac{7}{18}$	*Ans.* $\frac{149299}{18}$
$247\frac{5}{18}$	*Ans.* $\frac{4451}{18}$	(19)	$5289\frac{5}{27}$	*Ans.* $\frac{142808}{27}$
$196\frac{5}{9}$	*Ans.* $\frac{1769}{9}$	(20)	$6982\frac{9}{17}$	*Ans.* $\frac{118703}{17}$
$785\frac{4}{39}$	*Ans.* $\frac{30619}{39}$	(21)	$3895\frac{5}{19}$	*Ans.* $\frac{74010}{19}$
$568\frac{2}{29}$	*Ans.* $\frac{16474}{29}$	(22)	$6298\frac{1}{37}$	*Ans.* $\frac{233027}{37}$
$17968\frac{4}{23}$	*Ans.* $\frac{413268}{23}$	(23)	$2957\frac{3}{35}$	*Ans.* $\frac{103498}{35}$
$2984\frac{15}{31}$	*Ans.* $\frac{92519}{31}$	(24)	$12998\frac{9}{28}$	*Ans.* $\frac{363953}{28}$
$1794\frac{18}{35}$	*Ans.* $\frac{62808}{35}$	(25)	$4296\frac{5}{18}$	*Ans.* $\frac{77333}{18}$
$2938\frac{12}{49}$	*Ans.* $\frac{143974}{49}$	(26)	$7482\frac{7}{23}$	*Ans.* $\frac{127199}{17}$
$19\frac{3}{5}$	*Ans.* $\frac{98}{5}$	(27)	$6294\frac{5}{17}$	*Ans.* $\frac{107003}{17}$
$144\frac{5}{9}$	*Ans.* $\frac{1301}{9}$	(28)	$2986\frac{2}{37}$	*Ans.* $\frac{110484}{37}$
$673\frac{8}{13}$	*Ans.* $\frac{8757}{13}$	(29)	$2965\frac{11}{13}$	*Ans.* $\frac{38556}{13}$
$2095\frac{5}{7}$	*Ans.* $\frac{14670}{7}$	(30)	$7824\frac{10}{19}$	*Ans.* $\frac{148666}{19}$

CASE 4.

To reduce an Improper Fraction to a mixed Number.

ULE.—Divide the numerator of the improper fraction by its ninator for the integral part, placing the remainder *over* the ninator of the improper fraction, for the fractional part.

—Reduce $\frac{4253}{12}$ to a mixed number.

$= 354\frac{5}{12}$ Here we divide the 4253 (the numerator of the improper fraction) by 12, and obtain for quotient 354, and for remainder 5, under which we place 12, the denominator of the fractional part.

Reduce the following improper fractions to mixed numbers:—

1) $\frac{49395}{247}$	*Ans.* $199\frac{242}{247}$	(16) $\frac{69372}{932}$	*Ans.* $74\frac{404}{932}$
(2) $\frac{43294}{19}$	*Ans.* $2278\frac{12}{19}$	(17) $\frac{16739}{43}$	*Ans.* $389\frac{12}{43}$
(3) $\frac{79864}{247}$	*Ans.* $323\frac{83}{247}$	(18) $\frac{93917}{394}$	*Ans.* $238\frac{145}{394}$
(4) $\frac{62958}{371}$	*Ans.* $169\frac{259}{371}$	(19) $\frac{20968}{173}$	*Ans.* $121\frac{35}{173}$
(5) $\frac{56734}{396}$	*Ans.* $143\frac{53}{198}$	(20) $\frac{76938}{5821}$	*Ans.* $13\frac{1265}{5821}$
(6) $\frac{29694}{698}$	*Ans.* $42\frac{189}{349}$	(21) $\frac{7968}{14}$	*Ans.* $569\frac{1}{7}$
(7) $\frac{29831}{692}$	*Ans.* $43\frac{75}{692}$	(22) $\frac{5987}{29}$	*Ans.* $206\frac{13}{29}$
(8) $\frac{40973}{2968}$	*Ans.* $13\frac{2389}{2968}$	(23) $\frac{78349}{129}$	*Ans.* $607\frac{46}{129}$
(9) $\frac{45291}{7329}$	*Ans.* $6\frac{1317}{7329}$	(24) $\frac{2964}{38}$	*Ans.* 78
(10) $\frac{62949}{196}$	*Ans.* $321\frac{33}{196}$	(25) $\frac{21738}{239}$	*Ans.* $90\frac{228}{239}$
(11) $\frac{39682}{493}$	*Ans.* $80\frac{242}{493}$	(26) $\frac{3895}{47}$	*Ans.* $82\frac{41}{47}$
(12) $\frac{86943}{493}$	*Ans.* $176\frac{175}{493}$	(27) $\frac{5968}{35}$	*Ans.* $170\frac{18}{35}$
(13) $\frac{71968}{293}$	*Ans.* $245\frac{183}{293}$	(28) $\frac{798457}{1396}$	*Ans.* $571\frac{1341}{1396}$
(14) $\frac{28695}{473}$	*Ans.* $60\frac{315}{473}$	(29) $\frac{2847}{32}$	*Ans.* $88\frac{31}{32}$
(15) $\frac{29674}{939}$	*Ans.* $74\frac{404}{939}$	(30) $\frac{3964}{25}$	*Ans.* $158\frac{14}{25}$

CASE 5.

To find the greatest Common Measure of a Fraction.

Rule.—Divide the greater of the two terms by the less, and the last remainder by the divisor last used, and so on, continually performing the like operation till there is either 1 for remainder or *no* remainder. In the first instance, if 1 be remainder, the fraction is already in its lowest terms; in the second instance, *the last divisor* is the common measure.

Note.—This rule is merely employed as a means of effecting the next Rule.

Ex. Find the greatest common measure to $\frac{36}{132}$

```
36)132(3
   108
   ‾‾24)36(1
        24
        ‾‾12)24(2
             24
```

Here we divide 132 by 36, agreeably to the rule, and then 24 into 36; then 12 into 24, and get no remainder; the last divisor being 12, which is the common measure.

Find the greatest common measure of the following fractions:—

(1) $\frac{1344}{2304}$	*Ans.* 192	(4) $\frac{10368}{1440}$	*Ans.* 288
(2) $\frac{584}{9264}$	*Ans.* 4	(5) $\frac{1008}{2520}$	*Ans.* 504
(3) $\frac{1330}{3575}$	*Ans.* 5	(6) $\frac{648}{9018}$	*Ans.* 54

$\frac{456}{304}$.	*Ans.* 16	(19) $\frac{6892}{8964}$	*Ans.* 4
$\frac{5912}{5552}$	*Ans.* 8	(20) $\frac{5384}{2876}$	*Ans.* 4
$\frac{5280}{1120}$	*Ans.* 5280	(21) $\frac{3895}{7260}$	*Ans.* 5
$\frac{5184}{8654}$.	*Ans.* 6	(22) $\frac{216}{384}$	*Ans.* 24
$\frac{298}{694}$.	*Ans.* 2	(23) $\frac{4125}{7680}$	*Ans.* 15
$\frac{382}{026}$.	*Ans.* 2	(24) $\frac{15542}{48384}$	*Ans.* 2
$\frac{482}{966}$.	*Ans.* 2	(25) $\frac{512}{6912}$	*Ans.* 256
$\frac{569}{798}$.	*Ans.* $\frac{569}{6798}$	(26) $\frac{324}{3888}$	*Ans.* 324
$\frac{324}{308}$.	*Ans.* 4	(27) $\frac{3888}{8748}$	*Ans.* 972
$\frac{6724}{3496}$	*Ans.* 4	(28) $\frac{812}{7620}$	*Ans.* 4
$\frac{907}{292}$	*Ans.* 9	(29) $\frac{1218}{5580}$	*Ans.* 42
$\frac{5829}{4975}$	*Ans.* 3	(30) $\frac{2475}{8640}$	*Ans.* 45

CASE 6.

To reduce a Fraction to its lowest terms.

le.—Continually divide both numerator and denominator
number which is a factor of both; when the terms of the
n become prime to each other, the fraction is in its lowest
;

, find the common measure, and divide the given fraction
common measure; the reduced fraction is in its lowest

—Reduce $\frac{48}{176}$ to its lowest terms.

by 8. Div. both by 2.

$= 2)\frac{6}{22} = \frac{3}{11}$

$16)\frac{48}{176} = \frac{3}{11}$

48)176(3
144
32)48(1
32
16)32(2
32

(1) Here we perceive that 48 and 176 are each divisible by 8, and we obtain the fraction $\frac{6}{22}$. We then perceive that 6 and 22 are each divisible by 2, and we obtain the fraction $\frac{3}{11}$. The terms of which being prime to each other, contain no factors that are common; the fraction is ∴ reduced to its lowest term.

(2) We find the greatest common measure to be 16; and on dividing this into each of the terms of the fraction, we obtain the same reduced result, $\frac{3}{11}$, as before.

educe the following fractions to their lowest terms:—

$\frac{84}{62}$	*Ans.* $\frac{1492}{1981}$	(4) $\frac{41535}{74485}$	*Ans.* $\frac{923}{1635}$
$\frac{08}{23}$	*Ans.* $\frac{2636}{641}$	(5) $\frac{4284}{9900}$	*Ans.* $\frac{119}{275}$
$\frac{378}{370}$	*Ans.* $\frac{1521}{965}$	(6) $\frac{69}{8280}$	*Ans.* $\frac{1}{120}$

(7) $\frac{512}{1728}$ *Ans.* $\frac{8}{27}$
(8) $\frac{178104}{197808}$ *Ans.* $\frac{7421}{8242}$
(9) $\frac{6264}{71838}$ *Ans.* $\frac{348}{3991}$
(10) $\frac{23079}{35679}$ *Ans.* $\frac{1099}{1699}$
(11) $\frac{26856}{53712}$ *Ans.* $\frac{1}{2}$
(12) $\frac{2719}{8026}$ *Ans.* $\frac{2719}{8026}$
(13) $\frac{37296}{63532}$ *Ans.* $\frac{1332}{2269}$
(14) $\frac{51219}{132363}$ *Ans.* $\frac{813}{2101}$
(15) $\frac{12392}{25136}$ *Ans.* $\frac{1549}{3142}$
(16) $\frac{78912}{109184}$ *Ans.* $\frac{1233}{1706}$
(17) $\frac{53712}{65961}$ *Ans.* $\frac{5968}{7329}$
(18) $\frac{25192}{59712}$ *Ans.* $\frac{3149}{7464}$
(19) $\frac{21987}{799719}$ *Ans.* $\frac{7329}{266573}$
(20) $\frac{576}{864}$ *Ans.* $\frac{2}{3}$
(21) $\frac{2698}{4976}$ *Ans.* $\frac{1349}{2488}$
(22) $\frac{596}{742}$ *Ans.* $\frac{298}{371}$
(23) $\frac{3294}{8926}$ *Ans.* $\frac{1647}{4463}$
(24) $\frac{5938}{9674}$ *Ans.* $\frac{2969}{4837}$
(25) $\frac{2468}{7964}$ *Ans.* $\frac{617}{1991}$
(26) $\frac{2958}{4176}$ *Ans.* $\frac{17}{24}$
(27) $\frac{4160}{7390}$ *Ans.* $\frac{416}{739}$

CASE 7.

To reduce one Fraction to another of the same value, having given either the Numerator or Denominator of the proposed Fraction.

As the numerator of the given fraction is to its denominator so is the numerator of the proposed fraction to its denominator;

Or, as the denominator of the given fraction is to its numerator, so is the denominator of the intended fraction to its numerator.

Ex.—Reduce $\frac{7}{8}$ to a fraction of the same value, whose numerator is 42.

$7 : 8 :: \underset{6}{\cancel{42}} : 48$

Hence the proposed fraction is $\frac{42}{48}$.

This example is so simple, that no explanation is deemed necessary.

Ex.—Reduce $\frac{42}{48}$ to a fraction of the same value, whose denominator is 8.

As $48 : 42 :: 8 : 7$

Hence the proposed fraction is $\frac{7}{8}$.

The same remark applies to this question as in the last.

Reduce the following fractions to fractions of the same value, whose numerators and denominators shall be as below:—

(1) $\frac{5}{9}$ .. n. 15 *Ans.* $\frac{15}{27}$
(2) $\frac{7}{8}$ .. n. 35 *Ans.* $\frac{35}{40}$
(3) $\frac{6}{13}$ .. n. 24 *Ans.* $\frac{24}{52}$
(4) $\frac{8}{11}$ .. n. 7 *Ans.* $\frac{7}{9\frac{5}{8}}$
(5) $\frac{3}{5}$ .. n. 6 *Ans.* $\frac{6}{10}$
(6) $\frac{8}{15}$ .. n. 9 *Ans.* $\frac{9}{16\frac{7}{8}}$
(7) $\frac{4}{13}$ .. n. 7 *Ans.* $\frac{7}{20\frac{1}{4}}$
(8) $\frac{3}{7}$ .. d. 4 *Ans.* $\frac{\frac{12}{7}}{4}$

9) $\frac{24}{13}$ .. d. 6 *Ans.* $\frac{11\frac{1}{13}}{6}$ | (10) $\frac{4}{9}$ .. d. 3 *Ans.* $\frac{4}{3}$

(11) $\frac{6}{5}$ .. d. $2\frac{1}{2}$ *Ans.* $\frac{3}{2\frac{1}{2}}$

CASE 8

To reduce a Compound Fraction to a Simple one.

Rule.—Place the fractions with the sign of multiplication tween them; then cancel any numerator into any denominator, *vice versâ*, or cancel by any multiple which is common to a merator or denominator; the product of all the uncancelled merators will give the numerator of the simple fraction in a duced state; and the product of all the uncancelled denominators ll give the denominator of the simple fraction in its reduced ite.

Ex.—Reduce $\frac{5}{9}$ of $\frac{11}{12}$ of $\frac{13}{19}$ of $\frac{19}{25}$ of $\frac{12}{33}$ of $\frac{18}{36}$ to a simple fraction.

$\times\frac{\cancel{11}}{\cancel{12}}\times\frac{13}{\cancel{19}}\times\frac{\cancel{19}}{\underset{5}{\cancel{25}}}\times\frac{\cancel{12}}{\cancel{33}}\times\frac{\cancel{18}}{\underset{2}{\cancel{36}}}$

First we cancel 18 numerator into 36 denominator, giving a quotient 2; then 12 numerator into 12 denominator; then 11 numerator into 33 denominator, giving a quotient then 19 numerator into 19 denominator; then 5 numerator to 25 denominator, giving 5 for quotient; we then perceive that is a prime, and that it will not divide into any denominator, r will any denominator divide into it. We therefore find that is the numerator of the simple fraction, and 9. 5. 3. 2, or 270, e denominator, ∴ the simple fraction is $\frac{13}{270}$.

To Reduce a Compound Fraction to a Single one.

Reduce the following fractions to single fractions:—

1) $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{6}{7}$ of $\frac{9}{13}$ of $\frac{14}{27}$ of $\frac{13}{19}$. *Ans.* $\frac{40}{513}$.

2) $\frac{1}{5}$ of $\frac{8}{17}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{7}{9}$. *Ans.* $\frac{56}{1377}$.

3) $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{8}{13}$ of $\frac{6}{11}$ of $\frac{10}{13}$ of $\frac{13}{17}$. *Ans.* $\frac{1600}{21879}$.

4) $\frac{5}{9}$ of $\frac{6}{7}$ of $\frac{2}{3}$ of $\frac{3}{16}$. *Ans.* $\frac{5}{84}$.

5) $\frac{7}{8}$ of $\frac{6}{13}$ of $\frac{5}{9}$ of $\frac{8}{11}$. *Ans.* $\frac{70}{409}$.

6) $\frac{3}{5}$ of $\frac{21}{105}$ of $\frac{32}{35}$. *Ans.* $\frac{728}{1575}$.

7) $\frac{4}{7}$ of $\frac{6}{7}$ of $\frac{9}{15}$. *Ans.* $\frac{72}{245}$.

8) $\frac{5}{9}$ of $\frac{2}{3}$ of $\frac{5}{16}$. *Ans.* $\frac{25}{216}$.

(9) $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{9}{13}$. *Ans.* $\frac{9}{52}$.

(10) $\frac{1}{7}$ of $\frac{2}{3}$ of $\frac{7}{10}$ of $\frac{3}{11}$. *Ans.* $\frac{1}{55}$.

(11) $\frac{2}{5}$ of $\frac{6}{7}$ of $\frac{9}{13}$ of $\frac{8}{17}$. *Ans.* $\frac{864}{7735}$.

(12) $\frac{5}{9}$ of $\frac{6}{7}$ of $\frac{13}{19}$ of $\frac{7}{11}$ of $\frac{7}{9}$ of $\frac{19}{15}$ of $\frac{11}{13}$ of $\frac{10}{19}$. *Ans.* $\frac{140}{1539}$.

(13) $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{6}{15}$ of $\frac{8}{9}$ of $\frac{5}{13}$ of $\frac{9}{15}$ of $\frac{8}{17}$ of $\frac{2}{5}$. *Ans.* $\frac{448}{49725}$.

(14) $\frac{5}{9}$ of $\frac{6}{13}$ of $\frac{7}{8}$ of $\frac{5}{7}$ of $\frac{8}{13}$ of $\frac{18}{40}$ of $\frac{13}{21}$ of $\frac{6}{9}$. *Ans.* $\frac{5}{273}$.

(15) $\frac{1}{6}$ of $\frac{9}{11}$ of $\frac{6}{18}$ of $\frac{7}{43}$ of $\frac{9}{17}$ of $\frac{8}{15}$ of $\frac{18}{29}$ of $\frac{5}{7}$. *Ans.* $\frac{1296}{233189}$.

(16) $\frac{2}{9}$ of $\frac{4}{7}$ of $\frac{6}{9}$ of $\frac{5}{8}$ of $\frac{7}{15}$ of $\frac{6}{17}$ of $\frac{42}{57}$ of $\frac{6}{9}$. *Ans.* $\frac{112}{26163}$.

(17) $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{5}{11}$ of $\frac{13}{19}$ of $\frac{8}{19}$ of $\frac{24}{91}$ of $\frac{19}{13}$. *Ans.* $\frac{1800}{86721}$.

(18) $\frac{4}{5}$ of $\frac{2}{9}$ of $\frac{7}{9}$ of $\frac{3}{92}$ of $\frac{27}{29}$ of $\frac{5}{18}$ of $\frac{46}{7}$. *Ans.* $\frac{2}{261}$.

(19) $\frac{2}{3}$ of $\frac{2}{7}$ of $\frac{3}{19}$ of $\frac{4}{11}$ of $\frac{12}{13}$ of $\frac{16}{27}$ of $\frac{9}{12}$. *Ans.* $\frac{256}{57057}$.

(20) $\frac{3}{7}$ of $\frac{5}{9}$ of $\frac{2}{9}$ of $\frac{12}{27}$ of $\frac{5}{13}$ of $\frac{7}{19}$ of $\frac{3}{8}$. *Ans.* $\frac{25}{20007}$.

(21) $\frac{4}{5}$ of $\frac{2}{7}$ of $\frac{2}{13}$ of $\frac{4}{11}$ of $\frac{5}{9}$ of $\frac{6}{13}$ of $\frac{11}{24}$. *Ans.* $\frac{16}{10647}$.

(22) $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{9}{10}$ of $\frac{12}{16}$ of $\frac{14}{27}$ of $\frac{5}{7}$ of $\frac{10}{11}$ of $\frac{13}{14}$. *Ans.* $\frac{65}{924}$.

(23) $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{7}{5}$ of $\frac{9}{10}$ of $\frac{6}{13}$ of $\frac{5}{7}$ of $\frac{5}{3}$ of $\frac{7}{9}$. *Ans.* $\frac{8}{117}$.

(24) $\frac{2}{3}$ of $\frac{1}{9}$ of $\frac{5}{6}$ of $\frac{12}{19}$ of $\frac{17}{45}$ of $\frac{34}{36}$ of $\frac{32}{15}$ of $\frac{19}{6}$ of $\frac{3}{34}$ of $\frac{5}{9}$.

Ans. $\frac{272}{64049}$.

(25) $\frac{5}{12}$ of $\frac{3}{13}$ of $\frac{7}{8}$ of $\frac{}{29}$ of $\frac{45}{19}$ of $\frac{16}{17}$ of $\frac{13}{19}$ of $\frac{29}{14}$ of $\frac{3}{13}$ of $\frac{5}{7}$.

Ans. $\frac{3375}{1116934}$.

(26) $\frac{4}{9}$ of $\frac{6}{13}$ of $\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{6}{10}$ of $\frac{7}{18}$ of $\frac{8}{13}$ of $\frac{9}{12}$ of $\frac{12}{7}$ of $\frac{5}{9}$.

Ans. $\frac{28}{845}$.

(27) $\frac{3}{9}$ of $\frac{5}{7}$ of $\frac{6}{13}$ of $\frac{4}{5}$ of $\frac{2}{9}$ of $\frac{8}{29}$ of $\frac{3}{4}$ of $\frac{9}{10}$ of $\frac{17}{45}$ of $\frac{29}{32}$.

Ans. $\frac{17}{20475}$.

(28) $\frac{7}{9}$ of $\frac{6}{13}$ of $\frac{5}{10}$ of $\frac{9}{11}$ of $\frac{7}{18}$ of $\frac{6}{9}$ of $\frac{1}{6}$ of $\frac{2}{7}$ of $\frac{5}{8}$. *Ans.* $\frac{35}{308885}$.

CASE 9

Reduction of Fractions.

Rule.—Place the quantity to be reduced on the left hand, and reduce it to its lowest term or fractional part, if it contains any. Then place on the right hand the term *to which it is to be reduced to the fraction of;* reduce this to precisely the same term or fractional part as the preceding quantity. 2. Place this reduced number underneath the preceding quantity, reducing, if necessary, the fraction to its lowest terms by Case 6.

Ex. Reduce 2*s*. $5\frac{1}{2}$*d*. to the fraction of a crown.

2*s*. $5\frac{1}{2}$*d*.	60*d*.
12	2
29	120
2	
59	
120	

This is already in its lowest term.

Ex. 2. Reduce 5*s*. $6\frac{1}{4}$*d*.$+\frac{2}{3}$ to the fraction of a guinea.

5*s*. $6\frac{1}{4}$*d*.$+\frac{2}{3}$	21
12	12
66	252
4	4
265	1008
3	3
797 reduced to thirds of farthings.	3024 reduced to thirds of farthings.
3024	

Here again we reduce the left hand quantity to thirds of farthings; we must therefore reduce the right hand one to the same denomination. We then place the latter under the former, and find that the terms of the fraction contain no common factor; and we therefore perceive it is in its lowest term.

Reduce

(1)	$\frac{2}{3}$ of a lb. to the fraction of a cwt.		*Ans*. $\frac{1}{168}$.
(2)	14*s*. $8\frac{1}{2}+\frac{2}{3}$ to	,, a £.	*Ans*. $\frac{53}{72}$.
(3)	7*s*. 8*d*. to	,, a guinea.	*Ans*. $\frac{23}{63}$.
(4)	9*s*. $10\frac{1}{2}$*d*. to	,, a moidore.	*Ans*. $\frac{79}{216}$.
(5)	16*s*. $11\frac{3}{4}$*d*.$+\frac{1}{9}$ to	,, a crown.	*Ans*. $3\frac{107}{270}$.
(6)	$\frac{1}{5}$ of $9\frac{1}{2}$*d*. to	,, a shilling.	*Ans*. $\frac{19}{120}$.
(7)	$4\frac{3}{4}$*d*. to	,, a half guinea.	*Ans*. $\frac{19}{504}$.
(8)	$\frac{1}{2}$*d*. to	,, of $\frac{5}{9}$ of a £.	*Ans*. $\frac{3}{1000}$.
(9)	$1\frac{3}{4}$*d*. to	,, a guinea.	*Ans*. $\frac{7}{1008}$.
(10)	$\frac{1}{5}$ of $1\frac{1}{2}$*d*. to	,, a noble.	*Ans*. $\frac{3}{800}$.
(11)	5*s*. $9\frac{1}{2}$*d*. to	,, a half crown.	*Ans*. $2\frac{19}{60}$.
(12)	$\frac{5}{960}$ of a guinea to	,, a penny.	*Ans*. $\frac{21}{16}$.

Reduce

(13) $\frac{2}{1980}$ of a £. to the fraction of a farthing. *Ans.* $\frac{32}{33}$.

(14) $\frac{4}{1932}$ of a cwt. to ,, a lb. *Ans.* $\frac{16}{69}$.

(15) $\frac{5}{2968}$ of a tun Wine Measure to ,, a pint. *Ans.* $\frac{180}{53}$.

(16) $\frac{3}{152}$ of a cwt. to ,, a half ounce. *Ans.* $\frac{1344}{19}$.

(17) $\frac{27}{372}$ of a moidore to ,, a half farthing. *Ans.* $\frac{5832}{31}$.

(18) $\frac{13}{729}$ of a half guinea to ,, 3 pence. *Ans.* $\frac{182}{243}$.

(19) $\frac{36}{7924}$ of a wey to ,, a pint. *Ans.* $\frac{2340}{1981}$.

(20) $\frac{14}{79}$ of a bushel to ,, a quart. *Ans.* $\frac{448}{79}$.

(21) $\frac{2}{1738}$ of a mile to ,, a half yard. *Ans.* $\frac{3520}{869}=4\frac{54}{869}$.

(22) $\frac{12}{735}$ of a cwt. to ,, a half ounce. *Ans.* $58\frac{18}{35}$.

(23) $\frac{29}{196}$ of a tun of wine to ,, a gill. *Ans.* $1328\frac{8}{11}$.

(24) $\frac{34}{294}$ of a yard to ,, $\frac{1}{20}$ of an inch. *Ans.* $83\frac{13}{49}$.

(25) $\frac{4}{9}$ of a quarter of a lb. Avoir. to ,, $\frac{3}{10}$ of an ounce. *Ans.* $5\frac{25}{27}$.

(26) $\frac{7}{8}$ of a $\frac{1}{2}$ pint to ,, $\frac{1}{44}$ of a gill. *Ans.* $\frac{77}{1}$.

(27) $(\frac{2}{3}+\frac{1}{9})$ of a hhd. Wine Measure to ,, $\frac{3}{17}$ of a $\frac{1}{4}$ of a pint. *Ans.* $\frac{26656}{3}$.

(28) $(\frac{1}{8}$ of $\frac{6}{7})$ of a firkin to ,, $\frac{2}{9}$ of $\frac{1}{2}$ a pint. *Ans.* $\frac{486}{7}$.

(29) $(\frac{2}{3}+\frac{1}{9}$ of $\frac{6}{7})$ of a ton to ,, $\frac{2}{5}$ of $(\frac{3}{4}+\frac{1}{9})$ of a $\frac{1}{2}$ oz. *Ans.* $\frac{4915200}{31}$.

(30) $\frac{2}{9}$ of a gallon to ,, $\frac{1}{5}$ of $\frac{2}{3}$ of a gill. *Ans.* $\frac{160}{3}$.

(31) 2*l*. 18*s*. 6½*d*. to ,, a crown. *Ans.* $\frac{281}{24}$.

(32) 7 half-crowns to ,, half a farthing. *Ans.* $\frac{1680}{1}$.

(33) 1¼*d*. to ,, 2*s*. 8½*d*. *Ans.* $\frac{1}{26}$.

(34) 1*s*. 7¼*d*. to ,, a £. sterling. *Ans.* $\frac{77}{960}$.

(35) 3¾*d*. to ,, $\frac{1}{5}$ of 9*s*. 6*d*. *Ans.* $\frac{25}{152}$.

(36) 7*s*. 8½*d*. to ,, a moidore. *Ans.* $\frac{185}{648}$.

(37) 12 drs. Avoirdupoise to ,, a half cwt. *Ans.* $\frac{3}{3584}$.

(38) 15 lbs. 7 drs. to ,, $\frac{3}{4}$ of 5½ cwt. *Ans.* $\frac{11541}{143360}$.

(39) 2 cwt. 2 qrs. 18 lbs. to ,, $\frac{3}{7}$ of $\frac{1}{9}$ of a ton. *Ans.* $\frac{447}{160}$.

Reduce

(40) $2\frac{3}{4}$ lbs. to the fraction of $27\frac{1}{2}$ lbs. *Ans.* $\frac{1}{10}$.

(41) 5 hhds. Wine Measure to " $\frac{1}{2}$ of $\frac{1}{8}$ of a pint. *Ans.* 40320.

(42) $2\frac{1}{2}$ hhds. Wine Measure to " 842 gal. *Ans.* $\frac{315}{1684}$.

(43) 21 qs. 1 pint Wine Measure to " $\frac{1}{2}$ a hhd. *Ans.* $\frac{43}{252}$.

(44) $\frac{35}{798}$ of a rundlet to " $\frac{3}{5}$ of a tun. *Ans.* $\frac{25}{4788}$.

(45) $\frac{7}{9}$ of 2 pecks to " $\frac{5}{9}$ of a chaldron. *Ans.* $\frac{7}{360}$.

CASE 10.

Addition of Fractions.

RULE.—Prepare the fractions; *i.e.*, reduce mixed numbers to mproper fractions, or compound to simple ones. Reduce the ractions to a common denominator by Case 1; and add all the umerators together, reducing the fraction (if requisite) to a nixed number.

Ex. Add $\frac{5}{9}$, $\frac{7}{12}$, and $\frac{4}{15}$ together.

$$3)\underline{9.\ 12.\ 15}\ \ \ 180\text{ C. M.}$$
$$3.\ 4.\ 5$$

$$\frac{20}{180}+\frac{105}{180}+\frac{48}{180}=\frac{173}{180}$$

Here we reduce the fractions to a common denominator, $\frac{20}{180}$, $\frac{105}{180}$, $\frac{48}{180}$; and we then add the numerators together, and find the sum to be $\frac{173}{180}$.

But if there are any mixed numbers in the question, it is better o add the integers together in one sum, and the fractions in nother; the sum of the 2 parts is the sum required.

Ex. Add $5\frac{1}{2}$, $3\frac{3}{4}$, $7\frac{1}{5}$, and $19\frac{2}{3}$ together.

$$5+3+7+19 = 34$$

$$\frac{1}{2}+\frac{3}{4}+\frac{1}{5}+\frac{2}{3}=\frac{30}{60}+\frac{45}{60}+\frac{12}{60}+\frac{40}{60}+\frac{127}{60}=2\frac{7}{60}$$

Here we first add the integers together, and obtain for heir sum 34. We then add the fractions together by the above ule, and obtain for sum $2\frac{7}{60}$, which added to the 34 before obained $=36\frac{7}{60}$.

Add

(1) $\frac{1}{5}$, $\frac{2}{3}$, $\frac{7}{9}$, and $68\frac{1}{4}$ together. *Ans.* $69\frac{161}{180}$.

(2) $\frac{1}{2}$, $\frac{2}{9}$, $\frac{5}{6}$, and $\frac{8}{13}$ together. *Ans.* $1\frac{1217}{1872}$.

(3) $\frac{5}{9}$ of 12 to $\frac{3}{5}$ of 19. *Ans.* $18\frac{1}{15}$.

(4) $\frac{1}{9}$ of 17 to $\frac{8}{7}$ of 35. *Ans.* $41\frac{8}{9}$.

Add

(5) $\frac{1}{5}$, $\frac{2}{7}$, and $8\frac{1}{4}$ together. *Ans.* $8\frac{103}{140}$.

(6) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{2}{5}$ together. *Ans.* $2\frac{29}{210}$.

(7) $\frac{2}{7}$ of $\frac{1}{2}$ of $\frac{5}{9}$ to $8\frac{3}{17}$. *Ans.* $8\frac{114}{1091}$.

(8) $\frac{2}{3}$ of $9\frac{1}{2}$ to $\frac{4}{9}$ of $\frac{2}{6}$ of $\frac{1}{2}$ of $\frac{12}{17}$. *Ans.* $6\frac{59}{153}$.

(9) $\frac{1}{5}$ of $\frac{1}{16}$ to $\frac{2}{16}$ of $\frac{7}{8}$ of $\frac{6}{13}$ of $\frac{2}{16}$ of $\frac{8}{9}$ of $\frac{7}{12}$ of $\frac{2}{13}$ of $\frac{5}{7}$.

Ans. $\frac{2917}{67600}$.

(10) $\frac{1}{5}$ of 27 to $\frac{2}{9}$ of $6\frac{1}{2}$. *Ans.* $6\frac{38}{45}$.

(11) $\frac{4}{7}$ of $96\frac{1}{4}$ to $\frac{5}{9}$ of $\frac{11}{12}$ of $5\frac{1}{6}$. *Ans.* $59\frac{118}{567}$.

(12) $\frac{1}{7}$ to $\frac{2}{3}$ of $\frac{5}{9}$ of $6\frac{1}{2}$. *Ans.* $2\frac{104}{189}$.

(13) $\frac{1}{2}$, $\frac{2}{7}$, and $\frac{11}{12}$ together. *Ans.* $2\frac{213}{420}$.

(14) $\frac{2}{7}$ of 6 to $\frac{4}{5}$ of $2\frac{1}{2}$. *Ans.* $3\frac{5}{7}$.

(15) $\frac{1}{9}$ of $6\frac{3}{4}$ to $\frac{8}{17}$ to $\frac{6}{7}$ of $8\frac{1}{2}$. *Ans.* $4\frac{5}{28}$.

(16) $\frac{2}{3}$ of $\frac{6}{7}$ to $\frac{8}{9}$ of $\frac{2}{3}$ of $9\frac{1}{4}$. *Ans.* $6\frac{10}{189}$.

(17) $\frac{2}{3}$, $\frac{5}{7}$, $\frac{6}{8}$, $\frac{5}{9}$, and $\frac{9}{12}$ together. *Ans.* $3\frac{55}{126}$.

(18) $\frac{9}{10}$ of 5 to $\frac{3}{5}$ of 16 of $\frac{2}{5}$ of 8. *Ans.* $144\frac{1}{25}$.

(19) $\frac{1}{2}$ of 96 of $\frac{8}{13}$ to $\frac{9}{19}$ of $\frac{12}{13}$ of $\frac{19}{8}$. *Ans.* $30\frac{15}{26}$.

(20) $\frac{1}{2}$ of $6\frac{1}{4}$ to $\frac{1}{2}$ of 63. *Ans.* $34\frac{5}{8}$.

(21) $\frac{2}{9}$ of 5 to $\frac{1}{2}$ of $\frac{6}{7}$ of $9\frac{1}{4}$. *Ans.* $5\frac{19}{252}$.

(22) $\frac{6}{7}$ of $\frac{5}{9}$ to $\frac{3}{5}$ of $\frac{13}{8}$ of $\frac{9}{16}$ to 1. *Ans.* $1\frac{38}{63}$.

(23) $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{5}{6}$ to $3\frac{3}{4}$. *Ans.* $3\frac{13}{42}$.

(24) $\frac{2}{7}$ of 5 to $\frac{6}{13}$ of 98 of $\frac{4}{7}$. *Ans.* $27\frac{25}{91}$.

(25) $\frac{7}{9}$, $\frac{5}{8}$, $\frac{6}{12}$, $\frac{8}{18}$, and $\frac{5}{7}$ together. *Ans.* $3\frac{31}{504}$.

(26) $\frac{1}{2}$ of 9, $\frac{2}{3}$ of 7, and $\frac{5}{9}$ together. *Ans.* $9\frac{13}{18}$.

(27) $\frac{1}{5}$, $\frac{2}{3}$, $8\frac{1}{2}$, and $\frac{5}{7}$ together. *Ans.* $10\frac{17}{210}$.

(28) $\frac{2}{3}$ of $\frac{5}{9}$ of $2\frac{1}{2}$ to $\frac{5}{6}$ of 8. *Ans.* $7\frac{16}{27}$.

(29) $\frac{2}{3}$ of 21 to $\frac{9}{13}$ of $6\frac{1}{2}$. *Ans.* $18\frac{1}{2}$.

(30) $\frac{2}{3}$ of a hhd. of wine to $\frac{5}{9}$ of a gallon.

Ans. 42 gal. 2 qt. 0 pt. 1 gill. + $\frac{7}{9}$.

(31) $\frac{5}{9}$ of $2\frac{1}{2}$ butts of beer to $\frac{8}{9}$ of a kilderkin.

Ans. 1 butt 1 b. 1 k. 4 galls.

(32) $\frac{6}{13}$ of a cwt. to $\frac{7}{9}$ of 1 oz. *Ans.* 51 lbs. 11 oz. 13 drs. + $\frac{79}{117}$.

(33) $\frac{2}{3}$ of $\frac{5}{9}$ of $5\frac{1}{2}$ tons to $4\frac{1}{2}$ oz.

Ans. 3 t. 5 c. 0 q. 21 lbs. 0 oz. 1 dr. $\frac{17}{27}$.

Add

34) $\frac{1}{4}$ of a butt of beer to $\frac{7}{9}$ of a gal. *Ans.* 18 gal. $3\frac{1}{9}$ qt.

35) $\frac{2}{3}$ of a rundlet to $\frac{4}{5}$ of a pipe. *Ans.* 112 gal. $3\frac{1}{5}$ qt.

36) $\frac{2}{5}$ of a pint to $\frac{2}{7}$ of a tun of wine. *Ans.* 72 gal. $\frac{1}{5}$ qt.

37) $\frac{4}{9}$ of $\frac{1}{2}$ a hhd. to $\frac{2}{5}$ of $\frac{3}{4}$ of a gallon. *Ans.* 14 gal. $1\frac{1}{5}$ qt.

38) $\frac{1}{15}$ of a tun to $\frac{7}{8}$ of $\frac{1}{2}$ a gallon. *Ans.* 17 gal. $\frac{19}{80}$ qt.

39) $\frac{1}{7}$ of $\frac{2}{9}$ of a barrel of wine to $\frac{1}{9}$ of a pipe. *Ans.* 14 gal.

40) $\frac{2}{5}$ of $\frac{1}{2}$ a hhd. to $\frac{2}{9}$ of $\frac{1}{4}$ of a gallon. *Ans.* 12 gal. $2\frac{28}{45}$ qt.

Subtraction of Fractions.

RULE 1.—Prepare the fractions as in addition, then subtract he less numerator from the greater, underneath which place the ommon denominator.

Ex. From $5\frac{1}{6}$ take $\frac{5}{7}$.

$\frac{1}{6}=\frac{31}{6}$

$\therefore \frac{31}{6}-\frac{5}{7}=\frac{217}{42}-\frac{30}{42}=\frac{187}{42}=4\frac{19}{42}$

Here we first reduce the mixed number to an improper fraction, and then we place he fraction with the sign — before the one that is to be subtracted, r as in common subtraction, place the subtrahend after the ninuend; then we reduce the fractions to a common denominator, aking the numerator of the subtrahend from the numerator of he minuend, and subfixing the common denominator. We then btain the improper fraction $\frac{187}{42}$ which we turn into a mixed umber $4\frac{19}{42}$. In cases where there are several quanties with the ign + before them; and several others with the sign — before hem; it is better to collect all the quantities with the sign + efore them, into one simple fraction; and all the quantities with he sign — into another; then proceed as before.

Ex. From $\frac{2}{3}+\frac{5}{6}-\frac{3}{4}-\frac{5}{9}$ take $\frac{5}{9}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$.

$\frac{2}{3}+\frac{5}{6}=\frac{12+15-37}{18}=\frac{37}{18}$

and $(\frac{3}{4}+\frac{5}{9})=-(\frac{27+20-47}{36})=-\frac{47}{36}$.

Ience $\frac{37}{18}{}^{7}-\frac{47}{36}=\frac{74-47}{36}=\frac{27}{36}=\frac{3}{4}$ minuend.

2dly.

$\frac{5}{9}+\frac{1}{3}=\frac{8}{9}$

$-(\frac{1}{2}+\frac{1}{4}=\frac{3}{4})$

Ience $\frac{8}{9}-\frac{3}{4}=\frac{32-27}{36}=\frac{5}{36}$ subtrahend.

$\therefore \frac{3}{4}-\frac{5}{36}=\frac{27}{36}-\frac{5}{36}=\frac{22}{36}=\frac{11}{18}$. *Ans.*

Here we see that the minuend contains quantities affected with the sign +, and others with the sign —. We therefore add together all those with +; and also all those with —. We find the former = $\frac{7}{8}$, and the latter $\frac{47}{36}$; and obtain for difference $\frac{3}{4}$; which is the minuend of the simple fraction. We proceed exactly in the same manner with the subtrahend, and obtain $\frac{5}{36}$. We have then to find the difference between $\frac{3}{4}$ and $\frac{5}{36}$, which is $\frac{11}{8}$, or $1\frac{3}{8}$.

(1) From $\frac{1}{3}$ of 9 take $\frac{2}{3}$ of 2. *Ans.* $3\frac{1}{16}$
(2) " $\frac{2}{3}$ take $\frac{1}{19}$ of $\frac{1}{14}$. *Ans.* $\frac{529}{798}$.
(3) " $\frac{1}{6}$ of $9\frac{1}{2}$ take $\frac{2}{3}$ of $2\frac{1}{6}$. *Ans.* $\frac{7}{36}$.
(4) " $\frac{2}{3}$ of $76\frac{1}{2}$ take $\frac{2}{9}$ of $13\frac{1}{4}$. *Ans.* $48\frac{1}{8}$.
(5) " $\frac{1}{8}$ of 96 take $\frac{3}{14}$ of $37\frac{1}{8}$. *Ans.* $2\frac{117}{560}$.
(6) " 26 take $3\frac{1}{2}$. *Ans.* $22\frac{1}{2}$.
(7) " $22\frac{1}{4}$ take $\frac{1}{2}$ of 6. *Ans.* $19\frac{1}{4}$.
(8) " $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{16}$ of $\frac{12}{4}$ take $\frac{1}{2}$. *Ans.* $\frac{11}{16}$.
(9) " $2\frac{1}{2}$ take $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{2}$. *Ans.* $2\frac{1}{3}$.
(10) " $5\frac{1}{4}$ take $\frac{1}{2}$ of $9\frac{1}{4}$. *Ans.* $\frac{5}{8}$.
(11) " $6\frac{3}{4}$ take $\frac{5}{9} + 2\frac{1}{4}$. *Ans.* $3\frac{17}{18}$.
(12) " $\frac{2}{3}$ of $\frac{2}{7}$ of $\frac{2}{3}$ take 1. *Ans.* $-\frac{55}{63}$.
(13) " $\frac{5}{9}$ of 2 take $\frac{4}{7}$ of $\frac{1}{3} + \frac{1}{9}$. *Ans.* $\frac{29}{35}$.
(14) " $\frac{1}{7}$ of $24\frac{1}{2}$ take $\frac{1}{9}$ of $\frac{5}{6}$. *Ans.* $3\frac{3}{14}$.
(15) " $\frac{2}{3}$ of 9 take $\frac{5}{6}$ of $12\frac{1}{4}$. *Ans.* $-\frac{29}{36}$.
(16) " 99 take $\frac{1}{9}$ of 99 of $\frac{1}{99}$. *Ans.* $98\frac{8}{9}$.
(17) " $\frac{2}{3}$ of 71 take $5\frac{1}{4}$. *Ans.* $42\frac{1}{12}$.
(18) " $\frac{1}{3}$ of $34\frac{1}{4}$ take $\frac{2}{7}$ of 9. *Ans.* $8\frac{71}{84}$.
(19) " $\frac{2}{3}$ of $\frac{3}{5}$ of $18\frac{1}{4}$ take $\frac{1}{6}$ of $13\frac{1}{4}$. *Ans.* $5\frac{149}{180}$.
(20) " $\frac{5}{7}$ of $16\frac{1}{2}$ take $\frac{2}{15}$ of $7\frac{1}{4}$. *Ans.* $10\frac{61}{91}$.
(21) " $\frac{5}{9}$ of 13 take $\frac{2}{7}$ of 9. *Ans.* $4\frac{41}{63}$.
(22) " $\frac{3}{5}$ take $\frac{1}{3}$. *Ans.* $\frac{4}{15}$.
(23) " $\frac{1}{9}$ of 16 take $\frac{1}{2}$. *Ans.* $1\frac{5}{18}$.
(24) " $\frac{2}{9}$ of $\frac{3}{4}$ of 6 take $\frac{1}{4}$ of 9. *Ans.* $-1\frac{1}{4}$.
(25) " $\frac{25}{9}$ take $\frac{2}{7}$ of $\frac{6}{9}$ of $\frac{5}{2}$. *Ans.* $2\frac{19}{63}$.
(26) " $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{9}$ take $1\frac{1}{7}$. *Ans.* $-\frac{109}{126}$.
(27) " $\frac{2}{7}$ of $91\frac{1}{2}$ take $\frac{2}{5}$ of $6\frac{1}{4}$. *Ans.* $23\frac{9}{14}$.
(28) " $\frac{5}{7}$ take $\frac{1}{2}$ of $\frac{2}{9}$ of $\frac{3}{14}$. *Ans.* $\frac{29}{42}$.
(29) " 16 take $\frac{1}{3}$ of $22\frac{1}{2}$. *Ans.* 8.
(30) " $\frac{2}{3}$ of $\frac{135}{4}$ take $\frac{1}{18}$ of $28\frac{1}{2}$. *Ans.* $5\frac{23}{36}$.

) From $\frac{2}{3}$ of $\frac{3}{4}$ take $\frac{1}{7}$. *Ans.* $\frac{5}{14}$.

2) ,, $\frac{1}{7}$ of $24\frac{1}{2}$ take $\frac{3}{4}$. *Ans.* $2\frac{3}{4}$.

3) ,, $\frac{1}{7}$ of $\frac{1}{5}$ of $\frac{20}{3}$ take 2. *Ans.* $-1\frac{17}{21}$.

4) ,, $\frac{1}{8}$ of 96 take $\frac{1}{2}$ of 12. *Ans.* 6.

5) ,, 5 take $\frac{2}{5}$ of $6\frac{1}{2}$. *Ans.* $2\frac{2}{5}$.

6) ,, $73\frac{1}{4}$ take 9 *Ans.* $64\frac{1}{4}$.

7) ,, $3\frac{1}{2}$ take $\frac{5}{9}$ of $\frac{2}{7}$. *Ans.* $3\frac{43}{126}$.

8) ,, $54\frac{1}{2}$ take $\frac{1}{5}$ of 7. *Ans.* $53\frac{1}{10}$.

9) ,, $\frac{1}{5}$ of 6 take $\frac{4}{7}$ of $2\frac{3}{4}$. *Ans.* $-\frac{13}{35}$.

0) ,, $\frac{2}{3}$ of a lb. Avoirdupoise take $\frac{8}{9}$ of a dram.
Ans. 10 oz. $9\frac{7}{9}$ drs.

1) ,, $\frac{4}{7}$ of $\frac{5}{9}$ of a lb. Troy take $\frac{2}{9}$ of a grain.
Ans. 3 oz. 16 dwts. $4\frac{22}{63}$ grs.

2) ,, $\frac{5}{9}$ of a bushel take $\frac{2}{3}$ of a peck. *Ans.* 1 p. $1\frac{1}{9}$ g.

3) ,, $\frac{2}{3}$ of a pipe of wine take $\frac{7}{8}$ of a pint.
Ans. 83 g. 3 q. 1 p. + $\frac{1}{8}$.

4) ,, $\frac{5}{6}$ of a tun take $\frac{7}{9}$ of a rundlet. *Ans.* 3 hhds. 7 gals.

5) ,, $\frac{9}{17}$ of a cwt. take $\frac{2}{3}$ of an oz. *Ans.* 59 lbs. 4 ozs. + $\frac{2}{51}$.

6) ,, $\frac{5}{12}$ of a mile take $\frac{6}{25}$ of a foot. *Ans.* 733 yds. $\frac{19}{25}$.

7) ,, $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{4}{9}$ of $\frac{6}{7}$ of a barrel of beer take $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{7}$ of $9\frac{1}{2}$. *Ans.* 8 gal. 2 qu. $1\frac{31}{42}$ p.

8) ,, $\frac{6}{7}$ of $\frac{9}{10}$ of $\frac{5}{6}$ of $8\frac{1}{2}$ of a pipe take $\frac{2}{9}$ of 5 gallons.
Ans. 10 hhds. 57 gals. + $\frac{7}{18}$.

9) ,, $\frac{7}{12}$ of 6 lbs. tea take $\frac{2}{5}$ of $\frac{8}{9}$ of an oz. *Ans.* $55\frac{29}{45}$ gs.

0) ,, $\frac{9}{16}$ of a gallon take $\frac{3}{7}$ of $\frac{2}{3}$ of a gill. *Ans.* $17\frac{5}{7}$ gills.

MULTIPLICATION.

Rule.—1. If the fractions are simple fractions, place them th the sign for multiplication between them, then cancel any merator into a denominator, or *vice versa*. The product of all uncancelled numerators will be the numerator of the reduced ction, and the product of all the uncancelled denominators will the denominator.

Ex.—Multiply $\frac{5}{9}$, $\frac{8}{17}$, $\frac{9}{32}$, $\frac{4}{19}$, $\frac{19}{24}$, $\frac{64}{9}$, together.

Here we perceive tnat 32 denominator will divide into 64 numerator, and give quotient 2; then 19 denominator into 19 numerator; 9 denominator into 9 numerator; 8 numerator into 24 denominator. The numerators and denominators being prime to each other, we therefore multiply all the uncancelled numerators together, and obtain 40; we also multiply all the uncancelled denominators together, and obtain 459: the product therefore is $\frac{40}{459}$.

$$\frac{5}{9}\times\frac{\cancel{9}}{17}\times\frac{\cancel{8}}{\cancel{24}}\times\frac{4}{\cancel{19}}\times\frac{\cancel{19}}{\underset{3}{\cancel{32}}}\times\frac{\overset{2}{\cancel{64}}}{\cancel{9}}=\frac{40}{459}$$

RULE 2.—But if the given fractions are mixed numbers, reduce them to improper fractions, and cancel as before.

Ex.—Multiply $4\frac{2}{3}$, $4\frac{1}{11}$, and $12\frac{5}{6}$ together.

Here we reduce the mixed numbers and obtain $\frac{14}{3}$, $\frac{45}{11}$, and $\frac{77}{6}$; we perceive that 11 denominator will divide into 77 numerator, giving 7 for quotient. We also perceive that 6 contains a factor 2, which is common to 14 numerator, we therefore divide each of these by 2 and obtain for numerator 7 and for denominator 3; we then divide the 3 denominator into 45 numerator, and obtain for quotient 15; we then divide 15 numerator by 3 denominator, and obtain for quotient 5. We then perceive we have cancelled all the denominators; we therefore multiply all the numerators together and obtain for product $\frac{245}{1}$ or 245, which is the answer.

$$4\tfrac{2}{3},\quad 4\tfrac{1}{11},\quad 12\tfrac{5}{6}$$

$$\frac{\overset{7}{\cancel{14}}}{\cancel{3}}\times\frac{\overset{\overset{5}{\cancel{15}}}{\cancel{45}}}{\cancel{11}}\times\frac{\overset{7}{\cancel{77}}}{\underset{\cancel{3}}{\cancel{6}}}=245$$

(1) Multiply $\frac{1}{9}$ of 7 by $\frac{2}{3}$ of 16. *Ans.* $8\frac{8}{27}$.

(2) ,, $\frac{2}{3}$ of 16 by $\frac{1}{4}$ of $99\frac{1}{2}$. *Ans.* $265\frac{1}{3}$.

(3) ,, $23\frac{1}{2}$ by $12\frac{1}{4}$. *Ans.* $287\frac{7}{8}$.

(4) ,, $67\frac{3}{4}$ by $\frac{2}{3}$ of $\frac{9}{10}$ of $6\frac{1}{4}$. *Ans.* $508\frac{1}{8}$.

(5) ,, $13\frac{3}{4}$ by $\frac{1}{5}$ of $\frac{9}{10}$ of $\frac{11}{12}$. *Ans.* $2\frac{43}{160}$.

(6) ,, $19\frac{1}{2}$ by $\frac{5}{6}$ of $\frac{7}{8}$ of $19\frac{1}{2}$. *Ans.* $\frac{17745}{64}$.

(7) ,, $\frac{2}{3}$ of $6\frac{1}{4}$ by $\frac{1}{5}$ of $9\frac{3}{4}$. *Ans.* $271\frac{1}{64}$.

(8) ,, $2\frac{1}{8}$ by $\frac{1}{7}$ of 9. *Ans.* $8\frac{1}{6}$.

(9) ,, $\frac{2}{3}$ of 6 by $\frac{7}{9}$ of $14\frac{3}{4}$. *Ans.* $2\frac{41}{56}$.

(10) ,, 199 by $\frac{5}{9}$ of $\frac{192}{3}$. *Ans.* $45\frac{8}{9}$.

(11) ,, $26\frac{1}{9}$ by $\frac{2}{9}$ of $\frac{11}{13}$ of $\frac{9}{6}$. *Ans.* $7\frac{128}{351}$.

(12) ,, $\frac{5}{9}$ of $\frac{2}{3}$ of $\frac{1}{7}$ of $92\frac{1}{2}$ by $\frac{3}{5}$. *Ans.* $1\frac{471}{504}$.

(13) ,, $47\frac{1}{4}$ by $4\frac{1}{47}$. *Ans.* $190\frac{1}{188}$.

(14) ,, $269\frac{1}{4}$ by $365\frac{1}{9}$. *Ans.* $98366\frac{1}{6}$.

(15) ,, $32\frac{1}{8}$ by $\frac{1}{5}$ of 16. *Ans.* $102\frac{4}{5}$.

) Multiply $\frac{2}{7}$ of 4 by $\frac{2}{4}$ of 7. *Ans.* 4.
) „ $19\frac{1}{2}$ by $\frac{7}{9}$ of $62\frac{1}{9}$. *Ans.* $942\frac{1}{54}$.
) „ $4\frac{1}{2}$ by $\frac{2}{3}$ of 13. *Ans.* 39.
) „ $49\frac{3}{4}$ by $\frac{3}{49}$. *Ans.* $3\frac{9}{196}$.
) „ $79\frac{1}{2}$ by $\frac{5}{8}$ of $16\frac{3}{8}$. *Ans.* $813\frac{81}{128}$
) „ $51\frac{1}{2}$ by $\frac{3}{5}$ of $9\frac{1}{2}$. *Ans.* $293\frac{11}{20}$.
) „ $2\frac{3}{4}$ by $\frac{5}{7}$ of $6\frac{1}{2}$. *Ans.* $12\frac{43}{56}$.
) „ $28\frac{1}{2}$ by $\frac{6}{7}$ of $9\frac{1}{6}$. *Ans.* $233\frac{7}{10}$.
) „ $72\frac{3}{8}$ by $\frac{4}{5}$ of $26\frac{1}{2}$. *Ans.* $1505\frac{3}{5}$.
) „ $\frac{7}{8}$ of $9\frac{1}{2}$ by $\frac{5}{8}$ of 2. *Ans.* $11\frac{7}{8}$.
) „ $\frac{6}{32}$ of $\frac{7}{9}$ of $\frac{16}{14}$ by $\frac{1}{5}$ of 6. *Ans.* $\frac{1}{5}$.
) „ $\frac{2}{3}$ of 7 by $\frac{5}{3}$ of $18\frac{3}{4}$. *Ans.* $145\frac{5}{6}$.
) „ $\frac{7}{9}$ by $\frac{5}{6}$ of $9\frac{1}{2}$. *Ans.* $6\frac{17}{108}$.
) „ $\frac{2}{9}$ by $\frac{3}{4}$ of $7\frac{1}{4}$ of $\frac{5}{6}$. *Ans.* $1\frac{1}{144}$.
) „ $\frac{1}{7}$ of $9\frac{1}{2}$ by $3\frac{3}{4}$. *Ans.* $5\frac{5}{56}$.
) „ $\frac{2}{3}$ of $19\frac{1}{4}$ by $\frac{5}{9}$ of 6. *Ans.* $42\frac{7}{9}$.
) „ $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{2}{7}$ together. *Ans.* $\frac{16}{189}$.
) „ $\frac{1}{5}$, $\frac{7}{8}$, and $\frac{2}{3}$ together. *Ans.* $\frac{7}{60}$.
) „ $\frac{2}{9}$, $\frac{6}{13}$, and $2\frac{1}{3}$ together. *Ans.* $\frac{28}{117}$.
) „ $\frac{1}{4}$ of 8 by $\frac{2}{7}$ of 19. *Ans.* $10\frac{6}{7}$.
) „ $\frac{2}{3}$ by $\frac{1}{2}$ of $16\frac{1}{4}$. *Ans.* $5\frac{5}{12}$.
) „ $\frac{1}{8}$ by $\frac{2}{3}$ of $\frac{1}{6}$ of 7. *Ans.* $\frac{7}{72}$.

DIVISION OF FRACTIONS.

Rule.—Reduce the fractions, if necesssary, as in Multiplica- ; then invert the divisor and cancel; multiply the uncancelled erators for a new numerator and the uncancelled denominators a new denominator.

Ex.—Divide $\frac{4}{5}$ of $\frac{7}{8}$ by $\frac{2}{3} + \frac{9}{10}$.

$\frac{7}{\underset{2}{\cancel{8}}} = \frac{7}{10}$ Dividend.

$\frac{9}{10} = \frac{20}{30} + \frac{27}{30} = \frac{47}{30}$.

$\frac{\overset{3}{\cancel{30}}}{47} = \frac{21}{47}$ Quotient.

Here we first reduce the compound fraction of the dividend into a simple fraction, obtaining for result $\frac{7}{10}$. 2ndly. We then add the two fractions of the divisor together, and obtain for sum

We then invert the divisor, cancel, multiply, and obtain $\frac{21}{47}$.

(1) Divide $26\frac{1}{2}$ by $\frac{5}{9}$. *Ans.* $47\frac{7}{10}$.

(2) „ $12\frac{1}{4}$ by $\frac{8}{9}$. *Ans.* $13\frac{25}{32}$.

(3) „ $\frac{3}{4}$ by $\frac{2}{9}$ of $\frac{5}{8}$. *Ans.* $5\frac{2}{5}$.

(4) „ $\frac{2}{9}$ of $84\frac{1}{4}$ by $\frac{1}{7}$. *Ans.* $131\frac{1}{18}$.

(5) „ $\frac{3}{5}$ of 6 by $\frac{2}{9}$ of 12. *Ans.* $1\frac{1}{20}$.

(6) „ $28\frac{1}{2}$ by $\frac{2}{7}$ of $54\frac{1}{2}$. *Ans.* $1\frac{181}{218}$.

(7) „ $\frac{1}{2}$ of 6 by $\frac{3}{4}$ of 9. *Ans.* $\frac{4}{9}$.

(8) „ $\frac{2}{5}$ of 12 by $\frac{4}{7}$ of 8. *Ans.* $1\frac{1}{20}$.

(9) „ $\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{5}{9}$ of $\frac{11}{12}$. *Ans.* $2\frac{5}{11}$.

(10) „ $36\frac{1}{2}$ by $\frac{5}{9}$ of $\frac{13}{25}$. *Ans.* $126\frac{1}{26}$.

(11) „ $\frac{7}{8}$ of $\frac{16}{81}$ of $12\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{16}{1}$. *Ans.* $\frac{175}{486}$.

(12) „ $\frac{4}{5}$ by $63\frac{1}{2}$. *Ans.* $\frac{8}{635}$.

(13) „ $289\frac{3}{4}$ by $\frac{5}{6}$ of $\frac{7}{1}$. *Ans.* $49\frac{47}{70}$.

(14) „ $\frac{2}{9} \times 8\frac{1}{2}$ by $\frac{2}{3}+\frac{5}{6}$. *Ans.* $1\frac{7}{27}$.

(15) „ $\frac{1}{2}$ of $6\frac{1}{2}$ by 48. *Ans.* $\frac{13}{192}$.

(16) „ 6 by $\frac{5}{9}$ of $\frac{34}{75}$ of 9. *Ans.* $10\frac{3}{7}$.

(17) „ $6\frac{3}{4}$ by $\frac{2}{3}$ of 12. *Ans.* $\frac{27}{32}$.

(18) „ $\frac{1}{9}$ of 8 by $\frac{2}{3}+9$. *Ans.* $\frac{8}{87}$.

(19) „ $6\frac{1}{2}$ by 5. *Ans.* $1\frac{3}{10}$.

(20) „ $\dfrac{\frac{1}{2}}{9}$ by $\dfrac{\frac{5}{6}}{8}$. *Ans.* $\frac{8}{15}$.

(21) „ $\frac{2}{9}$ of 17 by $\frac{3}{8}+9$. *Ans.* $\frac{85}{316}$.

(22) „ $\frac{1}{5}$ of $\frac{7}{10}$ of $\frac{18}{19}$ by $\frac{2}{5}$ of $\frac{13}{19}$. *Ans.* $\frac{63}{130}$.

(23) „ $\frac{2}{3}$ of $\frac{11}{12}$ by $\frac{1}{5}$ of $(9-3\frac{1}{2})$. *Ans.* $\frac{5}{9}$.

(24) „ $\frac{1}{7}$ of $6\frac{1}{4}$ by $\frac{5}{7}$ of $6\frac{3}{4}$. *Ans.* $\frac{5}{27}$.

(25) „ $\frac{2}{3}$ of $6\frac{1}{4}$ by $\frac{5}{9}$ of $7\frac{1}{2}$. *Ans.* 1.

(26) „ $\frac{1}{7}$ of $\frac{3}{8}$ by $\frac{3}{5}+\frac{7}{9}+\frac{11}{12}$. *Ans.* $\frac{40}{3731}$.

(27) „ $\frac{4}{5}$ of $37\frac{1}{2}$ by $\frac{1}{9}$ of $19\frac{1}{2}$. *Ans.* $13\frac{11}{13}$.

(28) „ $84\frac{1}{4}$ by $\frac{15}{8}$. *Ans.* $44\frac{14}{15}$.

(29) „ $24\frac{1}{2}$ by $\frac{6}{7}$. *Ans.* $28\frac{7}{12}$.

(30) „ $4\frac{1}{2}$ by $\frac{5}{9}$ of 11. *Ans.* $\frac{81}{110}$.

(31) „ $3\frac{1}{2}$ by $\frac{5}{9}$ of $\frac{68}{3}$. *Ans.* $\frac{189}{680}$.

(32) „ $\frac{4}{7}$ of 18 by $\frac{1}{5}$ of 12. *Ans.* $4\frac{2}{7}$.

33) Divide $\frac{1}{9}$ of 84 by $\frac{2}{7}$ of $\frac{3}{4}$. *Ans.* $43\frac{5}{9}$.
34) „ $\frac{1}{7}$ of 18 by $\frac{1}{5}$ of 12. *Ans.* $1\frac{1}{14}$.
35) „ $\frac{2}{9}$ of 6 by $\frac{1}{5}$ of $27\frac{1}{4}$. *Ans.* $\frac{80}{327}$.
36) „ $\frac{1}{7}$ of 8 by $\frac{2}{3}$ of $12\frac{1}{4}$. *Ans.* $\frac{48}{343}$.
37) „ 24 by $\frac{1}{5}$ of $\frac{7}{8}$ of $\frac{2}{3}$. *Ans.* $205\frac{5}{7}$.
38) „ $\frac{1}{5}$ of 9 by $\frac{2}{7}$ of 96. *Ans.* $\frac{21}{320}$.
39) „ $48\frac{1}{2}$ by $\frac{2}{9} + \frac{3}{5}$ of 6. *Ans.* $19\frac{55}{59}$.
40) „ $6\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{9}{10} + \frac{8}{17}$. *Ans.* $6\frac{371}{559}$.
41) „ $2\frac{1}{3}$ (by $\frac{5}{9} + \frac{6}{35}$ of 9.) *Ans.* $7\frac{7}{80}$.
42) „ $43\frac{1}{4}$ by $\frac{5}{7}$ of $29\frac{1}{2}$. *Ans.* $2\frac{31}{590}$.
43) „ $\frac{2}{7}$ of $9\frac{1}{2}$ by $\frac{1}{5}$ of 6. *Ans.* $2\frac{11}{42}$.
44) „ $\frac{5}{7}$ of 12 by $\frac{1}{2}$. *Ans.* $17\frac{1}{7}$.
45) „ $369\frac{13}{21}$ by $79\frac{3}{5}$. *Ans.* $4\frac{2789}{4179}$.
46) „ $2098\frac{12}{19}$ by $139\frac{4}{17}$. *Ans.* $15\frac{32578}{12564}$.
47) „ $3695\frac{15}{27}$ by $356\frac{7}{19}$. *Ans.* $10\frac{22550}{60939}$.
48) „ $4984\frac{11}{19}$ by $298\frac{9}{16}$. *Ans.* $16\frac{63104}{90763}$.
49) „ $2749\frac{12}{13}$ by $396\frac{11}{17}$. *Ans.* $6\frac{81770}{87659}$.

RULES FOR COMPLEX FRACTIONS.

Rule.—Reduce all the fractions contained in the numerator and denominator to simple ones; and the best method to do this is to link the different numerators and denominators together by a brace, as seen below.

$$\text{Ex.—Add } \frac{1}{13} \text{ of } \left.\frac{\left.\frac{3\frac{1}{4}}{\frac{1}{5}}\right\}}{\frac{7\frac{1}{2}}{\left.\frac{3\frac{1}{7}}{\frac{5}{9}}\right\}}}\right\} \text{ to } \frac{475}{7} \text{ of } \left.\frac{\left.\frac{2\frac{1}{2}}{\frac{1}{6}\text{ of }\frac{3}{5}}\right\}}{\frac{6\frac{1}{4}}{\left.\frac{\frac{7}{8}}{9\frac{1}{2}}\right\}}}\right\}$$

Here the first fraction is reduced as in the Rule.

$$\frac{\frac{13}{4}\cdot\frac{5}{1}}{\frac{15}{2}+\frac{22}{7}\cdot\frac{9}{5}} = \frac{\frac{13}{4}\cdot\frac{5}{1}}{\frac{\cancel{315}}{2}\cdot\frac{7}{22}\cdot\frac{5}{\frac{9}{5}}} = \frac{\frac{13}{\cancel{4}}}{\cancel{2}}\cdot\frac{\cancel{5}}{1}\cdot\frac{\cancel{2}}{3}\cdot\frac{\overset{11}{\cancel{22}}}{7}\cdot\frac{\cancel{2}}{5}\text{; but we}$$

are to take $\frac{1}{\cancel{13}}$ of $\frac{\cancel{13}\cdot11\cdot3}{5\cdot7} = \frac{11\cdot3}{5\cdot7} = \frac{33}{35}$. 'Reduced 1st fraction.

The second fraction is to be reduced in the same way.

$\dfrac{\cancel{\frac{6}{2}} \cdot \frac{1}{3} \cdot \frac{1}{5}}{\frac{25}{4} \div \frac{7}{8} \cdot \frac{2}{19}} = \dfrac{\frac{1}{2.3}}{\frac{25}{\cancel{4}} \cdot \frac{\cancel{8}}{7} \cdot \frac{19}{\cancel{2}}} = \frac{1}{2.3} \cdot \frac{7}{25.19}$; but we are to take $\cancel{\frac{475}{7}}$ of $\frac{1}{4.3} \cdot \frac{7}{\cancel{25.19}} = \frac{1}{2.3} = \frac{1}{6}$ 2nd fraction.

$\therefore \frac{33}{35} + \frac{1}{6} = \frac{198}{210} + \frac{35}{210} = \frac{233}{210} = 1\frac{23}{210}$.

Rule for Subtraction, Multiplication, and Division, performed in precisely the same manner as in Fractions, after reducing the Complex Fractions to simple ones.

Ex. in Division.

Divide $\dfrac{\left.\frac{12\frac{1}{4}}{\frac{3}{5}}\right\}}{\frac{7}{8}}$ by $\dfrac{\left.\frac{\frac{4}{5} \text{ of } 2\frac{1}{2}}{3}\right\}}{9\frac{1}{3}}$

Here $\frac{\overset{7}{\cancel{49}}}{\cancel{4}} \cdot \frac{5}{3} \cdot \frac{\overset{2}{\cancel{8}}}{\cancel{7}} = \frac{70}{3}$ is the 1st fraction.

And $\frac{\overset{2}{\cancel{4}}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{2}} \cdot \frac{1}{\cancel{3}} \cdot \frac{\cancel{2}}{\underset{14}{\cancel{28}}} = \frac{1}{14}$ is 2nd fraction.

$\therefore \frac{70}{3} \div \frac{1}{14} = \frac{14}{3} \cdot \frac{70}{1} = \frac{980}{3} = 326\frac{2}{3}$ *Ans.*

ADDITION OF COMPLEX FRACTIONS.

(1) Add $\dfrac{\frac{2}{3}}{\frac{4}{9}}$ of $4\frac{1}{2}$ to $\frac{1}{9}$ of $\dfrac{\frac{6\frac{1}{2}}{3\frac{1}{7}}}{9\frac{1}{2}}$ *Ans.* $6\frac{5825}{7524}$.

(2) Add $\dfrac{\frac{\frac{5}{7}}{6\frac{1}{2}}}{5\frac{3}{4}}$ to $\frac{5}{9}$ of $\dfrac{6\frac{1}{2}}{\frac{7\frac{1}{4}}{5}}$ *Ans.* $2\frac{278344}{346275}$.

(3) Add $\dfrac{\frac{\frac{1}{7} \text{ of } 2}{\frac{5}{9}}}{7\frac{1}{2}}$ to $\dfrac{\frac{\frac{\frac{27}{9}}{6\frac{1}{2}}}{5}}{7\frac{1}{2}}$ *Ans.* $2\frac{856}{2275}$.

(4) Add $\dfrac{\frac{\frac{12}{19}}{\frac{7}{8}}}{\frac{9\frac{1}{2}}{3\frac{3}{8}}}$ to 6. *Ans.* $6\frac{648}{2527}$

(5) Add $\dfrac{\frac{2}{9} \text{ of } \frac{53}{8}}{\frac{2\frac{1}{2}}{15}}$ to $\frac{1}{9}$ of $\dfrac{\frac{6\frac{1}{2}}{7}}{\frac{13\frac{1}{4}}{9}}$ *Ans.* $8\frac{6053}{6675}$.

(6) Add $\dfrac{\dfrac{2\frac{1}{5}}{7}}{\dfrac{12\frac{1}{4}}{3}}$ to $\dfrac{\dfrac{\frac{2}{7}\text{ of }6\frac{1}{2}}{\frac{3}{7}}}{\frac{5}{9}}$ *Ans.* $7\frac{1504}{1715}$.

(7) Add $\dfrac{\dfrac{\frac{3}{4}\text{ of }2\frac{1}{2}}{7\frac{1}{2}}}{\dfrac{1\frac{1}{4}}{3}}$ to $\dfrac{\dfrac{\frac{1}{2}}{\frac{7}{9}}}{\dfrac{\frac{3}{4}}{\dfrac{5\frac{1}{2}}{2}}}$ *Ans.* $\frac{351}{385}$.

(8) Add $\frac{2}{3}$ of $\dfrac{\dfrac{6\frac{1}{8}}{7}}{\dfrac{3\frac{1}{4}}{9}}$ to $\frac{1}{7}$ of $\frac{2}{5}$ of $\dfrac{\dfrac{9}{8\frac{1}{2}}}{\dfrac{5\frac{1}{2}}{\dfrac{\frac{6}{7}}{3\frac{1}{2}}}}$ *Ans.* $1\frac{261832}{416975}$.

(9) Add $\frac{1}{2}+\dfrac{\dfrac{2\frac{3}{4}}{7}}{\dfrac{12}{\frac{2}{3}}}$ to $\dfrac{\frac{5}{6}}{\frac{7}{8}}$ of $2\frac{1}{2}+\dfrac{3\frac{1}{2}}{\dfrac{7}{2\frac{1}{4}}}$ *Ans.* $3\frac{521}{1008}$.

(10) Add $7+\left(\frac{2}{3}\text{ of }\dfrac{9\frac{1}{2}}{\dfrac{7}{\frac{3}{4}}}\right)$ to $\dfrac{\dfrac{\frac{3}{2}}{5}}{\dfrac{6\frac{1}{2}}{\dfrac{2\frac{1}{8}}{3}}}$ *Ans.* $7\frac{2589}{3640}$.

(11) Add $\dfrac{\dfrac{\frac{2}{3}}{7}}{\dfrac{\frac{2}{3}\text{ of }7\frac{1}{2}}{\dfrac{5}{\frac{13}{5}}}}$ to $\frac{2}{9}$ of $\dfrac{\dfrac{\frac{5}{6}}{7}}{\dfrac{\dfrac{3\frac{1}{4}}{\frac{5}{9}}}{\dfrac{6\frac{1}{2}}{3}}}$ *Ans.* $\frac{21742}{464373}$.

(12) Add $\frac{2}{3}$ of 6 to $\frac{3}{4}$ of $\dfrac{\dfrac{8\frac{1}{2}}{6}}{\dfrac{7\frac{1}{4}}{\dfrac{5\frac{1}{8}}{3}}}$ *Ans.* $4\frac{697}{2784}$.

(13) Add $\dfrac{\dfrac{\frac{1}{5} \text{ of } \dfrac{\frac{3}{2}}{7}}{22\frac{1}{8}}}{\dfrac{\frac{3}{5} \text{ of } (\frac{4}{9}+\frac{7}{8})}{\dfrac{\frac{2}{3}}{\frac{5}{9}}}}$ to $6\frac{1}{13}$ *Ans.* $6\frac{203663}{2550275}$.

(14) Add $\dfrac{57\frac{1}{4}}{\dfrac{\frac{3}{5} \text{ of } 8}{\frac{2}{3}}}$ to $\frac{5}{7}$ of $\dfrac{\dfrac{\frac{3}{4}}{\dfrac{64\frac{1}{2}}{32\frac{1}{4}}}}{\dfrac{7}{\dfrac{3\frac{1}{2}}{2\frac{1}{9}}}}$ *Ans.* $8\frac{71}{4788}$.

(15) Add $\frac{1}{7}$ of $\dfrac{\dfrac{99\frac{1}{2}}{6}}{\dfrac{3\frac{1}{4}}{5}}$ to $\frac{2}{3}$ of $\dfrac{\dfrac{14}{\frac{3}{5}}}{\dfrac{\frac{1}{7}}{2}}$ *Ans.* $221\frac{351}{819}$.

(16) Add $\dfrac{\frac{2}{3}}{\dfrac{5}{7\frac{1}{2}}}$ to $\dfrac{3\frac{1}{2}}{\dfrac{6}{5\frac{1}{2}}}$ *Ans.* $\frac{77}{24} = 3\frac{5}{24}$.

(17) Add $\dfrac{\dfrac{3\frac{1}{4}}{\frac{1}{5}}}{\dfrac{7\frac{1}{2}}{\dfrac{3\frac{1}{7}}{\frac{5}{9}}}}$ to $\dfrac{\dfrac{\dfrac{2\frac{1}{2}}{3}}{5}}{\dfrac{6\frac{1}{4}}{\dfrac{\frac{7}{8}}{9\frac{1}{2}}}}$ *Ans.* $1\frac{13}{210}$.

SUBTRACTION OF COMPLEX FRACTIONS.

(1) From $\dfrac{\dfrac{13\frac{1}{9}}{\frac{2}{3}}}{\dfrac{\frac{2}{9} \text{ of } 5\frac{1}{4}}{\dfrac{3\frac{1}{4}}{2}}}$ take $\dfrac{\dfrac{2\frac{1}{5}}{7\frac{1}{8}}}{\dfrac{\dfrac{3\frac{1}{2}}{6}}{2\frac{1}{7}}}$

Ans. $26\frac{963}{5724}$

(2) From $\frac{5}{7}$ of $6\frac{1}{2}$ $\bullet\left\{\dfrac{\dfrac{\dfrac{3\frac{1}{4}}{\frac{5}{2}}}{13\frac{1}{4}}}{\dfrac{16\frac{1}{8}}{5\frac{1}{2}}}\right.$ take $\dfrac{\dfrac{\frac{2}{3}\text{ of }6\frac{1}{2}}{5}}{\dfrac{\dfrac{\frac{1}{7}\text{ of }9\frac{1}{4}}{3\frac{1}{4}}}{7}}$

Ans. $1\frac{774117}{1811964}$

(3) From $\frac{5}{9}$ of 7 $\bullet\left\{\dfrac{\dfrac{2\frac{1}{8}}{3}}{6\frac{1}{4}}\right\}$ take $\dfrac{\dfrac{\frac{3}{4}\bullet\left\{\frac{2}{5}\text{ of }9\right\}}{\frac{7}{3}}}{2\frac{1}{2}\bullet\left\{\dfrac{\dfrac{4\frac{1}{4}}{3}}{\dfrac{2\frac{1}{2}}{7}}\right\}}$

Ans. $\frac{72883}{224910}$

(4) From $\dfrac{\dfrac{\frac{5}{6}+\frac{8}{7}}{3}}{\dfrac{\dfrac{2\frac{1}{2}}{5\frac{1}{3}}}{\frac{7}{8}}}$ take $\dfrac{\dfrac{\frac{6}{7}-\frac{3}{8}}{7}}{\dfrac{2\frac{1}{2}}{9}}$

Ans. $1\frac{66883}{85200}$

(5) From $\dfrac{\dfrac{5\frac{1}{7}}{\frac{3}{2}}}{\dfrac{\frac{1}{2}}{19\frac{1}{3}}}$ take $\dfrac{\dfrac{\frac{2}{5}\text{ of }\frac{6}{15}}{3\frac{1}{4}}}{\dfrac{2\frac{1}{2}}{7}}$

Ans. $132\frac{4932}{11375}$

(6) From $\dfrac{\dfrac{6\frac{3}{4}}{5\frac{1}{4}}}{\dfrac{9}{\frac{2}{3}}}$ take $\dfrac{\dfrac{20\frac{1}{4}}{5\frac{1}{2}}}{\dfrac{3\frac{1}{8}}{7}}$

Ans. $8\frac{478}{5575}$

(7) From $\frac{5}{6}$ of $\left.\dfrac{18\frac{1}{2}}{5}\right\}$ take $\frac{1}{9}$ of $\frac{7}{10}$ of $\dfrac{6\frac{1}{4}}{\dfrac{7}{3\frac{1}{2}}}$

Ans. $3\frac{4}{65}$

(8) From $\cfrac{\cfrac{\frac{1}{3}\text{ of }25}{6\frac{1}{2}}}{\cfrac{\frac{2}{3}}{\cfrac{7\frac{1}{4}}{9}}}$ take $\frac{2}{3}$ of $(\frac{1}{2}+\frac{1}{7})$ of $\cfrac{\cfrac{6\frac{1}{2}+2}{7\frac{1}{3}}}{\cfrac{5\frac{1}{2}}{9}}$

Ans.

MULTIPLICATION OF COMPLEX FRACTIONS.

(9) Multiply $\cfrac{\frac{2}{3}}{9}$ by $\cfrac{\frac{5}{6}}{\frac{7}{8}}$ $\cfrac{\cfrac{2\frac{1}{2}}{3}}{\cfrac{9}{\frac{1}{5}}}$

Ans. $\frac{20}{11907}$

(2) Multiply $\cfrac{\cfrac{5}{\cfrac{\frac{2}{5}}{7\frac{1}{2}}}}{\cfrac{\cfrac{3\frac{3}{4}}{6}}{\cfrac{\frac{2}{3}}{9}}}$ by $\cfrac{\cfrac{\frac{2}{5}}{\frac{3}{9}}}{\cfrac{\frac{7}{8}}{2}}$ *Ans.* $24\frac{24}{35}$

(3) Multiply $\cfrac{\cfrac{5\frac{1}{2}}{3\frac{1}{4}}}{\cfrac{\cfrac{7\frac{1}{2}}{3}}{\cfrac{2}{\frac{1}{9}}}}$ by $\cfrac{\cfrac{\frac{1}{2}\text{ of }6\frac{1}{2}}{3}}{\cfrac{5\frac{1}{4}}{2}}$ *Ans.* $5\frac{13}{455}$

(4) Multiply $\cfrac{\cfrac{\cfrac{3\frac{1}{2}}{5}}{\cfrac{6}{3\frac{1}{3}}}}{\cfrac{\cfrac{2\frac{1}{3}}{8\frac{1}{2}}}{\cfrac{5}{\frac{2}{3}}}}$ by $\cfrac{\cfrac{\cfrac{5\frac{1}{2}}{7\frac{1}{4}}}{3\frac{1}{2}}}{\cfrac{2\frac{1}{8}}{\cfrac{3\frac{1}{4}}{\frac{1}{5}}}}$ *Ans.* $\frac{13728}{13995}=1\frac{533}{1819}$

(5) Multiply $\dfrac{5\frac{1}{2}}{3\frac{1}{4}}$ by $\dfrac{\frac{2}{3}\text{ of }96}{3\frac{1}{2}}$ *Ans.* $\frac{2816}{91}=30\frac{86}{91}$

(6) Multiply $\dfrac{\;\dfrac{\frac{2}{3}}{9}\;}{\frac{1}{2}}$ by $\frac{5}{9}$ of $\dfrac{\;\dfrac{6\frac{1}{2}}{7}\;}{\frac{2}{5}}$ *Ans.* $\frac{65}{3103}$

(7) Multiply $\dfrac{\;\dfrac{3\frac{1}{2}}{7}\;}{\dfrac{3}{2\frac{1}{2}}}$ by $\dfrac{\;\dfrac{6\frac{1}{2}}{2\frac{1}{4}}\;}{\dfrac{5}{\frac{1}{7}}}$ *Ans.*

DIVISION OF COMPLEX FRACTIONS.

(1) Divide $\dfrac{\;\dfrac{3\frac{3}{4}}{6}\;}{\dfrac{9}{\frac{3\frac{1}{2}}{4}}}$ by $\dfrac{\;\dfrac{\frac{2\frac{3}{4}}{6\frac{1}{2}}}{3}\;}{\dfrac{3\frac{1}{4}}{\frac{1}{8}}}$ *Ans.* $\frac{[illegible]35}{19008}$

(2) Divide $38\frac{1}{2}$ by $\dfrac{\;\dfrac{3\frac{1}{2}}{6}\;}{\dfrac{7\frac{1}{2}}{5}}$ *Ans.* 99

(3) Divide $\dfrac{\;\dfrac{3\frac{3}{4}}{6}\;}{\dfrac{9}{\frac{3\frac{1}{2}}{4}}}$ by $\dfrac{\;\dfrac{2\frac{3}{4}}{\frac{6\frac{1}{2}}{3}}\;}{\dfrac{3\frac{1}{4}}{\frac{1}{8}}}$ *Ans.* $\frac{35}{1188}$

(4) Divide $\dfrac{\;\dfrac{2\frac{3}{4}}{\frac{6\frac{1}{2}}{7}}\;}{\dfrac{5\frac{1}{2}}{3}}$ by $\dfrac{\;\dfrac{\frac{1}{5}\text{ of }6\frac{1}{2}}{\frac{2}{3}}\;}{\dfrac{\frac{3}{4}}{\frac{2}{7}\text{ of }\frac{16}{25}}}$ *Ans.* $3\frac{3151}{3408}$

(5) Divide $\dfrac{\dfrac{\dfrac{12\frac{1}{4}}{7}}{3\frac{1}{4}}}{\dfrac{\dfrac{\dfrac{\dfrac{9}{\frac{2}{3}}}{\frac{7}{9}}}{5}}{4\frac{1}{2}}}$ by $\dfrac{\dfrac{\frac{2}{3}\text{ of }32}{\frac{7}{3}}}{\dfrac{9\frac{1}{4}}{\dfrac{8\frac{1}{2}}{\dfrac{2}{\frac{1}{3}}}}}$ *Ans.* $\frac{9065}{151632}$

(6) Divide $\dfrac{\dfrac{12\frac{1}{2}}{5\frac{1}{4}}}{\dfrac{3\frac{3}{4}}{5\frac{1}{2}}}$ by $\dfrac{\dfrac{\dfrac{2\frac{1}{4}}{5}}{6\frac{1}{2}}}{\dfrac{3\frac{3}{4}}{\dfrac{\frac{1}{3}}{\frac{1}{2}}}}$ *Ans.* $\frac{17875}{63}$

(7) Divide $\dfrac{3}{\dfrac{\frac{1}{2}}{\dfrac{\dfrac{3\frac{1}{2}}{7}}{5\frac{1}{4}}}}$ by $\dfrac{\dfrac{3\frac{1}{4}}{2\frac{1}{8}}}{\dfrac{3\frac{1}{2}}{3}}$ *Ans.* $\frac{17}{29}$

(8) Divide $\dfrac{\dfrac{\dfrac{\frac{1}{3}}{7}}{6\frac{1}{2}}}{\dfrac{9\frac{1}{2}}{\dfrac{3}{\frac{1}{2}}}}$ by $\dfrac{\frac{5}{6}}{7}$ *Ans.* $\frac{836}{8645}$

THE RULE OF THREE, OR SINGLE PROPORTION.

DEFINITION.

Ratio is a comparison of two quantities, Proportion of three or more.

If the ratio between two quantities arises from Addition or Subtraction, it is called Arithmetical ratio. But, if from Multiplication, or Division of two quantities, it is called Geometrical ratio. Ex.—2, 3, 4, 5, or 4, 3, 2, 1 are examples of *Arithmetical ratio.*

The first series is an increasing series, and formed by the ddition of unity to each preceding term; the latter is a decreasing eries, and formed by the subtraction of unity from each preceding erm. As an example of Geometrical Ratio, take the following eries, 2, 4, 8, 16, 32, &c., and 24, 12, 6, 3, ½. The first series s an increasing geometrical series, and the second is a decreasing geometrical series. In the first series the increase arises from doubling the preceding term, in the second the decrease arises from dividing the preceding term by 2. The 2 in each instance is called the *ratio*.

Now, in order to be a proportion, there must be an equality of ratios subsisting between the terms. Thus, there is an equality of ratio subsisting between 2 and 3 and 4 and 6.

There is an equality of ratio existing between the two ratios 7 : 14 and 4 : 8, as they can be expressed by the fractions $\frac{7}{14}$ and $\frac{4}{8}$, which are equal. When this is the case these four numbers form a proportion, and are written 7 : 14 :: 4 : 8. The signs between being read, As 7 is to 14 so is 4 to 8.

The numbers 7, 14, 4, and 8 are called the *Terms* of the *Proportions:* they are also called Antecedent and Consequent. Thus, 7 and 4 are the antecedents, and 14 and 8 are the consequents. In every proportion thus expressible the exterior numbers, 7 and 8, are called *Extremes*, and the interior numbers, 14 and 4, are called the *Means*.

Now if the above fractions which express an equality of ratios be each multiplied by their common denominator, viz., 112.

Thus, $\frac{7}{14} \bullet 112 = \frac{8}{4} \bullet 112$ we perceive by cancelling that $\frac{7}{\cancel{14}} \bullet \overset{8}{\cancel{112}} = \frac{4}{\cancel{8}} \bullet \overset{144}{\cancel{112}}$ and that we have obtained on the left-hand side of the equation 7 • 8, and on the right-hand side 4 • 14; and on looking at our proportion, viz., 7 : 14 :: 4 : 8, we perceive that 7 and 8 are the extremes and that 4 and 14 are the means; we thence infer that the product of the Extremes is equal to the product of the Means. And that if the product of both the Extremes be divided by either of the Means, we obtain the other Mean. Thus, to find the Mean 4, if we multiply the 7 and 8 together, and divide by the 14, we obtain the Mean 4, thus,

$\frac{\cancel{7} \bullet \overset{4}{\cancel{8}}}{\underset{\cancel{2}}{\cancel{14}}} = 4.$ Either of the Extremes is found in the same manner. *Hence the proof of the rule for the Rule of Three*, viz., multiplying the two Means together and dividing by the first Extreme gives the fourth term, or other Extreme.

RULE. 1. Put the term of demand in the third place of the ratio.

2. Put the cause of, or what you get, or what you give for the

third term, or the *greater* or the *less* of the two terms which are alike, according as the answer is to be greater or less than the third term, in the first place.

3. Put the remaining term in the second place.

4. Bring the first and second terms to the same denomination, and the third to the lowest term named in it; then having cancelled (if the question will admit of it) multiply the second and third terms together and divide the product by the first; the quotient will be of the same denomination as that in which the third term was left.

Ex.—If 7 yds. of cloth cost 9*s.* 2½*d.* what will 84 yds. cost?

$$\begin{array}{cccccccc} \text{yds.} & & \text{yds.} & & s. & d. & & \\ \cancel{7} & : & 84 & :: & 9 & 2\frac{1}{2} & : & x \\ & & 12 & & & 12 & & \\ \hline & & & & \pounds 5\ \ 10 & 6 & & \end{array}$$

Here we take the question according to the rule 1st. On reading the question we perceive that the term of demand (or term required) is money. We therefore place money, (the 9*s.* 2½*d.*) in the third place, we then ask the question, What do we give or what do we get for this third term? The answer is 7 yds. We then place this 7 yds. in the first term, and the remaining one, viz., 84, in the second. We then perceive that the first term will divide into the second, giving 12 for a quotient; we then multiply the second and third terms together, which is done by Compound Multiplication.

Note.—Whenever the second term is less than 100, it is better, if the third term be pounds, shillings, and pence, not to reduce them to a lower denomination. At first sight to multiply by any number less than 100 may appear a difficult operation, but a little practice will easily surmount it.

Ex.—Required the value of 360 yds. of cloth when 8 yds. cost 7*l.* 9*s.* 10*d.*

$$\begin{array}{ccccccc} & & & & \pounds & s. & d. \\ \cancel{8} & : & \cancel{360} & :: & 7 & 9 & 10 \\ & & 45 & & & & 45 \\ \hline & & & & 337 & 2 & 6 \end{array}$$

Here we find "money" is the required term; we therefore place it in the third place, and we also see that we get 8 yds. for this money, therefore we put this first and the remaining term, or 360 yds. in the second; we then cancel by dividing the 360 yds. by the first term, giving a quotient of 45, by which we multiply the 7*l.* 9*s.* 10*d.*, and obtain for a product 337*l.* 2*s.* 6*d.*

It is not always practicable to employ cancelling, but wherever it can be introduced it should be used in the following manner:—Divide the first term into either the second or third terms, or, either of the second or third terms into the first. Or if the first will not divide into the second or the third, or if the second or the third will not divide into the first, perhaps some factor of the first will divide into the same factor of the second or third; and in these instances it may be expunged from the first term and either the second or the third, but not from both, as beginners generally mistake.

(1) If 16¾ lbs. cost 1*l*. 18*s*. 6*d*., what cost 3 cwt. 0 qrs. 18 lbs.?
Ans. 40*l*. 13*s*. 8*d*.+4.

(2) Gave 7½*d*. for 3½ lbs., what cost 19 cwt. 0 qrs. 18 lbs.?
Ans. 19*l*. 3*s*. 2½*d*.+$\frac{1}{7}$.

(3) Bought 76¼ yards for 16*l*. 19*s*. 6*d*., what cost 3¾ yards?
Ans. 16*s*. 8*d*.+22.

(4) Sold 7¼ yards for 8*s*. 9*d*., what did I get for 59 Flemish ells?
Ans. 2*l*. 13*s*. 4*d*.+$\frac{25}{29}$.

(5) At 4*s*. 6*d*. per yard, what must I give for 97 French ells?
Ans. 32*l*. 14*s*. 9*d*.

(6) If 3½ lbs. cost 6*s*. 8*d*., what cost 17 cwt. 3 qrs. 18 lbs.?
Ans. 191*l*. 0*s*. 11½*d*.+5.

(7) Bought 263 cwt. 18 lbs. for 386*l*. 18*s*. 9½*d*., what cost 17¾ lbs.?
Ans. 4*s*. 7¾*d*.+822994.

(8) Gave 9½*d*. per lb. for cheese, what cost 56 cwt. 3 qrs. 18 lbs.?
Ans. 252*l*. 6*s*. 1*d*.

(9) Suppose 4968 men dig a trench in 48 days, how long will it take 793958 men to dig the same?
Ans. 7 h. 12 m. 30 sec. + 151500.

(10) What cost 5 hhds. of sugar, each weighing 12 cwt. 3 qrs. at 1*l*. 19*s*. 6*d*. for 184 lbs.?
Ans. 76*l*. 12*s*. 9¼*d*. + 8.

(11) If 9¾ lbs. of indigo cost 4*l*. 18*s*. 9*d*., what cost 5 cwt. 2 qrs. 16 lbs.?
Ans. 320*l*. 1*s*. 0¼*d*. + 9.

(12) Gave 86*l*. 13*s*. 4*d*. for 72 cwt. 1 qr. 13 lbs., what cost 2 qrs. 16 lbs.?
Ans. 1*l*. 13*s*. 1¾*d*. + 7345.

(13) If 6½ lbs. of cheese cost 8*s*. 9*d*. what cost 3 cwt. 2 qrs. 18 lbs.?
Ans. 27*l*. 11*s*. 1*d*. + 1.

(14) If I can have 17 cwt. 3 qrs. 18 lbs. carried 79 miles for 1*l*. 8*s*. 6*d*., what ought I to give for the carriage of 79 cwt. 3 qrs. 16 lbs. 128 miles?
Ans. 10*l*. 5*s*. 11*d*. + 118394 rem.

(15) If 7 lbs. cost 4*s*. 9½*d*., what cost 4 cwt.?
Ans. 15*l*. 6*s*. 8*d*.

(16) Gave 3*s*. 9*d*. for 6 lbs., what cost 78 lbs.?
Ans. 2*l*. 8*s*. 9*d*.

(17) If 3¾ lbs. cost 9*l*. 8*s*. 6*d*., what cost 180 lbs.?
Ans. 452*l*. 8*s*. 0*d*.

(18) Gave 3*s*. 3*d*. for 7½ lbs., what cost 330 lbs.?
Ans. 7*l*. 3*s*. 0*d*.

(19) Sold 67½ lbs. for 3*l*. 10*s*. 6*d*., what cost 9 lbs.? *Ans*. 9*s*. 4¾*d*. + 3.

(20) Bought 3 qrs. 14 lbs. for 10*l*. 9*s*. 0*d*., what cost 6 cwt. 0 qr. 14 lbs.? *Ans*. 73*l*. 3*s*. 0*d*.

(21) If 17½ lbs. cost 3*l*. 14*s*. 9½*d*., what cost 245 lbs.? *Ans*. 52*l*. 7*s*. 1*d*.

(22) If 304 lbs. cost 19*l*. 18*s*. 10½*d*., what cost 16 lbs.? *Ans*. 1*l*. 0*s*. 11¼*d*. + $\frac{11}{19}$.

(23) Gave 9*s*. 6*d*. for 7½ lbs., what cost 90 lbs.? *Ans*. 5*l*. 14*s*. 0*d*.

(24) If 6¾ ozs. cost 7*s*. 10½*d*., what cost 324 lbs.? *Ans*. 302*l*. 8*s*. 0*d*.

(25) Sold 28 lbs. for 4*l*. 10*s*. 6*d*., what cost 3½ lbs.? *Ans*. 11*s*. 3¾*d*.

(26) Gave 4*l*. 18*s*. 6*d*. for 27¾ lbs., what cost 111 lbs.? *Ans*. 19*l*. 14*s*. 0*d*.

(27) If 6 cwt. of sugar cost 15*l*. 8*s*. 9*d*., what cost 14 lbs.? *Ans*. 6*s*. 6½*d*. + ¾.

(28) Gave 2*l*. 6*s*. 10*d*. for 5 stone, what cost 60 cwt.? *Ans*. 224*l*. 16*s*. 0*d*.

(29) If 6*s*. 6*d*. be given for 24 lbs., what cost 1½ lbs.? *Ans*. 4¾*d*.

(30) Gave 7*s*. 9*d*. for 2½ lbs., what cost 70 lbs.? *Ans*. 20*l*. 12*s*. 0*d*.

(31) If 23¼ lbs. cost 5*l*. 18*s*. 6*d*., what cost 63 lbs. 9 ozs.? *Ans*. 16*l*. 3*s*. 11*d*. + 210.

(32) Gave 209*l*. 12*s*. 4*d*. for 172 cwt. 3 qrs. 19 lbs., what cost 2½ lbs.? *Ans*. 6¼*d*. + 18881.

(33) If 48 oranges cost 1*s*. 5*d*., what must I give for 892? *Ans*. 1*l*. 6*s*. 3¾*d*. + ⅓.

(34) If 72 pecks cost 92*l*. 18*s*. 6*d*., what cost 3 gallons? *Ans*. 1*l*. 18*s*. 8½*d*. + 1.

(35) Gave 1*s*. 2½*d*. for 19 lbs. of potatoes, what cost 1 ton? *Ans*. 7*l*. 2*s*. 5¼*d*. + 17.

(36) Gave 9*d*. per lb. for soap, what cost 33 cwt. 2 qrs.? *Ans*. 35*l*. 3*s*. 8*d*.

(37) Bought 23 cwt. 1 qr. 19 lbs. for 42*l*. 18*s*. 6*d*., what cost 3½ lbs.? *Ans*. 13½*d*. + 2576.

(38) If a gallon of beer cost 1*s*. 4*d*., what cost 96 hhds.? *Ans*. 345*l*. 12*s*. 0*d*.

(39) Bought 3 English ells for 9*s*. 2½*d*., what cost 496 yds.? *Ans*. 6*l*. 0*s*. 3¾*d*. + 7.

(40) Sold 2 French ells for 1*l*. 18*s*. 6*d*., what cost 39 Flemish ells? *Ans*. 18*s*. 15*s*. 4½*d*.

(41) Bought 10 lbs. of raisins for 7*s*. 6*d*., what cost 39 cwt. 2 qrs. 18 lbs.? *Ans*. 166*l*. 11*s*. 6*d*.

(42) One bag of hops, weighing 264 lbs., for 16*l*. 18*s*. 9*d*., what did 1 lb. cost? *Ans*. 15¼*d*. + $\frac{13}{22}$.

(43) Bought 69 pockets of hops for 758*l*. 19*s*. 6*d*., what was that per 3 lbs.? *Ans*. 5*s*. 10½*d*. + 1092.

(44) If 68½ lbs. cost 2*l*. 8*s*. 9½*d*., what is the value of 72 cwt. 3 qrs. 27 lbs.? *Ans*. 291*l*. 2*s*. 11¼*d*. + $\frac{62}{137}$.

(45) If 4963 men eat 1792 bushels of wheat in 18 days, how many men will it take to eat 14968 quarters in 74 days? *Ans*. 10083 men, 63848 rem.

(46) If 5 men can dig a trench in 48 days of 6928 solid feet, how many men must be employed to dig another ten times as large in 20 days? *Ans*. 120.

(47) Gave 10*s*. 6*d*. for 4 lbs. of coffee, what cost 234 lbs.? *Ans*. 42*l*. 10*s*. 6*d*.

(48) What cost 796 cwt. 2 qrs. 18 lbs. at 3*l*. 18*s*. 6*d*. per cwt.? *Ans*. 3126*l*. 17*s*. 10¼*d*. + 64.

(49) If 68 hhds. of sugar, each weighing 12¼ cwt., cost 1837*l*. 14*s*. 9*d*., what is the value of 21¾ lbs.? *Ans*. 8*s*. 4¾. + 25436.

(50) What cost 3¾ lbs. of cheese when 78 cwt. 3 qrs. 19 lbs. cost 127*l*. 18*s*. 9½*d*.? *Ans*. 1*s*. 1*d*. + 3818.

(51) What cost 726 cwt. 3 qrs. 18 lbs. at 2*l*. 16*s*. 3*d*. per cwt.? *Ans*. 2044*l*. 8*s*. 8½*d*. + 104 rem.

(52) If 16½ lbs. cost 1*l*. 2*s*. 4*d*. what will 5½ cwt. come to at the same rate? *Ans*. 41*l*. 13*s*. 9¼*d*. + 11.

(53) If 68 cwt. 1 qr. 18 lbs. cost 286*l*. 14*s*. 11½*d*., what will 3⅓ lbs. cost? *Ans*. 2*s*. 7¼*d*. + 5723.

(54) What cost 17 tons of lead at 14*l*. 6*s*. 8*d*., per cwt.? *Ans*. 4873. 6*s*. 8*d*.

(55) What cost 19½ cwt. of lead when 3¾ lbs. are sold for 7¼*d*.? *Ans*. 17*l*. 11*s*. 10¼*d*.

(56) What cost 5 hhds. of sugar, each weighing 5 cwt. 2 qrs. 18 lbs. at 2*s*. 13*s*. 6½*d*. per cwt.? *Ans*. 75*l*. 15*s*. 5*d*. + 20.

(57) 84 men reap 36 acres, how many men will be required to reap 432? *Ans*. 432.

(58) Gave 3*l*. 18*s*. 6½*d*. for 4½ cwt., what cost ¾ of a lb.? *Ans*. 1¼*d*. + $\frac{412}{672}$.

(59) If 7½ yds. cost 13¼*d*., what cost 593 Eng. ells? *Ans*. 5*l*. 9*s*. 1½*d*. + 1.

60) I wish to gain 14*l*. 10*s*. on the sale of 2 cwt. of indigo, which cost 112*l*. 10*s*., what must I sell it at per lb.? *Ans*. 11*s*. 4$\frac{1}{14}$*d*.

(61) A school of 44 boys and 4 masters eat 350 lbs. of bread in a week, what will another school of 90 boys and 6 masters eat in the same time, supposing each boy to eat twice as much as a master? *Ans.* $707\frac{14}{23}$ lbs.

(62) Sold 27½ yds. for 32*l.* 18*s.* 6*d.*, what cost 5 yds.? *Ans.* 5*l.* 19*s.* 8½*d.* + $\frac{10}{11}$.

(63) Received 3½*d.* for half a pint of ale, what cost 52 hhds.? *Ans.* 655*l.* 4*s.* 0*d.*

(64) 36 lbs. of lead cost 4*s.* 6*d.*, how much must I give for 3 cwt. 1 qr.? *Ans.* 3*l.* 10*s.* 0*d.*

(65) In selling 52 handkerchiefs for 10*l.* 18*s.* 6*d.*, what was the price of 576? *Ans.* 121*l.* 0*s.* 3½*d.* + 10.

(66) If 23 half handkerchiefs cost 4*l.* 8*s.* 6*d.*, what cost 259 whole ones? *Ans.* 99*l.* 13*s.* 2*d.* + $\frac{2}{23}$.

(67) Sold 56 bottles of wine for 11*l.* 4*s.* 2*d.*, what cost 3½ bottles? *Ans.* 14*s.* 0*d.* + ½.

(68) Gave 3*s.* 4*d.* for 1 lb., how many cwt. can I get for 72*l.* 6*s.* 8*d.*? *Ans.* 3 cwt. 3 qrs. 14 lbs.

(69) 4½ lbs. cost 7*s.* 9½*d.*, what shall I get for 144 lbs.? *Ans.* 12*l.* 9*s.* 4*d.*

(70) If 7¾ yds. cost 8*s.* 9¼*d.*, what cost 296¼ yds.? *Ans.* 16*l.* 15*s.* 3¼*d.* + 2.

(71) If 36 English ells cost 15*s.* 8½*d.*, what cost 375¾ yds.? *Ans.* 62*l.* 15*s.* 11¾*d.* + 47.

(72) If 274¼ yds. cost 179*l.* 19*s.* 6*d.*, what cost 3¾ yds.? *Ans.* 2*l.* 9*s.* 8*d.* + 218.

(73) If 27¼ lbs. cost 1*l.* 8*s.* 9*d.*, what cost 3 ozs.? *Ans.* 3*d.* + ⅕.

(74) Gave 2*s.* 9*d.* for 3 ozs., what cost 27¾ lbs.? *Ans.* 20*l.* 7*s.* 0*d.*

(75) Gave 5*s.* 8½*d.* for 7 lbs., what cost 36¼ lbs.? *Ans.* 1*l.* 9*s.* 6½*d.* + 26.

(76) If 2¾ ozs. cost 7*s.* 9½*d.*, what cost 427¼ lbs.? *Ans.* 968*l.* 8*s.* 8*d.*

(77) If 3¼ ozs. cost 6*s.* 10¾*d.*, what cost 596½ lbs.? *Ans.* 1012*l.* 10*s.* 3*d.* + 9

(78) If 6 lbs. of cheese cost 5*s.* 6½*d.*, what will 15 cwt. 1 qr. 19 lbs. cost? *Ans.* 79*l.* 15*s.* 0¾*d.*

(79) Gave 6*s.* 8*d.* for 15¼ lbs. of rice, what cost 17 cwt. 2 qrs. 18 lbs.? *Ans.* 43*l.* 4*s.* 8¼*d.*

(80) If I sell 6 oz. of coffee for 1*s.*, what shall I get for 76 cwt. 3 qrs.? *Ans.* 1330*l.* 4*s.* 2½*d.* + 4.

(81) Gave 34*l.* 12*s.* for 17 cwt. of cheese, what must I sell it at per lb. to gain 12*l.* 8*s.* on the whole? *Ans.* 5¾*d.* + 1328.

(82) If I buy 34 cwt. of indigo for 1680*l.*, what must I sell it at per lb. to lose 84*l.* on the whole? *Ans.* 8*s.* 4½*d.*+1344.

(83) Gave 5*l.* 9*s.* 0½*d.* for 32 yards of drugget, what cost 288 yards? *Ans.* 49*l.* 8*s.* 10½*d.*

(84) Sold 3½ lbs. of tobacco for 7*s.* 8½*d.*, what cost 42 lbs.? *Ans.* 4*l.* 12*s.* 6*d.*

(85) Sold 13½ lbs. of cheese for 4*s.* 6*d.*, what cost 94½ lbs.? *Ans.* 1*l.* 11*s.* 6*d.*

(86) What cost 3 pairs of shoes, when 36½ dozens cost 98*l.* 16*s.*? *Ans.* 13*s.* 6¼*d.*+94.

(87) Gave 2*s.* 9*d.* for 3 lbs. of butter, what cost 84 firkins, each weighing 72 lbs.? *Ans.* 277*l.* 4*s.*

(88) If 84 lbs. of steel cost 5*l.* 18*s.* 6*d.*, what cost 3 oz.? *Ans.* 3*d.*+78 remainder.

(89) If 6½ yards of cloth cost 3*l.* 14*s.* 6*d.*, what cost 55 English ells? *Ans.* 39*l.* 7*s.* 11¾*d.*+1.

(90) How many pieces of money, each valued at 3*s.* 4½*d.*, can I get for 260*l.* *Ans.* 1520+$\frac{20}{27}$.

(91) If I give 6*s.* 8*d.* for ½ lb. of indigo, what must I give for 3 cwt. 2 qrs. 19 lbs.? *Ans.* 274*l.* 0*s.* 0*d.*

(92) Gave 2*l.* 18*s.* 6½*d.* for 1 cwt. of sugar, what is the price of 36 cwt. 3 qrs. 19 lbs.? *Ans.* 108*l.* 1*s.* 3¾*d.*

USEFUL PROPERTIES ON RATIOS AND PROPORTIONS.

(1) ***Ratio*** is a comparison between two quantities, as 5 : 7. The 5 is called the *antecedent* and the 7 the *consequent*. And when these quantities are expressed by a fraction, as $\frac{5}{7}$, the 5 and the 7 are called the *terms*.

(2) *Proportion* is a comparison between three or more quantities; as 2 : 4 :: 4 : 8, or 14 : 7 :: 8 : 4. In the first of these proportions the 8 is said to be a *third* proportional to the 2 and 4, and the second term (the 4) is called a *mean proportional*.

(3) If *all* the *antecedents* of any ratio or proportion be multiplied together, and also *all* the *consequents*, the ratio is said to be *compounded*, and assumes different names, according to the number of ratios introduced; thus, when of two equal ratios only, it is called *duplicate* ratio.

Antecedents. Consequents.
$$\begin{array}{lccc} \text{As} & 4 & : & 8 \\ & 12 & : & 24 \end{array}\Bigg\}\ \text{Here if 4 be multiplied by 12, and 8 by 24}\ \Bigg\}\ \text{we obtain } 48 : 192$$

Again, when compounded of three ratios it is called a *triplicate* ratio.

$$\begin{array}{lrcl} & \text{Ant.} & & \text{Con.} \\ \text{As} & 2 & : & 6 \\ & 3 & : & 9 \\ & 8 & : & 24 \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\}\begin{array}{l}\text{Here if 2, 3, and 8 be} \\ \text{multiplied together, and} \\ \text{6, 9, and 24}\end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\}\text{ we obtain } 48 : 1296$$

(4) When the antecedent and consequent are prime to each other, the ratio is said to be in its lowest terms, as 5 : 9, or $\frac{5}{9}$; where 5 is prime to 9.

(5) If the *antecedent* and *consequent* be equal the ratio is said to be a ratio of equality, as 2 : 2 or 9 : 9.

(6) If the *antecedent* be greater than the *consequent*, the ratio is said to be one of greater *inequality;* but if the *antecedent* be less than the consequent the ratio is said to be one of *less inequality.*

7 : 5 is an example of *greater inequality.*
3 : 5 is an example of *less inequality.*

(7) When in a proportion the extremes and means are displaced, the operation is called *Alternando.*

$$\begin{array}{l} \text{As } 2 : 3 :: 4 : 6 \\ \quad\;\; 6 : 4 :: 3 : 2 \\ \text{Or } 6 : 3 :: 4 : 2 \end{array}$$

(8) When the extremes are put in the place of the means the operation is called Invertendo, as,

$$3 : 2 :: 6 : 4.$$

(9) If the antecedents or the consequents be multiplied by the same number the proportion will still subsist,

$$\begin{array}{lrcrcrcr} \text{As} & 2 & : & 3 & :: & 4 & : & 6 \\ & 2\cdot5 & : & 3 & :: & 4\cdot5 & : & 6 \\ \text{Or} & 10 & : & 3 & :: & 20 & : & 6 \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\}\begin{array}{l}\text{If we multiply the antecedents by} \\ \text{any number, as 5, the proportion} \\ \text{still subsists.}\end{array}$$

$$\begin{array}{lrcrcrcr} \text{Again,} & 2 & : & 3 & :: & 4 & : & 6 \\ & 2 & : & 3\cdot8 & :: & 4 & : & 6\cdot8 \\ \text{Or,} & 2 & : & 24 & :: & 4 & : & 48 \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\}\begin{array}{l}\text{If we multiply the consequents} \\ \text{by any number, as 8, the pro-} \\ \text{portion still subsists.}\end{array}$$

(10) In any number of proportions, if the antecedents be added together or subtracted, and also if the consequents be added together, the sum or difference of the former is to the sum or difference of the latter, as any antecedent is to its consequent.

$$\begin{array}{l}\text{Ex.,—}2 : 4 :: 8 : 16 \\ \qquad\;\; 6 : 12 :: 9 : 18\end{array}\left.\vphantom{\begin{array}{l}1\\1\end{array}}\right\}\begin{array}{l}\text{Then } 2+6 : 4+12 :: 2 : 4 \\ \text{That is, } \quad 8 : 16 \quad :: 2 : 4\end{array}$$

$$\begin{array}{l}\text{Or, } 6-2 : 12-4 :: 2 : 4 \\ \text{That is, } \quad 4 : 8 \quad :: 2 : 4\end{array}$$

(11) If any two proportions be multiplied together, antecedent by antecedent, and consequent by consequent, the products are proportional.

$$\begin{array}{l}\text{Thus, } 3 : 4 :: 9 : 12 \\ \qquad\;\; 2 : 3 :: 4 : 6\end{array}\left.\vphantom{\begin{array}{l}1\\1\end{array}}\right\}\text{ or }\left\{\begin{array}{lcl} 3\cdot2 : 4\cdot3 & :: & 9\cdot4 : 12\cdot6 \\ 6 \quad : 12 & :: & 36 \quad : 72\end{array}\right.$$

(12) The squares, cubes, biquadrates, &c. of any proportion, are proportional.

$$\text{Thus, }\left.\begin{array}{l} 2\ :3\ ::4\ :6 \\ 2^2:3^2::4^2:6^2 \\ 2^3:3^3::4^3:6^3 \\ 2^4:3^4::4^4:6^4 \end{array}\right\}\text{ or }\left\{\begin{array}{r} 4:\ 9::\ 16:\ 36 \\ 8:27::\ 64:216 \\ 16:81::216:864 \end{array}\right.$$

HARMONICAL PROPORTION AND PROGRESSION.

(1) If three numbers in Arithmetical Progression be made reciprocals, then these reciprocals will be in harmonical proportion, *i. e.*, the first term will bear the same proportion to the third that the difference between the first and second bears to the difference between the second and third.

Thus in the series 7, 9, 11, if the 7, 9, and 11 be made reciprocals $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$.

Then $\frac{1}{7}:\frac{1}{11}::(\frac{1}{7}-\frac{1}{9}):(\frac{1}{9}-\frac{1}{11})$, or $\frac{1}{7}:\frac{1}{11}::\frac{2}{63}:\frac{2}{99}$.

From which, by multiplying the extremes and means together, we obtain $\frac{2}{693}=\frac{2}{693}$.

So of the series $\frac{4}{11}$, $\frac{4}{13}$, $\frac{4}{15}$; for $\frac{4}{11}:\frac{4}{15}::\frac{8}{143}:\frac{8}{195}$, or $\frac{32}{2145}=\frac{32}{2145}$.

(2) To find a harmonical mean between two numbers.

RULE.—Double the product of the two numbers and divide by the sum of them.

Ex.—Find a harmonical mean between $\frac{1}{12}$ and $\frac{1}{20}$.

Here $\frac{2\,(\frac{1}{12}\bullet\frac{1}{20})}{(\frac{1}{12}+\frac{1}{20}=)\frac{2}{15}}=\frac{\frac{1}{120}}{\frac{2}{15}}=\frac{1}{\underset{8}{\cancel{120}}}\bullet\frac{\cancel{15}}{\cancel{2}}=\frac{1}{16}$, $\therefore$ the three terms are $\frac{1}{12}$, $\frac{1}{16}$, and $\frac{1}{20}$.

(3) To find a third harmonical to two given terms.

RULE.—The product of the two given terms divided by the difference between twice the first term and the second, will give the term required.

Ex.—Find a third harmonical term to $\frac{1}{18}$, $\frac{1}{50}$.

Here $\frac{(\frac{1}{18}\bullet\frac{1}{50})}{2\,(\frac{1}{18})-\frac{1}{50}}=\frac{\frac{1}{900}}{\frac{41}{450}}=\frac{1}{\underset{2}{\cancel{900}}}\bullet\frac{\cancel{450}}{41}=\frac{1}{82}$.

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

In the preceding Rule, it will be perceived three terms only have been employed to find a fourth; but it frequently occurs that 5, 6, 7, 8, 9 to 20 terms are in the question. This is usually called the Double Rule of Three; but all questions may be solved by one Rule analogous to the preceding.

It is well known, circumstances being the same, that *causes* have a like ratio to the *effects* which they produce. Thus, if four men mow $3\frac{1}{2}$ acres of grass in a given time, eight men will mow 7 acres of grass in the same time; that is, supposing circumstances to be the same in both cases: from this principle originates the following Rule. In solving questions by this method, put x for the term sought.

RULE.—As first cause : second cause :: first effect : second effect.

Ex.—If a family, consisting of nine persons, eat 218 lbs. of bread in four weeks, how many persons will it require to eat 560 lbs. of bread in three weeks?

Here $\left.\begin{matrix}9\,.\,4\\ x\,.\,3\end{matrix}\right\}$ 1st & 2d $\left\{\text{Causes.}\right.$ Whence this proportion.
$: 9\,.\,4 : x\,.\,3 :: 218 : 560$

$\left.\begin{matrix}218\\ 560\end{matrix}\right\}$ 1st & 2d $\left\{\text{Effects.}\right.$ Multiplying extremes and means together; we obtain

$\underset{3}{\cancel{9}}\,.\,4\,.\,\underset{280}{\cancel{560}}=x\,.\,\cancel{3}\,.\,\underset{109}{\cancel{218}}$ and cancelling.

$3360=109x$ ÷ both sides by 109.

Persons $30\frac{90}{109}=\frac{3360}{109}=x$ (the term sought).

In this example we perceive, first, That the cause of the bread being eaten is the nine men, and time four weeks; and secondly, the second cause is x (an unknown number), and time three weeks; the first effect produced is the 218 lbs.; and the second effect 560 lbs. From these terms a proportion is formed, and we know from the preceding Rule that if the extremes and means be multiplied together, that the products resulting are equal. We then cancel, thus, the three in the numbers $x\,.\,3\,.\,218$ on the right side of the equation, will cancel into the 9 on the left; and 560 is divisible by 2, and also 218 on the other side is divisible by 2. Having thus cancelled all we can, we multiply all uncancelled numbers on each side together. We thus obtain 109 x on one side of the =, and 3360 on the other; we therefore divide both sides by 109, leaving x on one side, and $30\frac{90}{109}$ on the other: which is the answer.

If 480 pioneers can dig a trench 1600 feet long, 30 wide, and 14 deep, in 14 days of 12 hours long, how many men will be required to dig another 2480 feet long, 24 wide, and 18 deep, in 24 days of 8 hours long?

$$\left.\begin{matrix}480 & . & 14 & . & 12\\ x & . & 24 & . & 8\end{matrix}\right\}\text{1st and 2nd causes.}$$

$$\left.\begin{matrix}1600 & . & 30 & . & 14\\ 2480 & . & 24 & . & 18\end{matrix}\right\}\text{1st and 2nd effects.}$$

Hence the following proportion :—

$$480 . 14 . 12 : x . 24 . 8 :: 1600 . 30 . 14 : 2480 . 24 . 18$$

By multiplying means and extremes together.

$$x . \cancel{24} . \cancel{8} . \cancel{1600} . \cancel{30} . \cancel{14} = \cancel{480} . \cancel{14} . \cancel{12} . \cancel{2480} . \cancel{24} . 18 \text{ cancelling.}$$

$$\cancel{20} \qquad \cancel{60} \qquad 3 \quad 31$$

$$5 \qquad 2$$

$$5\,x = 3888 \div \text{both sides by } 5.$$

$$x = 777\tfrac{3}{5} \text{ men.}$$

Note.—It is quite unnecessary in practice, to form a proportion as is done above. When a competent knowledge of the Rule has been obtained, it is recommended to cancel the terms as they stand; but this method should *not* be attempted till practice has rendered the pupil expert in the method already given.

EXAMPLES.

(1) If 36 men dig a ditch 234 yards long, 36 broad, and 64 deep, in 24 days of 8 hours long, how many days of 6 hours long, would it take 500 men to dig another 436 yards long, 48 broad, and 96 deep? *Ans.* $8\frac{952}{1625}$.

(2) If 160 men can mow a field 1300 yards long, and 580 broad, in 38 days of 10 hours long, how many square yards will 34 men mow in 60 days of 8 hours long? *Ans.* $200389\frac{9}{19}$.

(3) If 360 men can throw up a rampart of a citadel in 56 days of 8 hours long, 400 feet long, 30 wide, and 20 deep, how many men would it require to throw up another 500 feet long, 30 wide, and 50 deep, in 48 days of 10 hours long? *Ans.* 1050.

(4) If 656 men can plant a nursery 500 feet long, 60 wide, and 18 deep, in 35 days of 6 hours long, how many days of 12 hours long, will it require 780 men to plant another 320 feet long, 54 wide, and 24 deep? *Ans.* $11\frac{493}{1625}$.

(5) If 23 men can reap a field 1400 yards long, and 780 wide, in 8 days of 14 hours long, how many men will it require to reap another field 1680 yards long, and 530 broad, in 63 days of 4 hours long? *Ans.* $8\frac{106}{585}$.

(6) How many horses will it require to draw up ⅌ crane 30 blocks of stone 30 feet long, 6 wide, and 4 deep, in 58 hours, when 568 horses can draw up 20 blocks of stone, each 14 feet long, 7 wide, and 3 deep, in 36 hours? *Ans.* $1295\frac{125}{1421}$.

(7) If 60 pioneers can dig a trench 500 feet long, 36 wide, and 40 deep, in 24 days 8 hours long, how many will be required to dig another 550 feet long, 68 wide, and 90 deep, in 56 days of 9 hours long? *Ans* $106\frac{6}{7}$.

(8) If 60 men can raise a stack of wood 680 feet long, 50

wide, and 40 deep, in 48 days of 12 hours each, how many men would be required to raise another 900 feet long, 54 wide, and 22 deep, in 78 days 9 hours long? *Ans.* 38$\frac{778}{1105}$.

(9) If 108 men eat 450 lbs. of bread in 6 days, how many days will 167 men be in eating 386 lbs. of bread? *Ans.* 3$\frac{1371}{4175}$.

(10) If 5 couriers, going different roads, travel 4969 miles in 48 days of 6 hours long, how many miles will 13 couriers travel in 16 days of 10 hours long? *Ans.* 7177$\frac{4}{9}$.

(11) If 5 men can dig a trench in 48 days of 6928 solid feet, how many men must be employed to dig another 10 times as large in 20 days? *Ans.* 120.

(12) If I can have 17 cwt. 3 qrs. 18 lbs. carried 79 miles for 1*l*. 8*s*. 6*d*., what ought I to give for the carriage of 79 cwt. 3 qrs. 16 lbs. for 128 miles? *Ans.* 10*l*. 5*s*. 11*d*. + 118394.

(13) If 4963 men eat 1792 bushels of wheat in 18 days, how many men will it take to eat 14968 quarters in 74 days?
Ans. 10083 men + 63848.

(14) Suppose 4968 men dig a trench in 48 days, how long will it take 793958 men to dig the same?
Ans. 7h. 12m. 30sec. + 151500.

(15) If 50*l*. gain 7*l*. 10*s*. in 9 months, what principal will gain 68*l*. in 15 months? *Ans.* 816*l*.

(16) If 68*l*. gain 5*l*. 10*s*. in 9 months, what principal will gain 473*l*. in 7 years and 4 months? *Ans.* 598*l*. 0*s*. 10$\frac{3}{4}$*d*. + 78.

(17) If 17260 men eat 7324 bushels of corn in 36 days, how many men will it require to eat 42968 bushels in 380 days?
Ans. 11046 + 10818 remainder.

(18) If 48 men dig a trench 298 yards long, 16 wide, and 18 deep, in 52 days of 5 hours long, how many men will it require to dig another 375 feet long, 36 wide, 16 deep, in 72 days 17 hours long? *Ans.* 70$\frac{115}{1045}$.

(19) If 480*l*. gain 36*l*. 10*s*. in 2$\frac{1}{2}$ years, what will 596*l*. gain in 6 years? *Ans.* 108*l*. 15*s*. 4$\frac{3}{4}$*d*. + $\frac{1}{5}$.

(20) If 264 men drink 48 barrels of beer in 36 days, how many men will it require to drink 54 barrels in 6 days? *Ans.* 1782.

(21) If 108 men consume 450 lbs. of meat in 6 days, how many days will 167 men be in eating 386 lbs.? *Ans.* 3 $\frac{1371}{4175}$.

(22) If 480*l*. gain 36*l*. 10*s*. in 2$\frac{1}{2}$ years, what will be gained in 6 years by lending 596*l*.? *Ans.* 108*l*. 15*s*. 4*d*$\frac{3}{4}$. + $\frac{1}{5}$.

(23) If 48 qrs. of wheat are eaten by 90 people in 33 weeks, how many people will it require to eat 320 qs. in 58 days?
Ans. 2389 + 19 rem.

(24) If a trench, 320 yds. long, 18 wide, and 0 deep, be dug by 48 men in 6 days of 4 hours long, now many men will be

required to dig another 144 yds. long, 26 wide, and 18 deep, in 14 days of 8 hours long? *Ans.* 12 men + 6 rem.

(25) If in 9 months 7*l.* 10*s.* be gained by lending 150*l.*, what principal must be lent to gain 68*l.* in 15 months? *Ans.* 816.

(26) If 72 men drink 148 barrels of beer in 71 days, how many men will it require to drink 49 barrels in 35 days? *Ans.* 48 $\frac{66}{185}$.

(27) 40*l.* was gained by lending 580*l.* for 3 years, what principal will be required to gain 300*l.* in 18 months? *Ans.* 8700*l.*

(28) If 47*l.* 12*s.* be gained by lending 864*l.* 10*s.* for 10 months, how much must I lend for 18 months to gain 326*l.* 12*s.*? *Ans.* 3295*l.* 7*s.* 0$\frac{1}{4}$*d.* + 273 rem.

(29) If 45 men drink 572 gallons of beer in 10 days, how many gallons will 3268 men require for 304 days' consumption? *Ans.* 1262813 gallons + 67 rem.

(30) If 83*l.* 16*s.* be gained by lending 532*l.* for 9 months, what ought to be lent to gain 375*l.* in 16 months? *Ans.* 1332 $\frac{943}{1675}$.

(31) If 380 men can dig a trench 2385 yds. long, 15 deep, and 16 wide, in 84 days of 7 hours long, how many days of 12 hours long will 560 men require to dig another 2430 yds. long, 25 wide, and 32 deep? *Ans.* 14 $\frac{1}{105}$.

(32) If 480 men eat 328 bushels of wheat in 14 days, how many men will it require to eat 48 bushels in 3 days? *Ans.* 327 $\frac{33}{41}$.

(33) If 650*l.* gain 38*l.* in 2 years, what principal will gain 63*l.* in one year? *Ans.* 2155*l.* 5*s.* 3*d.* + 3 rem.

(34) If 150*l.* gain 7*l.* 10*s.* in 9 months, what principal will gain 68*l.* in 15 months? *Ans.* 816*l.*

(35) If 250*l.* gain 48*l.* in 2$\frac{1}{2}$ years at 4 per cent., what will 984*l.* gain in 3$\frac{1}{4}$ years at 6 per cent.? *Ans.* 368*l.* 8*s.* 2$\frac{1}{4}$*d.* + $\frac{27}{125}$.

(36) If 640 yds. of velvet be woven in 39 days by 72 men, how many men will it require to weave 960 yds. in 56 days? *Ans.* 75 men + 3 rem.

(37) If 15*l.* 10*s.* be gained in 14 months from a loan of 680*l.*, what time will be required to gain 321*l.* by a loan of 598*l.*? *Ans.* 329 $\frac{6419}{9269}$.

(38) If 49 men eat 56 bushels of corn in 5 days, how many men will it require to eat 390 bushels in 18 days? *Ans.* 94 $\frac{19}{24}$.

(39) If 14 people drink 15$\frac{1}{2}$ gallons of beer in 2$\frac{1}{2}$ days, how many gallons will 65 people drink in 42$\frac{1}{4}$ days? *Ans.* 1216$\frac{11}{35}$.

(41) If 340 horses eat 1568 trusses of hay in 7 days, how many horses will it require to eat 7880 trusses in 40 days? *Ans.* 11$\frac{827}{925}$.

(40) If 40*l.* gain 4*l.* in 6 months at 20 per cent., what principal will gain 60*l.* in 16 months at 10 per cent.? *Ans.* 450*l.*

RULE OF THREE IN FRACTIONS.

RULE.—State the question as in common Rule of Three, then invert the first term of the proportion, cancel, and multiply all the uncancelled terms together, reduce, and the answer is obtained.

If $\frac{4}{5}$ of a yard cost $\frac{1}{6}l.$, what will $\frac{5}{9}$ of an English ell cost?

Here we state the question as in common numbers; then we invert the first term and cancel the 5 numerator into 5 denominator; then divide the 9 numerator and 6 denominator by 3; 3 being a factor of both, which gives a quotient 3 for numerator and 2 for denominator.

$$\frac{4}{5} \bullet \frac{4}{1} : \overset{\pounds}{\frac{1}{6}} :: \frac{5}{9} \bullet \frac{5}{1}$$

$$\frac{\cancel{5}}{4 \cdot 4} \bullet \frac{1}{\underset{2}{\cancel{6}}} \bullet \frac{\overset{3}{\cancel{9}}}{5 \cdot \cancel{5}} = \frac{3}{80}$$

We then perceive that the numerators and denominators are prime to each other. We then multiply all numerators together, and obtain 3 for N numerator, and all the denominators together, and obtain 80 for N denominator. The fraction is $\frac{3}{80}$ of a pound, or $\frac{3}{4}$ of a shilling, or 9*d.*

Ex. in Double Rule of Three.

If $1056\frac{1}{2}$ lbs. of bread are eaten by $16\frac{1}{2}$ men in $27\frac{1}{2}$ days, how many lbs. will be eaten in $45\frac{1}{3}$ days by $25\frac{1}{3}$ men?

$$\left.\begin{array}{lll} : 16\frac{1}{4} \bullet 27\frac{1}{2} & \text{or} & \frac{65}{4} \bullet \frac{55}{2} \\ : 1056\frac{1}{3} & \text{or} & \frac{3169}{3} \end{array}\right| \begin{array}{l} \text{1 and 2 Causes.} \\ \frac{65}{4} \bullet \frac{55}{2} \bullet x = \frac{3169}{3} \bullet \frac{76}{3} \bullet \frac{136}{3} \\ 65.55.3.3.3\,x = 3169.76.136.4.2 \end{array}$$

$$\left.\begin{array}{lll} :: 25\frac{1}{3} \bullet 45\frac{1}{3} & \text{or} & \frac{76}{3} \bullet \frac{136}{3} \\ : \quad x & & x \end{array}\right| \begin{array}{l} \text{1 and 2 Effects.} \\ 96525\,x = 262038272 \\ x = \frac{262038272}{96525} = 271\frac{45597}{96525}. \end{array}$$

(1) Gave $\frac{5}{7}$ of a shilling for $\frac{9}{13}$ of a lb. Avoirdupoise, what cost $\frac{3}{5}$ of an oz.? *Ans.* $\frac{1}{4}d. + \frac{6}{7}$.

(2) Gave $\frac{2}{7}$ of 1*d.* for $\frac{1}{2}$ of $\frac{3}{8}$ of an oz., what cost $72\frac{1}{2}$ lbs. Avoirdupoise? *Ans.* 7*l.* 7*s.* $3\frac{1}{2}d. + \frac{10}{21}$.

(3) Bought $3\frac{1}{2}$ ozs. of cheese for $4\frac{3}{4}d. + \frac{3}{8}$, what cost $12\frac{3}{8}$ cwt.? *Ans.* 127*l.* 17*s.* 6*d.*

(4) Sold $68\frac{1}{2}$ cwt. of tobacco for $2968\frac{5}{9}l.$, what cost $\frac{1}{7}$ of $\frac{3}{4}$ of $2\frac{3}{8}$ cwt.? *Ans.* 11*l.* 0*s.* $6\frac{1}{4}d. \frac{486}{959}$.

(5) Bought $7\frac{1}{2}$ lb. of indigo for 2*l.* 6*s.* 8*d.*, what cost $3\frac{5}{9}$ ozs.? *Ans.* 1*s.* $4\frac{1}{2}d. \frac{10}{27}$.

(6) Sold $2\frac{1}{2}$ cwt. of cheese for 5*l.* 18*s.* 9*d.*, what cost $\frac{7}{8}$ of $\frac{3}{4}$ of a lb.? *Ans.* $3\frac{1}{4}d. \frac{23}{64}$.

(7) Gave 6*s.* $9\frac{1}{2}d.$ for $2\frac{3}{4}$ lbs., what cost $\frac{5}{9}$ of $\frac{3}{5}$ of a cwt.? *Ans.* 4*l.* 12*s.* $2\frac{1}{4}d. \frac{23}{33}$.

(8) If $6\frac{1}{2}$ yards cost $3\frac{5}{8}l.$, what cost $72\frac{3}{4}$ lbs.? *Ans.* 40*l.* 11*s.* $5\frac{1}{4}d. + \frac{6}{13}$.

(9) Gave $98\frac{5}{9}l.$ for $3\frac{7}{8}$ cwt., what cost $\frac{3}{4}$ of a cwt.?
Ans. 19*l.* 1*s.* 6*d.*$\frac{2}{31}$.

(10) Bought $98\frac{1}{4}$ barrels of figs, each 36 lbs., for $158\frac{5}{12}l.$, what was that per lb.? *Ans.* $10\frac{1}{2}d.+\frac{3526}{3537}$.

(11) If $7\frac{1}{2}$ lbs. cost $92\frac{5}{7}s.$, what cost $29\frac{3}{4}$ lbs.?
Ans. 18*l.* 7*s.* $9\frac{1}{5}d.$

(12) If $62\frac{1}{2}$ lbs. cost $49\frac{5}{13}l.$, what cost $676\frac{3}{4}$ lbs.?
Ans. 534*l.* 14*s.* $8\frac{3}{4}d.+\frac{40}{385}$.

(13) If $29\frac{1}{4}$ lbs. cost $3\frac{7}{9}l.$, what cost $2\frac{1}{8}$ lbs.?
Ans. 5*s.* $5\frac{3}{4}d.+501$.

(14) Gave $4\frac{5}{8}l.$ for $\frac{7}{8}$ of a lb. Avoirdupoise, what cost $\frac{5}{9}$ of an oz.? *Ans.* 3*s.* $8\frac{1}{21}d.$

(15) Bought $\frac{7}{8}$ of a lb. for $\frac{5}{9}$ of a shilling, what cost $\frac{3}{8}$ of $\frac{5}{6}$ of a cwt.? *Ans.* 1*l.* 2*s.* $2\frac{1}{2}d.+\frac{2}{3}$.

(16) If $3\frac{3}{4}$ yards cost $5\frac{7}{9}s.$, what must be given for $84\frac{1}{4}$ English ells? *Ans.* 8*l.* 3*s.* $2\frac{1}{2}d.+\frac{2}{3}$.

DOUBLE RULE OF THREE IN FRACTIONS.

(1) If I pay $\frac{5}{8}$ of a pound for the carriage of $16\frac{1}{2}$ cwt. $10\frac{1}{2}$ miles, what must I pay for the carriage of $13\frac{1}{4}$ cwt. $17\frac{1}{2}$ miles?
Ans. 16*s.* $8\frac{1}{4}d.+\frac{1}{33}$.

(2) If 5 people eat $16\frac{3}{4}$ lbs. of bread in $4\frac{1}{4}$ days, how many people will eat $69\frac{1}{8}$ lbs. of bread in $15\frac{1}{6}$ days? *Ans.* $5\frac{19075}{24388}$ persons.

(3) Gave 14*l.* 10*s.* for interest of 347*l.* 15*s.* for $2\frac{1}{2}$ years, what must I borrow for $4\frac{1}{4}$ years to pay $75\frac{1}{3}l.$ interest?
Ans. 1062*l.* 15*s.* $3\frac{1}{2}d.+\frac{1134}{1479}$.

(4) 54 pioneers dig a trench $104\frac{1}{2}$ feet long, $7\frac{1}{4}$ broad, $5\frac{1}{8}$ deep, in $14\frac{1}{4}$ days, of 9 hours long, how many days, $4\frac{1}{2}$ hours long, will it require 538 men to dig another trench $500\frac{1}{4}$ feet long, $78\frac{1}{2}$ wide, and $3\frac{1}{4}$ deep? *Ans.* $94\frac{1742813}{66846769}$ men.

(5) If $76\frac{1}{4}l.$ gain $3\frac{1}{2}l.$ in $7\frac{1}{2}$ months, how many months must $958\frac{1}{2}l.$ be in gaining $171\frac{1}{3}l.$? *Ans.* $30\frac{7655}{13419}$ months.

(6) If 47 painters paint a partition $790\frac{1}{2}$ inches long and 28 wide, in $5\frac{1}{4}$ days, how many yards, 37 in. wide, will 240 painters paint in $36\frac{1}{4}$ days? *Ans.* 585 yds. 2 ft. 8 in.$+\frac{212}{1739}$.

(7) If 37 horses eat $74\frac{1}{2}$ quarters of corn in $7\frac{1}{4}$ months, how many quarters will 340 horses eat in $12\frac{1}{4}$ months?
Ans. $1156\frac{782}{1073}$ quarters.

(8) If 120*l.* gain $12\frac{1}{4}l.$ in $3\frac{1}{4}$ years, in how many years will $740\frac{5}{9}l.$ gain $257\frac{1}{7}l.$? *Ans.* $11\frac{24991}{457219}$.

(9) If $37\frac{1}{2}$ lbs. of rice will suffice for the food of 4 Hindoos $2\frac{1}{2}$ days, how many Hindoos will it require to eat $596\frac{1}{4}$ lbs. of rice in $31\frac{1}{3}$ days? *Ans.* $5\frac{7}{94}$.

(10) If 45 weavers manufacture $75\frac{1}{2}$ yards of velvet in $36\frac{1}{4}$ hours, how many weavers will manufacture $965\frac{1}{4}$ yards in 7 days of 10 hours each? *Ans.* $297\frac{15741}{16912}$.

(11) If 4965 men eat $34978\frac{1}{4}$ lbs. of bread in $12\frac{1}{4}$ days, how many days will it require 5796 men to eat $36974\frac{1}{4}$ lbs. of bread? *Ans.* $44\frac{14283273}{38615988}$.

(12) If 140 cows eat $2576\frac{1}{2}$ lbs. of hay in $2\frac{1}{2}$ days, how many cows will eat $13764\frac{1}{8}$ lbs. of hay in $6\frac{1}{8}$ days? *Ans.* $305\frac{9645}{36071}$.

DECIMALS.

DEFINITION.

(1) Decimal Fractions are fractions having their denominators 10, 100, 1000, &c. Decimal Fractions differ from Vulgar Fractions by having their denominators 10, 100, 1000, &c.; whilst Vulgar Fractions have any numbers for their denominators.

(2) The *decimal point* is the point separating integers, or whole numbers, from decimals. Thus, the point between 8 and 3 in the number 48·347 is called the decimal point. The proper place for the point is in a *line* with the top of the numbers, as when placed lower it is liable to be mistaken for the sign of Multiplication.

(3) *Whole numbers*, or integers, are the numbers situated to the *left* of the point, thus 48 in the number 48·347, are the whole numbers.

(4) *Decimals* are numbers situated to the *right* hand of the point, as the ·347 in 48·347, are decimals.

(5) A *finite* decimal is a decimal which terminates, as ·0625.

(6) An *infinite* decimal is one that has no end, or that has its numbers recurring either in periods or in the same figure; as ·68181' or ·3333.

(7) Infinite decimals are divided into simple and compound repetends.

(8) A *simple or single repetend* is a decimal in which there is a repetition of one number only, as ·6666: generally written, ·6' or ·6̸.

(9) A *compound repetend* is a decimal in which there is a repetition of two or more numbers, as ·681'81'.

(10) A *pure repetend* consists solely of *repeating* figures, or repeating periods, as ·1616̸'.

(11) A *mixed repetend* consists of other figures besides the repeating periods, as ·68181, or ·3266, where there are numbers *before* the circulating decimal commences, viz., ·6 and ·32, which

are called the *non-recurring* part, as the 81 and 6 are called the *recurring* part.

(12) Integers, or decimals, are valued according to their proximity to the decimal point. Thus, in the following table, if the decimal point be supposed to be situated between the 1 and 2, the numbers are valued according to their proximity to this point. Thus, the 1 to the left of the point is called the unit's place of the integers, or whole numbers; the 2 on the left is called the tens' place of integers; the 3 the hundredths' place, and so on. Again, on the right of the point the 2 is called the *first* decimal, and is equivalent $\frac{2}{10}$ in Vulgar Fractions; the 3 is called the *second* decimal, and is equivalent to $\frac{3}{100}$; the 4 is called the *third* decimal, and is equivalent to $\frac{4}{1000}$.

(13) *Similar* repetends are circulating or recurring decimals, consisting of an equal number of repeating figures, as $\cdot1616=\frac{16}{99}$ and $\cdot3131=\frac{31}{99}$: where the repetend consists of 2 figures in each example, viz. $\cdot1\dot{6}$ and $\cdot3\dot{1}$.

(14) *Dissimilar* repetends consist of an unequal number of repeating figures, as ·16' and ·321'.

(15) When the repetends commence and terminate at the same place of decimals they are said also to be *conterminous*.

(16) The following table shows the notation of decimals; the value of the numbers arises from their proximity to the decimal point:—

	Tens of Thousds.	Thousds.	Hunds.	Tens.	Units.		Tens.	Hunds.	Thousds.	Tens of Thousds.	
&c.	5	4	3	2	1	·	2	3	4	5	&c.

(17) All decimal fractions may be expressed as Vulgar Fractions, by placing underneath them unity, or 1 with as many ciphers as there are decimals. Thus ·749 or ·00317 may be thus expressed.

$$\frac{749}{1000} \text{ and } \frac{317}{1000}, \text{ or} \begin{cases} \cdot749=\frac{7}{10}+\frac{4}{100}+\frac{9}{1000} \\ \cdot00317=\frac{0}{10}+\frac{0}{100}+\frac{3}{1000}+\frac{1}{10000}+\frac{7}{100000} \end{cases}$$

Each of these fractions being of the same value as the decimal in its corresponding place. Thus $\frac{7}{10}$ is = to ·7, agreeably to Art. 12; and $\frac{4}{100}$ is equivalent to ·04; and $\frac{9}{1000}$ is equivalent to ·009.

(18) Ciphers affixed to the right hand of decimals do not alter their value.

For $\cdot34=\frac{34}{100}$

So $\cdot3400=\frac{3400}{10000}=\frac{34}{100}$, as before.

(19) Every cipher prefixed to the left hand of any decimal diminishes its value in a tenfold proportion.

Thus
$$\cdot053 = \frac{53}{1000} \text{ or } \frac{53}{1000}$$
$$\cdot0053 = \frac{53}{10000} \text{ or } \frac{53}{1000 \times 10}$$
$$\cdot00053 = \frac{53}{100000} \text{ or } \frac{53}{1000 \times 10 \times 10}$$

Where it will be perceived, that every fraction is diminished in value in a tenfold proportion.

(20) Any Vulgar Fraction can *generally* be accurately expressed by a decimal.

$$\text{Thus } \frac{4}{5} = \frac{40}{50} = \frac{\frac{1}{5}(40)}{10} = \frac{8}{10} = \cdot8.$$

$$5\tfrac{3}{625} = 5 + \frac{30000}{6250000} = 5 + \left(\frac{\frac{1}{625}(30000)}{10000} = \frac{48}{10000}\right) + \cdot0048 = 5 + \cdot0048 = 5\cdot0048.$$

Addition of Decimals.

Rule.—Place the numbers so that point may fall under point, and consequently the first decimal under the first decimal, the second under second, and the third under third, &c., then add as in integers.

Add 2·5729, 427·15, 1076·1, and ·000729 together.

```
   2·5729
 427·15
1076·1
     ·000729
-----------
1505·823629
```

Here we place all the points in a vertical line, so that the first decimal (viz. the 1 of 427·15) may fall under the first decimal (viz. the 5) of 2·5729; and so of the other numbers: we then add as in common numbers.

Proof by Vulgar Fractions.

$$2\cdot5729 = \frac{25729}{10000} = \frac{2572900}{1000000}$$
$$427\cdot15 = \frac{42715}{100} = \frac{427150000}{1000000}$$
$$1076\cdot1 = \frac{10761}{10} = \frac{1076100000}{1000000}$$
$$\cdot000729 = \frac{729}{1000000}$$

Here we turn the decimals into their corresponding fractions, which we reduce to a common denominator, and add together as in addition of Vulgar Fractions, and obtain $\frac{1505823629}{1000000} = 1505\cdot823629$.

$$\therefore \frac{2572900 + 427150000 + 1076100000 + 729}{1000000} = \frac{1505823629}{1000000} = 1505\cdot823629.$$

Add

(1) 2744·1, 48·2968, ·007215, and 2·74329 together.
Ans. 2795·147305.

(2) ·0072, 4·29, 5732·, 5·21721, and ·0009856 together.
Ans. 5741·5153956.

(3) ·000729, ·00005, ·00000629, ·000000952 and 464· together.
Ans. 464·000786242.

(4) 2·07961, 48·2, 796·35, 4178·293, and ·002749 together.
Ans. 5024·925359.

Add

(5) 6294·, 42·196, 3·82965, ·0296, and 524·3817 together.
Ans. 6864·43695.

(6) ·009, 4·07329, ·168, 716·3821, and 15967·331 together.
Ans. 16687·96339.

(7) ·0000925, ·0317, 409·21, 2·81, and 43179·6829 together.
Ans. 43591·7346925.

(8) ·01275, 408296·31, 728·43, and 52968·713 together.
Ans. 461993·46575.

(9) 409·216, 20821·3, ·0498693, and 5217·219 together.
Ans. 26447·7848693.

(10) ·072948, 32·15, ·7096, 42·145, and ·08296 together.
Ans. 75·160508.

(11) ·04872, 2·98645, 257·329 and ·072968 together.
Ans. 260·437138.

(12) 519·68, 41·732, 519·698, 41·786, and 5176·82 together.
Ans. 6299·716.

(13) 1·7493, 249·682, 1417·31, and ·0971561 together.
Ans. 1668·8384561.

(14) 469·829, 4178·219, ·03219, and 17698·29 together.
Ans. 22346·37019.

(15) ·016, ·4719, 52·168, 41·7319, and 59681·31 together.
Ans. 59775·6978.

(16) 37171·72, 1409·8176, 1208·173, and 1596·309 together.
Ans. 41386·0196.

(17) 51674·219, 3176·3298, ·004674, and 2968·309 together.
Ans. 57818·862474.

SUBTRACTION OF DECIMALS.

RULE.—Place the numbers as in addition: subtract as in common subtraction, previously conceiving a cipher or ciphers to be affixed to the minuend, if no other number or numbers be there.

Ex. From 29·1 take 3·1729.

$$\begin{array}{r} 29{\cdot}1 \\ 3{\cdot}1729 \\ \hline 25\ 9271 \end{array}$$

Here we place the numbers as in addition, *i. e.* we put the *unit* whole number of the subtrahend under the unit of the whole number of the minuend, and commence by conceiving ciphers to be placed over 9, 2, and 7, and subtract as in common numbers.

Proof by Vulgar Fractions.

$$\left.\begin{array}{l} 29{\cdot}1 = \frac{291}{10} = \frac{291000}{10000} \\ 3{\cdot}1729 = \frac{31729}{10000} \end{array}\right\} = \frac{291000}{10000} - \frac{31729}{10000} = \frac{291000-31729}{10000}$$

$$= \frac{259271}{10000} = 25{\cdot}9271.$$

(1)	From ·2864 take ·02987.	*Ans.* ·25653.
(2)	„ ·00982 take ·000198.	*Ans.* ·009622.
(3)	„ 3·0729 take ·918245.	*Ans.* 2·154655.
(4)	„ 256·82 take 198·2572.	*Ans.* 58·5628.
(5)	„ ·09947 take ·0329.	*Ans.* ·06657.
(6)	„ ·32719 take ·0099.	*Ans.* ·31729.
(7)	„ 2·74 take 2·6983195.	*Ans.* ·0416805.
(8)	„ 269·81 take 36·5198.	*Ans.* 233·2902.
(9)	„ 5986·7 take 2986·493.	*Ans.* 3000·207.
(10)	„ 1·782539 take ·63200000.	*Ans.* 1·150539.
(11)	„ 2865·9 take 30·821957.	*Ans.* 2835·07804
(12)	„ ·0928625 take ·00928625.	*Ans.* ·08357625.
(13)	„ 21·74968 take ·3029.	*Ans.* 21.44678.
(14)	„ 1·0009829 take ·0097.	*Ans.* ·9912829.
(15)	„ 3·17698 take 1·5798.	*Ans.* 2·59718.
(16)	„ 52·389 take ·00093215.	*Ans.* 52·3880678
(17)	„ 31692·7 take ·000627.	*Ans.* 31692·6993
(18)	„ 327·1 take 43·81917.	*Ans.* 283·28083.
(19)	„ ·029 take ·021735.	*Ans.* ·007265.
(20)	„ ·258·97326 take ·0625.	*Ans.* 258·91076.
(21)	„ 329·7 take 42·986.	*Ans.* 286·714.
(22)	„ 60·91 take ·093215.	*Ans.* 60·816785.
(23)	„ 4·173 take 2·91863.	*Ans.* 1·25437.
(24)	„ ·0000925 take ·00008.	*Ans.* ·0000125
(25)	„ 51·67 take 3·819673.	*Ans.* 47·850327.
(26)	„ 1·6812 take ·298451.	*Ans.* 1·382749.
(27)	„ 0·098925 take ·0098925.	*Ans.* ·0890325.
(28)	„ 3·15682 take ·072984.	*Ans.* 3·083836.
(29)	„ 9·173 take ·00098159.	*Ans.* 9·17201841
(30)	„ 12·0986 take 5·1829098.	*Ans.* 6·9156902.

RULE FOR MULTIPLICATION OF DECIMALS.

RULE.—Place the numbers and multiply as in Simple Mu plication. Then point off as many decimals from the produc there are decimals in the multiplier and multiplicand.

20·73
5·12
4146
2 073
103 65
106·1376

Here we multiply as in Simple Multiplication, point off 4 decimal places; because, there must b many decimals in the product as there are in the n tiplier and multiplicand.

Proof by Vulgar Fractions.

$$\left.\begin{aligned}20{\cdot}73 &= \tfrac{2073}{100}.\\ 5{\cdot}12 &= \tfrac{512}{100}.\end{aligned}\right\} = \frac{2073\times512}{10000} = \frac{1061376}{10000} = 106{\cdot}1376.$$

Multiply 28·25 by 3·28	*Ans.* 92·66
" 2·083 by ·025	*Ans.* ·052075
" 3·2517 by ·023	*Ans.* ·0747891
" 5·169 by ·2159	*Ans.* 1·1159871
" ·00998 by ·6098	*Ans.* ·006085804
" ·371698 by ·037	*Ans.* ·013752826
" 5·1658 by ·0991	*Ans.* ·51193078
" ·00961 by ·0213	*Ans.* ·000204693
" 41·78 by ·0629	*Ans.* 2·627962
" 417·8 by 329·1	*Ans.* 137497·98
" 3·179 by ·9096	*Ans.* 2·8916184
" ·017 by ·609997	*Ans.* ·010369949
" 3·256 by ·00986	*Ans.* ·03210416
" 9·173 by 20·968	*Ans.* 192·352464
" ·003296 by 5·782	*Ans.* ·019057472
" 3782·4 by ·00917	*Ans.* 34·684608
" 296·4 by 1·72	*Ans.* 509·808
" 32·49 by 520·	*Ans.* 16894·8
" ·6319 by ·5821	*Ans.* ·36782899
" ·3219 by 6219·	*Ans.* 2001·8961
" 3·0721 by ·000965	*Ans.* ·0029645765
" ·00729 by 416·2	*Ans.* 3·034098
" 20·875 by 32·1	*Ans.* 670·0875
" 419·3 by 71·29	*Ans.* 29891·897
" ·0986 by ·0029	*Ans.* ·00028594
" ·06983 by 2·1497	*Ans.* ·150013551
" 169·81 by 3·217	*Ans.* 546·27877
" 21·381 by 4·198	*Ans.* 89·757438
" ·009829 by 42·374	*Ans.* ·416494046
" 3·2968 by 5·7382	*Ans.* 18·90769776
" ·009869 by 3·0274	*Ans.* ·0298773106
" 1973·8 by 51·782	*Ans.* 102207·3116

RULE FOR DIVISION OF DECIMALS.

RULE 1.—Divide as in Simple Division. Then point off as y decimals in the quotient as the dividend contains, more than divisor.

RULE 2.—The figure standing first in the quotient must be first, second, third, &c. integer or decimal; according as the first whole number (or place of it, if there be no whole number) in the divisor stands under the first, second, third, &c. decimal, or first, second, third, &c. whole number.

Ex. Divide 3247·1 by ·0092.

·0092)3247·10000(352945·6
276
487
460
271
184
870
828
420
368
520
460
600
552
28

Here we divide as in Simple Division, disregarding the decimal point, till we have terminated the division. We then perceive that the dividend has one more decimal place than the divisor; we therefore point off 1 decimal place in the quotient.

2nd Rule.—Here the place of the first whole number of the divisor falls under the sixth place of whole numbers of the dividend; a mark 0 has been put for the place of the first whole number of the dividend, represented by prefixing two dots. This is a much superior rule to the preceding, showing the number of integers or decimals by inspection.

·0092)··3247·1(352945·6
0·0276
&c. &c.

Secondly, It is evident that the product of the divisor and quotient, with the remainder added, must equal the dividend; and as we have learnt in multiplication of decimals, that there must be as many decimals in the product as there are in the multiplier and multiplicand, it follows if the divisor have a certain number, say 3, and the dividend 7, that the quotient must have 4; hence the Rule generally given, that "there will be as many decimals in the quotient as the dividend has more than the divisor."

Proof of Division by Vulgar Fractions.

Divide 1·82375 by ·07295.

Here $1{\cdot}82375 \div {\cdot}07295 = \frac{182375}{100000} \div \frac{7295}{100000} = \frac{182375}{100000} \times \frac{100000}{7295} = \frac{182375}{7295}$.

Divide

(1)	42·1796 by ·25 to 3 decimals	*Ans.* 1687·184
(2)	217·3 by 4,176 to 2 decimals	*Ans.* 51·55
(3)	27·4319 by ·0217 to 1 decimal	*Ans.* 126·4
(4)	·0325986 by ·0009 to 6 decimals	*Ans.* 36·220666
(5)	426·812 by 24·9 to 2 decimals	*Ans.* 17·14
(6)	2·149 by 500·78 to 5 decimals	*Ans.* ·42913
(7)	3176·82 by ·09317 to 5 decimals	*Ans.* 34097·02694
(8)	·08318 by 52· to 5 decimals	*Ans.* ·00159
(9)	519·62 by 7849· to 4 decimals	*Ans.* ·0662
(10)	171·98 by ·00372 to 1 decimal	*Ans.* 46231·1559
(11)	·00269582 by 5217· to 9 decimals	*Ans.* ·000000516
(12)	327·4 by ·0092849	*Ans.* 35261·
(13)	4981·74 by 2968·349	*Ans.* 1·98153
(14)	·002964 by 302·78	*Ans.* ·00000979258
(15)	32·495 by ·00376	*Ans.* ·86
(16)	·031705 by 2·476	*Ans.* ·012
(17)	329·495 by ·29	*Ans.* 1136·1
(18)	·09863 by 175· to 7 decimals	*Ans.* ·0005636
(19)	132·417 by ·032 to 2 decimals	*Ans.* 4138·03
(20)	5·09832 by ·581 to 7 decimals	*Ans.* ·8775077
(21)	2317· by ·00525 to 1 decimal	*Ans.* 441333·3
(22)	31·782 by 4·817 to 4 decimals	*Ans.* 7·5919
(23)	·00984 by 24·81 to 6 decimals	*Ans.* ·0003966
(24)	596· by ·6931 to 2 decimals	*Ans.* 859·904
(25)	1739·4 by 2·821 to 2 decimals	*Ans.* 616·58
(26)	·0516 by 282·91 to 6 decimals	*Ans.* ·000182
(27)	829·4 by 17·81 to 4 decimals	*Ans.* 46·5693
(28)	518·9 by 2·78314 to 2 decimals	*Ans.* 186·44
(29)	·0017984 by 68·34	*Ans.* ·000026315
(30)	5296·7 by 428·3	*Ans.* 12·
(31)	482·59 by 20·98	*Ans.* 23·
(32)	·00329 by ·000967	*Ans.* 3·402
(33)	32·09 by 42·796	*Ans.* ·749
(34)	47·298 by 6·029	*Ans.* 7·84508

REDUCTION OF DECIMALS.

To reduce a Fraction to a Decimal.

RULE.—Add ciphers to the numerator, and divide by the denominator; point off as many decimals in the quotient as there are ciphers added to the numerator.

Reduce the following fractions to decimals:—

					Ans.
(1)	$\frac{2}{3}$	*Ans.* ·6666$\cancel{6}$			
(2)	$\frac{5}{9}$	*Ans.* ·5555$\cancel{5}$	(13)	$\frac{415}{634}$	·65457
(3)	$\frac{7}{8}$	*Ans.* ·875	(14)	$\frac{718}{1296}$	·554012
(4)	$\frac{3}{4}$	*Ans.* ·75	(15)	$\frac{315}{729}$	·432098
(5)	$\frac{6}{7}$	*Ans.* ·857142+	(16)	$\frac{21}{36}$	·58333
(6)	$\frac{4}{9}$	*Ans.* ·44444	(17)	$\frac{14}{35}$	·4
(7)	$\frac{5}{12}$	*Ans.* ·4166	(18)	$\frac{17}{112}$	·151785714285+
(8)	$\frac{13}{60}$	*Ans.* ·2166	(19)	$\frac{13}{191}$	·06806+
(9)	$\frac{195}{14}$	*Ans.* 13·928571	(20)	$\frac{19}{265}$	·07169811+
(10)	$\frac{72}{135}$	*Ans.* ·533$\cancel{3}$	(21)	$\frac{7092}{8315}$	·85291+
(11)	$\frac{64}{73}$	*Ans.* ·876712	(22)	$\frac{621}{784}$	·79209183+
(12)	$\frac{2184}{7132}$	*Ans.* ·306225	(23)	$\frac{5}{67}$	·0747761+

To find the value of a Decimal in parts of an Integer.

Of the Decimal of a £.

RULE 1st.—Double the first place of decimals for shillings.

2nd.—If the second decimal be 5 or more than 5, add one shilling more.

3rd.—The second and third decimals are farthings, abating 1 when above 23, and 2 when above 37.

This rule, and its converse, page 144, were composed by one of my worthy instructors, T. Kemp, Esq., Buckingham Street, Fitzroy Square, whose classical, mathematical, and scientific attainments I have never yet seen surpassed.

Ex.—Required the integral values of the following decimals of a £:—

			s.	*d.*				*s.*	*d.*
(1)	·875	=	17	6	(8)	·375	=	7	6
(2)	·6	=	12	0	(9)	·4833$\cancel{3}$	=	9	8
(3)	·55	=	11	0	(10)	·13125	=	2	7½
(4)	·1666	=	3	4	(11)	·87608$\cancel{3}$	=	17	6¼
(5)	·003125	=	0	0¾	(12)	·9625	=	19	3
(6)	·0354166'	=	0	8½	(13)	·63958$\cancel{3}$	=	12	9½
(7)	·0447916	=	0	10½					

Of the Decimal Parts of any other Integer.

RULE.—Multiply by as many of the next less denomination as make that term, the decimal of which you have to find the value of; point off the requisite number of decimals; the integral numbers on the left hand will be the value required.

Ex.—Find the value of ·625 of a cwt.

cwt.
·625
4
2·500
28
14·000

Here we multiply by 4, because 4 qrs. make a cwt., and point off 3 decimals as directed; obtain 2 qrs. for integer, and the decimal ·500, which is in qrs.; we then multiply by 28, because 28 lbs. make a quarter, and obtain 14 lbs.: therefore 2 qrs. 14 lbs. is the value of ·625 of a cwt.

	What is the value of—	*Ans.*
(1)	·09734 of a dwt.?	2·33616 grs.
(2)	·275 of a bushel?	1·1 peck.
(3)	·02978 of a hhd.?	1 gal. 3·50456 gls.
(4)	·3215 of a tun?	81·018 gal.
(5)	·21714 of a ton?	4 cwt. 1 q. 10·3936 lbs.
(6)	·10986 of a cwt.?	12·30432 lbs.
(7)	·002915 of a year?	1·063975 days.
(8)	·3217 of an acre?	1 rood, 11·4720 poles.
(9)	·711215 of a rod?	3 yds. 2·735046 ft.
(10)	·05398 of an English ell?	1·0796 nails.
(11)	·31742 of a lb. Troy?	3 ozs. 16·1808 dwts.
(12)	·0298 of a lb. Avoirdupoise?	7·6288 drs.
(13)	·6845 of a scruple?	13·69 grs.
(14)	·13829 of a pipe of wine?	17·42454 galls.
(15)	·79864 of a yard?	2 ft. 4·75104 ins.
(16)	·02784 of an oz. Troy?	13·3632 grs.
(17)	·31495 of a qr. of wheat?	2 bls. 2·0784 pks.
(18)	·7148 of a rundlet?	12 galls. 3·4656 qts.
(19)	·13275 of a perch?	2·19036 ft.
(20)	·01974 of a ton?	1 cwt. 3·9792 grs.
(21)	·69835 of a league?	1 mile, 167·288 yds.
(22)	·02984 of a day?	42·9696 minutes.
(23)	·2179 of a mile?	383·504 yds.
(24)	·061728 of a furlong?	13·58016 yds.
(25)	·9165 of a peck?	1 gall. 3·332 qts.
(26)	·173159 of a foot?	2·077908 ins.
(27)	·2609 of a firkin?	2 galls. 2·7848 pts.
(28)	·9824 of a ton?	19 cwt. 2 qrs. 94720 lbs.
(29)	·62951 of a hhd. of wine?	39 galls. 2·63652 qts.
(30)	·3298 of an anker of brandy?	3 galls. 1·192 qts.
(31)	·65325 of a puncheon?	54 galls. 3·492 qts.
(32)	·209573 of an acre?	33 rods, 16·0833 yds.
(33)	·713829 of a yard?	2 ft. 1·697844 in.
(34)	·20983 of a league?	1107·9024 yds.
(35)	·79817 of a cwt.?	3 qrs. 5 lbs. 6·32 ozs.
(36)	·4095 of a barrel of beer?	14 galls. 2·9806 qts.

	What is the value of—	*Ans.*
(37)	·59867 of a pipe?	1 hhd. 12 gal. 1 qr. 1·45936 pts.
(38)	·6815 of a pint?	2·726 galls.
(39)	·309153 of an oz. Troy?	6 dwts. 4·393440 grs.
(40)	·79685 of a lb. Avoirdupoise?	12 ozs. 11·9936 dr.
(41)	·932915 of a pipe of wine?	117 galls. 1·8916 qts.
(42)	·275 of a hhd.?	17 galls. 1·3 qts.
(43)	·75 of a hhd.?	40 galls. 2 qts.
(44)	·2057 of a lb. Troy?	2 ozs. 9 dwts. 8·8 grs.
(45)	·3965 of a mile?	3 fur. 6 p. 4 yds. 6·24 ins
(46)	·9825 of a furlong?	39 pls. 1 yd. 0 ft. 3·6 ins.
(47)	·625 of a league?	1 mile, 7 furlongs.
(48)	·3025 of a gallon?	1 qt. 0 pt. 1·68 gill.
(49)	·70952 of a peck?	1 gall. 1 qt. 1·35232 pts.

Examples for Practice.

(1) Express ·25, ·0925, 127·3, ·0007, 3·000009 in the form of vulgar fractions.

$\frac{25}{100}$, $\frac{925}{10000}$, $\frac{1273}{10}$, $\frac{7}{10000}$, $\frac{3000009}{1000000}$.

To reduce Shillings, Pence, and Farthings, to the Decimal of a £.

Rule 1.—Take half the shillings for the first place of decimals.

2. Let the farthings contained in the pence and farthings supply the second and third places, adding 1 when above 23.

3. If the shillings be odd, the second decimal must be increased by 5.

4. Divide half the farthings contained in the pence and farthings by 12, decimally, abating 6*d.*, if there be 6*d.* in the pence.

Ex.—Reduce 17*s.* 9½*d.* to a decimal.

Here we take half the 17 for the 1st place of decimals, or ·8. We then turn the pence and farthings into decimals, and obtain 38, to which we add 1, making 39; we then, as the shillings (17) are odd, increase the second decimal by 5, making it 8; we thus have obtained ·889; we then abate 6*d.* of the 9½*d.*, and get for remainder 3½*d.* = 14 farthings, the half of which is 7, which we divide by 12 decimally, obtaining 5836, and affix these to ·889, obtaining ·8895836.

	s.	*d.*		
(1)	17	9½	=	·8895833
(2)	17	6½	=	·877083
(3)	16	4½	=	·81875
(4)	13	11¼	=	·696875
(5)	7	8½	=	·385416
(6)	0	6¾	=	·028125
(7)	19	0¼	=	·9510416
(8)	12	3¾	=	·615625
(9)	1	8½	=	·085416
(10)	7	3¼	=	·3835416

To reduce a given quantity to the Decimal of any other denomination.

RULE.—Place the terms to be reduced on the left-hand side, and the term to which they are to be reduced on the right; reduce the left-hand terms to the lowest terms mentioned, or if there be a fraction, to the value of this fraction; reduce the right-hand quantity to the same denomination as the left; place it, when reduced, as the denominator of the left-hand term, and divide numerator by denominator, and it will give the decimal required.

	Reduce		to the decimal.	*Ans.*
(1)	2 qrs. 16 lbs.	Avoirdupoise.	a cwt.	·65178571428
(2)	10 ozs. 12 dwts.		a lb.	·8833
(3)	3 qrs. 18 lbs.		a ton	·9107142
(4)	1 qr. 9 lbs.		a cwt.	·2580356
(5)	26 lbs. 11 ozs.		a ton	·0139134
(6)	3 dwts. 17 grs.	Troy weight.	a lb.	·015451388
(7)	2 do. 8 do.		an oz.	·11666
(8)	7 ozs. 3 dwts.		a lb.	·595833
(9)	23 grs.		a lb.	·0039903
(10)	7¾ ozs.		a lb.	·64583
(11)	22 galls. 3 qts.	Wine measure.	a tun	·091277
(12)	3 run. 1 gall.		½ a hhd.	1·74603
(13)	12½ galls.		a hhd.	·2836734
(14)	73 gall. 1 qt.		a pipe	·581349
(15)	34 miles		a degree	·5666
(16)	9 bar. br.		a butt	·0833
(17)	2 scru. 7 grs.		a lb. Apoth.	·00815975
(18)	7¾ ins.		a rod	·341309+
(19)	13¼ ins.		a yard	·37222
(20)	3 yds. 2 ft. 1 in.		a furlong	·01679+
(21)	34 hs. 12 min.		3 weeks	·067857
(22)	2 sq. yds. 4 ft.		a sq. acre	·0050505
(23)	⅞ of a league		geogl. deg.	·04375
(24)	⅔ of ½ a mile		3½ ins.	2413·7142
(25)	4/7 of a rod		¾ of a furlong	·019047619
(26)	⅔ of a guinea		of 7*s.* 2¼*d.*+⅛	1·9471114
(27)	⅕ of ⅔ of half a ¼ guinea		1/9 of 2*s.* 3¾*d.*	·3405405
(28)	⅔ of a £		⅕ of 6¾*d.*	118·5185
(29)	1/7 of half a crown		4*s.* 6¼*d.*+⅛	·078099
(30)	⅔ of (2*s.* 6½*d.*+1/7)		half a guinea	·1611
(31)	⅓ of ⅖ of 3¼ lbs. Avoir.		¾ of an oz.	9·24
(32)	half of half a crown		⅕ of 2*s.* 3¼*d.*	2·752
(33)	⅔ of ⅞ of a half moidore		⅓ of ⅔ of a guinea.	1·6875

	Reduce	to the decimal.	*Ans.*
(34)	$\frac{1}{9}$ of a £	$\frac{3}{7}$ of 5 farthings	74·66
(35)	$\frac{2}{7}$ of $\frac{1}{2}$ of 6$\frac{3}{4}$*d.*	$\frac{1}{8}$ of a £	·1446428
(36)	$\frac{1}{7}$ of a yard	$\frac{2}{5}$ of a mile	·0002029
(37)	$\frac{2}{5}$ of a furlong	$\frac{1}{9}$ of 7 leagues	·021428
(38)	$\frac{4}{5}$ of a shilling	$\frac{2}{7}$ of 3*s.* 2$\frac{1}{4}$*d.*+$\frac{1}{3}$	·0312
(39)	$\frac{1}{3}$ of 2*s.* 10*d.*	$\frac{7}{8}$ of a £	·04096
(40)	$\frac{2}{3}$ of half a guinea	$\frac{6}{11}$ of a crown	2·565
(41)	$\frac{3}{8}$ of 3*s.* 9$\frac{1}{2}$*d.*+$\frac{1}{3}$	$\frac{1}{2}$ of 1*s.* 6*d.*	126·305

Useful method of converting a Fraction, whose Denominator is a prime number, into its equivalent Decimal.

This rule was first given by Colson, in Sir I. NEWTON'S *Fluxions*, p. 162.; and the following is extracted from DAVIES' HUTTON'S *Course of Mathematics*, a work the student will find one of the best for obtaining a knowledge of practical mathematics:—

Let $\frac{1}{29}$ be the fraction. Then by dividing in the common way till the remainder becomes a single figure, we shall have $\frac{1}{29}=·03448\frac{8}{29}$ for the complete quotient, and this equation being multiplied by the numerator 8, will give $\frac{8}{29}=·27584\frac{64}{29}$, or rather $·27586\frac{6}{29}$; and if this be substituted instead of the fraction $\frac{8}{29}$, it will make $\frac{1}{29}=·0344827586\frac{6}{29}$. Again, let this equation be multiplied by 6, and it will give $\frac{6}{29}=·2068965517\frac{7}{29}$; and then substituting, as before, viz., for $\frac{6}{29}$, we obtain

$$\frac{1}{29}=·0344827586206896551 7\frac{7}{29};$$

and so on, as far as may be thought proper, each fresh multiplication doubling the number of figures in the decimal value of the fraction.

(2) Required the vulgar fractions, in their lowest terms, of these decimals:

·275, ·35, ·125, ·0725, ·042

$·275=\frac{11}{40}$, $·35=\frac{7}{20}$, $·125=\frac{1}{8}$, $·0725=\frac{29}{400}$ $·042=\frac{21}{500}$

Convert these vulgar fractions into decimals:

$\frac{3}{25}$, $\frac{4}{625}$, $\frac{6}{75}$

$\frac{3}{25}=·24$ $\frac{4}{625}=·0064$ $\frac{6}{75}=·08$

(3) Transform the mixed decimals, 2·0375, 15·25, 3·009175, 3·76, 17·0034, and 6·425.

$2·0375=\frac{163}{80}$, $15·25=\frac{61}{4}$, $3·009175=\frac{120367}{40000}$, $3·76=\frac{94}{25}$, $17·0034=\frac{850017}{50000}$, $6·425=\frac{257}{40}$.

(4) Give the approximate values of $\frac{7}{18}$, $\frac{4}{7\cdot5}$, $\frac{2}{175}$, $\frac{147}{275}$, $\frac{7}{325}$

$\frac{7}{18}=\cdot388\dot{8}$, $\frac{4}{7\cdot5}=\cdot5333$, $\frac{2}{175}=\cdot01\dot{1}4285\dot{7}$, $\frac{147}{275}=\cdot58\dot{4}\dot{5}$
$\frac{7}{325}=\cdot02\dot{1}5384\dot{6}$.

(5) Express ·003125*l.* as the decimals of a shilling and a half-crown. *Ans.* ·0625 and ·025.

(6) Express ·7625*l.*+$\frac{1}{4}$ of half-sovereign + ·9 of 8*s.* 6*d.*+ ·275 of 12*s.* 4*d.*+$\frac{1}{8}$ of 7$\frac{1}{2}$*d.*+$\frac{1}{4}$ of 3*d.*, as the decimal of 12*s.* 8*d.* *Ans.* 2·69376.

RECURRING DECIMALS.

Any vulgar fraction when reduced to a decimal, and the figures of whose quotient do not terminate, but continually recur, is called a *recurring or circulating decimal.*

The *period* is the quantity repeated, and is subdivided into simple or compound repetend: simple, when only *one* figure is repeated; *compound,* when *more* figures than *one* are repeated. The extent of the period (that is, the number in the circle, as it is called) is expressed by dotting the first and last numbers in it.

Ex.—·4166 is a simple repetend, and $71428\dot{5}$ is a compound repetend. The first is represented by $\cdot41\dot{6}$, and the second by $\cdot\dot{7}1428\dot{5}$.

Sometimes other figures besides the repetend are found in the quotient, in this case the decimal assumes the name of a *mixed* circulating or mixed repetend. The part not repeated is called the *non*-recurring part, and the part repeated is called the *recurring* part.

Ex.—Convert $\frac{5}{7}$ and $\frac{5}{9}$ into decimals.

7)5·000000 &c., this is called a *compound repetend,* and is
·714285 expressed thus, $\cdot\dot{7}1428\dot{5}$.

9)5·0000 &c., this is called a *simple repetend,* and is expressed
5555 thus, $\cdot\dot{5}$. These are also called *pure circulates.*

CASE 1.

To convert a pure recurring Decimal into a Vulgar Fraction.

Let the decimal be ·333, then let

$$\begin{aligned} x &= \cdot333 \\ 10x &= 3\cdot333 \\ \hline 9x &= 3\cdot \quad \text{by subtraction.} \\ x &= \tfrac{3}{9}=\tfrac{1}{3} \quad \text{the fraction required.} \end{aligned}$$

Rule.—Place the *repetend* as numerator of a fraction, and let its denominator consist of as many nines as the repetend consists of figures. Reduce it to its lowest terms, and this will be the fraction required.

Ascertain the vulgar fractions equivalent to the pure repetend ·3.

Here, agreeably to the rule, we perceive that the repetend consists of only one number. We, therefore, place as many nines as the repetend consists of figures underneath, and obtain $\frac{3}{9}$, which reduced, gives $\frac{1}{3}$ for the fraction required.

Here $\frac{3}{9} = \frac{1}{3}$

Case 2.

To find the Vulgar Fraction which will be equivalent in value to a mixed recurring *decimal.*

To find the vulgar fraction which shall be equivalent to ·41666.

Let
$$\begin{aligned} x &= .41\dot{6} \\ 10x &= 4{\cdot}16 \\ 100x &= 41{\cdot}66 \\ 1000x &= 416{\cdot}66 \\ \hline 900x &= 375 \quad \text{by subtraction.} \\ x &= \frac{375}{900} = \frac{75}{180} = \frac{5}{12} \end{aligned}$$

Let
$$\begin{aligned} x &= .25 \\ 10x &= 2{\cdot}5 \\ 100x &= 25{\cdot}5 \\ \hline 90x &= 23 \\ x &= \frac{23}{90} \left\{ \text{the fraction required.} \right. \end{aligned}$$

Hence the following rule.

Subtract the non-recurring part of the mixed recurring decimal from the non-recurring and recurring parts taken together, and make this the numerator of the fraction; then place as many nines as there are *recurring* figures, and as many noughts as there are *non-recurring* figures, for the denominator. The fraction being reduced, will give the fraction required.

Ex.—Ascertain the vulgar fraction equivalent to the mixed repetend ·68181.

Here '68181'= $\left\{\right.$the mixed repetend, of which 6 is the *non-recurring* part,
and 81 the *recurring* part ∴

681 the *non-recurring* and the *recurring* parts taken together.
6 the *non-recurring* part taken alone.
675 by subtraction, which gives the numerator of the fraction required.

Underneath which we are to place as many nines as there are recurring figures, and as many noughts as there are non-recurring figures, which, as there are two recurring figures, viz., 81, and only one non-recurring figure, we obtain 99 for the 81, and 0 for the 6, making for denominator 990; therefore our fraction is $\frac{675}{990}$, or $\frac{135}{198}$ when reduced.

Examples to Cases 1 *and* 2.

(1) Find the fractions corresponding to the following recurring decimals:—

$\cdot 8\dot{3}$, $\cdot\dot{3}$, $\cdot 214\dot{4}$, $\cdot 3266$

Ans. $\frac{5}{6}$, $\frac{1}{3}$, $\frac{193}{900}$, $\frac{147}{450}$

As the operations in addition, subtraction, multiplication, and division of recurring decimals, are generally very laborious, it is always better to turn the decimals into vulgar fractions, and perform the necessary operations upon these.

Note.—The decimal arising from the division of the numerator by the denominator of a fraction, will be *finite* or *infinite*, according as the numbers, 10, 100, 1000, 10000, &c., or their multiples, are *divisible* by the denominator or *not.* We thence find that as 2 and 5 are the only incomposite factors of 10, 100, &c., that any vulgar fraction, whose denominator is resolvable into the factors 2 and 5, will, when made a decimal fraction, be *finite*, and all others not possessing this property, will be *infinite.*

Ex. 1. $\frac{7}{80} = \frac{7}{2.5.8} = \frac{7}{2.2.2.2.5} = \cdot 0875$ finite decimal.

2. $\frac{8}{15} = \frac{8}{3.5}$ ———————— $\cdot 5\dot{3}$ recurring and infinite decimal.

PART III.

COMMERCIAL ARITHMETIC.—PRACTICE.

This rule in commerce is of more frequent occurrence than any other rule; hence its name. Its principles are frequently applied to other rules, as cross multiplication. It is performed by taking aliquot parts; the parts of the most use are those which follow.

Of a £.

s.	*d.*			*s.*	*d.*			*s.*	*d.*			*s.*	*d.*		
10	0	is	$\frac{1}{2}$	3	4	is	$\frac{1}{6}$	1	4	is	$\frac{1}{15}$	0	4	is	$\frac{1}{60}$
6	8	..	$\frac{1}{3}$	2	6	..	$\frac{1}{8}$	1	3	..	$\frac{1}{16}$	0	3	..	$\frac{1}{80}$
·5	0	..	$\frac{1}{4}$	2	0	..	$\frac{1}{10}$	1	0	..	$\frac{1}{20}$	0	2	..	$\frac{1}{120}$
4	0	..	$\frac{1}{5}$	1	8	..	$\frac{1}{12}$	0	6	..	$\frac{1}{40}$				

Of a shilling.	Of a ton.	Of a cwt.	Of a penny.	Of a qr.
6*d*. is $\frac{1}{2}$	10 cwt. is $\frac{1}{2}$	2 qrs. is $\frac{1}{2}$	$\frac{1}{2}$*d*. is $\frac{1}{2}$	14 lbs. is $\frac{1}{2}$
4 .. $\frac{1}{3}$	5 „ .. $\frac{1}{4}$	1 „ .. $\frac{1}{4}$	$\frac{1}{4}$ .. $\frac{1}{4}$	7 „ .. $\frac{1}{4}$
3 .. $\frac{1}{4}$	4 „ .. $\frac{1}{5}$	16 lbs. .. $\frac{1}{7}$		4 „ .. $\frac{1}{7}$
2 .. $\frac{1}{6}$	$2\frac{1}{2}$ „ .. $\frac{1}{8}$	14 „ .. $\frac{1}{8}$		$3\frac{1}{2}$ „ .. $\frac{1}{8}$
1 .. $\frac{1}{12}$	2 „ .. $\frac{1}{10}$	8 „ .. $\frac{1}{14}$		
$\frac{3}{4}$ .. $\frac{1}{16}$	1 „ .. $\frac{1}{20}$	7 „ .. $\frac{1}{16}$		
$\frac{1}{2}$ .. $\frac{1}{24}$		4 „ .. $\frac{1}{28}$		
$\frac{1}{4}$ .. $\frac{1}{48}$		$3\frac{1}{2}$ „ .. $\frac{1}{32}$		

The various cases resolve themselves into the following:—

1. When the price is less than a penny.
2. When more than a penny and less than a shilling.
3. When more than 1 shilling and less than 2 shillings.
4. When above 2 shillings and less than a £.
5. When an aliquot part of a £.
6. When *minus* an aliquot part of a £.
7. When the price consists of pounds, shillings, and pence.
8. When the quantity consists of several denominations, and the price, of £. *s*. *d*.

CASE 1.

When the price is less than a penny.

If the price be an aliquot part of a penny, divide the quantity by it; if not, take the greatest aliquot part of a penny in the price, and divide by it; then take the aliquot part of a penny of the remaining price; add the quotients together, divide by 12 and 20.

Ex.—32947 lbs. at $\frac{1}{4}$d.

$$\begin{array}{l|l|l} \frac{1}{4} & \frac{1}{4} & 32947 \\ \hline & & 12)8236\frac{3}{4} \\ \hline & & \;\;20)686\;\;4\frac{3}{4} \\ \hline & & \;\;\;\;\pounds 34\;\;6\;\;4\frac{3}{4} \end{array}$$

Here if the price had been 1*d*. per lb., the value of 32947 lbs. would have been 32947 pence; but as one farthing is the $\frac{1}{4}$ of a penny, the value must necessarily be only $\frac{1}{4}$ of what it would be when at 1*d*.; we therefore, first, take the value at 1*d*., and then take a fourth part of it, which gives us 8236$\frac{3}{4}$ pence, which we bring to pounds by dividing by 12 and 20, as in reduction.

Ex. 2.—47315 lbs. at $\frac{3}{4}$*d*.

Here we have $\frac{3}{4}$*d*. for the price; now $\frac{3}{4}$*d*. are not an aliquot part of a penny; we therefore take the *greatest* aliquot part out of the $\frac{3}{4}$*d*., viz. $\frac{1}{2}$*d*., and we find that $\frac{1}{2}$*d*. or 2 farthings are $\frac{1}{2}$ of a penny; we therefore divide the 47315 by 2, and obtain 23657$\frac{1}{2}$; we now perceive that we have the value of 47315 lbs. at

½	½	47315
¼	½	23657½
		11828¾
		12)35486¼
		20)2957 2
		£147 17 2¼

½d., but our price is ¾d.; we therefore find we are to take the ½ of the 23657½d., because ¼d. is the ½ part of a half-penny; therefore we have 47315 lbs. at ½d., and 47315 lbs. at ¼d., which, added together, give the value of 47315 at ¾d.; we then divide by 12 and 20, and obtain £147 17s. 2¼d.

Case 2.

When the price is more than a penny and less than a shilling.

Rule.—Take the greatest aliquot part of a shilling, contained in the price, and divide the quantity by it; then take the greatest aliquot part of the remaining price, either of a shilling or of the aliquot part already found; and so on till all the price has been taken. The sum of all the quotients, divided by 20, will be the price required.

Ex.—92945 at 8d.

6	½	92945
2	⅓	46472½
		15490¾
		20)61963¼
		£3098 3 0¼

Here 92945 articles at 1s. each is 92945 shillings, and 93945 articles at 6d. must be ½ of the price at a shilling; we therefore say 6d. is the ½, and divide the 92945 by 2; but we then see we have only taken 6d. of our price, and that 2d. remains; we therefore say 2 is the ⅓ of 6d., and take the ⅓ of the 6d. line: the sum of the quotients, divided by 20, gives 3098*l.* 3s. 0¼d.

Case 3.

When the price is more than a shilling and less than 2 shillings.

Rule.—Take aliquot parts of the pence and farthings, and add these to the given quantity taken as shillings; the sum divided by 20 will give the answer.

Ex.—32794 at 1s. 4½d.

4	⅓	10931 4
½	⅛	1366 5
		20)45091 9
		£2254 11 9

Here 32794 articles at 1s. each is 32794 shillings, and 32794 at 4d. each must be the third part of the preceding; also 32794 at ½ each must be the ⅛ part of the same number at 4d.; we therefore take ⅛ of the preceding line, and the sum of the quotients ÷ by 20, gives 2254*l.* 11s. 9d.

Case 4.

When the price is above 2 shillings and less than a £.

Rule 1.—If the price be an even number of shillings, multiply the quantity by half the number of shillings, doubling the product of the first number and the half number for shillings; then multiply the remaining numbers by the half number of shillings for pounds.

Ex.—31729 at 6*s*.
3
£9518 14

Here we multiply the 9 by 3 (because 3 is half the number of shillings), double this product and obtain 54, which is 2*l*. 14*s*.; we put down the 14 and carry the 2; we then multiply all the remaining numbers, 3172, by 3, adding in the 2 carried, and obtain 9518*l*. 14*s*.

Rule 2.—But if the shillings be not even, multiply the given quantity by the shillings, taking aliquot parts of the price out of the original number, and divide by 20, as in *Case* 3.

Ex.—95824 at 13*s*. 6¾*d*.
13
1225712
| 6 | ½ | 47912
| ¾ | ⅛ | 5989
20)1279613
£63980 13

Here we multiply the 95824 by 13, which gives 1225712 shillings; but 95824 articles at 6*d*. each must be half the price at 1*s*. each; we therefore divide the original quantity by 2; we then perceive that 3 farthings (the remainder of the price) is ⅛ of 6*d*.; we therefore divide the preceding line, viz. 47912, by 8; the sum of all their quotients is 1279613 shillings, or 63980*l*. 13*s*.

Case 5.

When the price is an aliquot part of a £.

Rule.—Divide by the aliquot part of a £, and the quotient is pounds.

Ex.—21748 at 3*s*. 4*d*.
| 3 4 | ⅙ | 3624 13 4

Here we perceive that 3*s*. 4*d*. is the ⅙ of a £; we therefore divide by 6, and obtain for quotient 3624*l*. 13*s*. 4*d*.

Note.—The answer is more quickly obtained in instances of this kind by multiplying the remainder into the aliquot part; thus, in this instance, 4 remains after dividing by 6; we therefore multiply 3*s*. 4*d*. by 4, which gives for product 13*s*. 4*d*. The reason is obvious, for after division we get for remainder 4/6, which shows that the £ is divided into 6 parts, and that 4 of them are to be taken for the fraction.

Case 6.

When the price is minus *an aliquot part of a shilling or a £.*

Rule.—Divide the aliquot part of a shilling or a £ into the given quantity, and subtract this quotient from the quantity.

Ex.—

2	$\frac{1}{6}$	3295 at 10d.		
		549	2	
		2,0)274,5	10	
		£137	5	10

Here 3295 articles at a shilling will be 3295 shillings; but we have reckoned 2*d.* too much for each one; therefore we subtract the value of 3295 at 2*d.* from 3295 at a shilling, and obtain 137*l.* 5*s.* 10*d.*

Ex.—

5	$\frac{1}{4}$	2498 at 15*s.*	
		624	10
		£1873	10

Here 2498 articles at 1*l.* for each would be 2498*l.*; but we have taken the price 5*s.* too great; we therefore subtract the value of 2498 articles at 5*s.* from the preceding, and obtain 1873*l.* 10*s.*

CASE 7.

When the price consists of pounds, shillings, pence, and farthings.

RULE.—Multiply the quantity by the pounds, and take aliquot parts of a pound for the rest of the price.

Ex.—3159 at 3*l.* 16*s.* 7½*d.*

		3		
10	$\frac{1}{2}$	9477		
5	$\frac{1}{2}$	1579	10	
1	$\frac{1}{5}$	789	15	
7½	$\frac{1}{8}$	157	19	
		98	19	4½
		£12103	3	4½

Here we multiply the quantity by the 3, and take parts of the shillings, pence, and farthings; the sum of all these quotients is the value sought.

CASE 8.

When the quantity consists of several denominations, and the price of £. s. d.

RULE.—Multiply the price of one of the given denominations by the number of that denomination, taking aliquot parts of the remaining denominations: the sum of all these quotients is the amount sought.

Ex.—What cost 25 cwt. 2 qrs. 16 lbs., at 2*l.* 14*s.* 9½*d.* per cwt.?

		2	14	9½
				5×5
		13	13	11½
				5
		68	9	9½
2	$\frac{1}{2}$	1	7	4¾
16	$\frac{1}{7}$		7	9¾
		£70	5	0

Here we multiply the value of one cwt. by the number of cwts., and take parts of the qrs. and lbs.

But the following rule is generally in its application much shorter.

1. Multiply the lbs. by $2\frac{1}{7}$, and call the product *pence*, which convert into shillings, putting down the pence and carrying the shillings; multiply the quarters by 5, and call the product shillings, which convert into pounds: the result will be the value or price of the cwts., qrs., and lbs., at 1*l.* per cwt. 2. Then multiply this by the pounds in the price, and finally, find the value of 1 cwt. at the price of the remaining shillings, pence, and farthings: the sum of all these is the price required.—See *Case* 21, MENTAL ARITHMETIC.

CASE 1.

	Required the value of		*d.*			£.	*s.*	*d.*
(1)	32964	apples	.. at $\frac{1}{4}$	each.	*Ans.*	34	6	9
(2)	72869	pears	.. at $\frac{1}{2}$	do.	*Ans.*	151	16	$2\frac{1}{2}$
(3)	32176	biscuits	.. at $\frac{3}{4}$	do.	*Ans.*	100	11	0
(4)	40987	onions	.. at $\frac{1}{4}$	do.	*Ans.*	42	13	$10\frac{3}{4}$
(5)	25864	cabbages	.. at $\frac{1}{4}$	do.	*Ans.*	26	18	10
(6)	62179	cauliflowers	.. at $\frac{1}{2}$	do.	*Ans.*	129	10	$9\frac{1}{2}$
(7)	46957	plums	.. at $\frac{3}{4}$	do.	*Ans.*	146	14	$9\frac{3}{4}$
(8)	59864	..	.. at $\frac{1}{4}$		*Ans.*	62	7	2
(9)	28477	..	.. at $\frac{1}{2}$		*Ans.*	59	6	$6\frac{1}{2}$
(10)	65248	..	.. at $\frac{3}{4}$		*Ans.*	203	18	0

CASE 2.

			d.			£.	*s.*	*d.*
(1)	99569	.. at	1	....	*Ans.*	414	17	5
(2)	72684	.. at	$1\frac{1}{4}$	....	*Ans.*	378	11	3
(3)	45926	.. at	$1\frac{1}{2}$	....	*Ans.*	287	0	9
(4)	89847	.. at	$1\frac{3}{4}$	....	*Ans.*	655	2	$8\frac{1}{4}$
(5)	67824	.. at	2	....	*Ans.*	564	4	0
(6)	42968	.. at	$2\frac{1}{2}$	....	*Ans.*	447	11	8
(7)	78964	.. at	$2\frac{3}{4}$	....	*Ans.*	904	15	11
(8)	49215	.. at	3	....	*Ans.*	615	3	9
(9)	78926	.. at	$3\frac{1}{4}$	....	*Ans.*	1068	15	$9\frac{1}{2}$
(10)	69245	.. at	$3\frac{1}{2}$	....	*Ans.*	1009	16	$5\frac{1}{2}$
(11)	76198	.. at	$3\frac{3}{4}$	....	*Ans.*	1190	11	$10\frac{1}{2}$
(12)	29164	.. at	4	....	*Ans.*	486	1	4
(13)	78196	.. at	$4\frac{1}{4}$	....	*Ans.*	1384	14	5
(14)	95972	.. at	$4\frac{1}{2}$	....	*Ans.*	1799	9	6
(15)	28479	.. at	$4\frac{3}{4}$	....	*Ans.*	563	10	$11\frac{1}{4}$
(16)	79648	.. at	5	....	*Ans.*	1659	6	8
(17)	68924	.. at	$5\frac{1}{4}$	....	*Ans.*	1502	14	3
(18)	21989	.. at	$5\frac{1}{2}$	....	*Ans.*	503	18	$3\frac{1}{2}$

			d.			£.	s.	d
(19)	59268 ..	at	5¾	....	*Ans.*	1419	19	3
(20)	78649 ..	at	6	....	*Ans.*	1966	4	6
(21)	59689 ..	at	6¼	....	*Ans.*	1554	8	0¼
(22)	89247 ..	at	6½	....	*Ans.*	2417	2	1½
(23)	72947 ..	at	6¾	....	*Ans.*	2051	10	8¼
(24)	82945 ..	at	7	....	*Ans.*	2424	7	2
(25)	92864 ..	at	7¼	....	*Ans.*	2805	5	4
(26)	25982 ..	at	7½	....	*Ans.*	757	16	2
(27)	78964 ..	at	7¾	....	*Ans.*	2549	17	7
(28)	82916 ..	at	8	....	*Ans.*	2763	17	4
(29)	72968 ..	at	8¼	....	*Ans.*	2508	5	6
(30)	91269 ..	at	8½	....	*Ans.*	3232	8	10½
(31)	82964 ..	at	8¾	....	*Ans.*	3024	14	7
(32)	97682 ..	at	9	....	*Ans.*	8547	3	6
(33)	72196 ..	at	9¼	....	*Ans.*	2782	11	1
(34)	82926 ..	at	9½	....	*Ans.*	3282	9	9
(35)	82796 ..	at	9¾	....	*Ans.*	3363	11	9
(36)	92714 ..	at	10	....	*Ans.*	3863	1	8
(37)	98965 ..	at	10¼	....	*Ans.*	4226	12	7¼
(38)	71496 ..	at	10½	....	*Ans.*	3127	19	0
(39)	82964 ..	at	10¾	....	*Ans.*	7864	5	11
(40)	52916 ..	at	11	....	*Ans.*	2425	6	4
(41)	78964 ..	at	11¼	....	*Ans.*	3701	8	9
(42)	42968 ..	at	11½	....	*Ans.*	2058	7	8
(43)	41726 ..	at	11¾	....	*Ans.*	2042	16	8½

CASE 3.

			£	s.	d.			£	s.	d.
(1)	52982	at	0	1	0	....	*Ans.*	2649	0	2
(2)	78296	at	0	1	0¼	....	*Ans.*	3996	7	1½
(3)	92809	at	0	1	0½	....	*Ans.*	4833	15	7½
(4)	78296	at	0	1	0¾	....	*Ans.*	4159	9	6
(5)	12968	at	0	1	1	....	*Ans.*	702	8	8
(6)	23892	at	0	1	1¼	....	*Ans.*	1319	0	9
(7)	17968	at	0	1	1½	....	*Ans.*	1010	14	0
(8)	52174	at	0	1	1¾	....	*Ans.*	2989	2	8½
(9)	62999	at	0	1	2	....	*Ans.*	3574	18	10
(10)	82968	at	0	1	2¼	....	*Ans.*	4296	4	6
(11)	79682	at	0	1	2½	....	*Ans.*	4814	2	5
(12)	42968	at	0	1	2¾	....	*Ans.*	2640	14	10
(13)	62973	at	0	1	3	....	*Ans.*	3935	16	3
(14)	78296	at	0	1	3¼	....	*Ans.*	4975	1	2
(15)	49639	at	0	1	3½	....	*Ans.*	3205	17	0½

But the following rule is generally in its application mu shorter.

1. Multiply the lbs. by 2¼, and call the product *pence*, whi convert into shillings, putting down the pence and carrying shillings; multiply the quarters by 5, and call the product sh lings, which convert into pounds: the result will be the value price of the cwts., qrs., and lbs., at 1*l*. per cwt. 2. Then m tiply this by the pounds in the price, and finally, find the value 1 cwt. at the price of the remaining shillings, pence, and fa things: the sum of all these is the price required.—See *Case* MENTAL ARITHMETIC.

CASE 1.

	Required the value of		*d.*		£.	*s.*	
(1)	32964 apples	..	at ¼ each.	*Ans.*	34	6	
(2)	72869 pears	..	at ½ do.	*Ans.*	151	16	
(3)	32176 biscuits	..	at ¾ do.	*Ans.*	100	11	
(4)	40987 onions	..	at ¼ do.	*Ans.*	42	13	1
(5)	25864 cabbages	..	at ¼ do.	*Ans.*	26	18	1
(6)	62179 cauliflowers	..	at ½ do.	*Ans.*	129	10	
(7)	46957 plums	..	at ¾ do.	*Ans.*	146	14	
(8)	59864 ..	..	at ¼	*Ans.*	62	7	
(9)	28477 ..	..	at ½	*Ans.*	59	6	
(10)	65248 ..	..	at ¾	*Ans.*	203	18	

CASE 2.

				d.			£.	*s.*	*d.*
(1)	99569	..	at	1	....	*Ans.*	414	17	5
(2)	72684	..	at	1¼	....	*Ans.*	378	11	3
(3)	45926	..	at	1½	....	*Ans.*	287	0	9
(4)	89847	..	at	1¾	....	*Ans.*	655	2	8¼
(5)	67824	..	at	2	....	*Ans.*	564	4	0
(6)	42968	..	at	2½	....	*Ans.*	447	11	8
(7)	78964	..	at	2¾	....	*Ans.*	904	15	11
(8)	49215	..	at	3	....	*Ans.*	615	3	9
(9)	78926	..	at	3¼	....	*Ans.*	1068	15	9½
(10)	69245	..	at	3½	....	*Ans.*	1009	16	5½
(11)	76198	..	at	3¾	....	*Ans.*	1190	11	10½
(12)	29164	..	at	4	....	*Ans.*	486	1	4
(13)	78196	..	at	4¼	....	*Ans.*	1384	14	5
(14)	95972	..	at	4½	....	*Ans.*	1799	9	6
(15)	28479	..	at	4¾	....	*Ans.*	563	10	11¼
(16)	79648	..	at	5	....	*Ans.*	1659	6	8
(17)	68924	..	at	5¼	....	*Ans.*	1502	14	3
(18)	21989	..	at	5½	....	*Ans.*	503	18	3½

				d.			£.	s.	d
(19)	59268	..	at	5¾	....	*Ans.*	1419	19	3
(20)	78649	..	at	6	....	*Ans.*	1966	4	6
(21)	59689	..	at	6¼	....	*Ans.*	1554	8	0¼
(22)	89247	..	at	6½	....	*Ans.*	2417	2	1½
(23)	72947	..	at	6¾	....	*Ans.*	2051	10	8¼
(24)	82945	..	at	7	....	*Ans.*	2424	7	2
(25)	92864	..	at	7¼	....	*Ans.*	2805	5	4
(26)	25982	..	at	7½	....	*Ans.*	757	16	2
(27)	78964	..	at	7¾	....	*Ans.*	2549	17	7
(28)	82916	..	at	8	....	*Ans.*	2763	17	4
(29)	72968	..	at	8¼	....	*Ans.*	2508	5	6
(30)	91269	..	at	8½	....	*Ans.*	3232	8	10½
(31)	82964	..	at	8¾	....	*Ans.*	3024	14	7
(32)	97682	..	at	9	....	*Ans.*	8547	3	6
(33)	72196	..	at	9¼	....	*Ans.*	2782	11	1
(34)	82926	..	at	9½	....	*Ans.*	3282	9	9
(35)	82796	..	at	9¾	....	*Ans.*	3363	11	9
(36)	92714	..	at	10	....	*Ans.*	3863	1	8
(37)	98965	..	at	10¼	....	*Ans.*	4226	12	7¼
(38)	71496	..	at	10½	....	*Ans.*	3127	19	0
(39)	82964	..	at	10¾	....	*Ans.*	7864	5	11
(40)	52916	..	at	11	....	*Ans.*	2425	6	4
(41)	78964	..	at	11¼	....	*Ans.*	3701	8	9
(42)	42968	..	at	11½	....	*Ans.*	2058	7	8
(43)	41726	..	at	11¾	....	*Ans.*	2042	16	8½

Case 3.

			£	s.	d.			£	s.	d.
(1)	52982	at	0	1	0	....	*Ans.*	2649	0	2
(2)	78296	at	0	1	0¼	....	*Ans.*	3996	7	1½
(3)	92809	at	0	1	0½	....	*Ans.*	4833	15	7½
(4)	78296	at	0	1	0¾	....	*Ans.*	4159	9	6
(5)	12968	at	0	1	1	....	*Ans.*	702	8	8
(6)	23892	at	0	1	1¼	....	*Ans.*	1319	0	9
(7)	17968	at	0	1	1½	....	*Ans.*	1010	14	0
(8)	52174	at	0	1	1¾	....	*Ans.*	2989	2	8½
(9)	62999	at	0	1	2	....	*Ans.*	3574	18	10
(10)	82968	at	0	1	2¼	....	*Ans.*	4296	4	6
(11)	79682	at	0	1	2½	....	*Ans.*	4814	2	5
(12)	42968	at	0	1	2¾	....	*Ans.*	2640	14	10
(13)	62973	at	0	1	3	....	*Ans.*	3935	16	3
(14)	78296	at	0	1	3¼	....	*Ans.*	4975	1	2
(15)	49639	at	0	1	3½	....	*Ans.*	3205	17	0½

	Reqd. the value of		£	s.	d.			£	s.	d.
(16)	73295	at	0	1	3¾	....	*Ans.*	4809	19	8¼
(17)	29516	at	0	1	4	....	*Ans.*	1967	14	8
(18)	78649	at	0	1	4¼	....	*Ans.*	5325	3	10¼
(19)	65987	at	0	1	4½	....	*Ans.*	4536	12	1½
(20)	43829	at	0	1	4¾	....	*Ans.*	3058	17	11¾
(21)	94937	at	0	1	5	....	*Ans.*	6724	14	1
(22)	62973	at	0	1	5¼	....	*Ans.*	4513	1	2½
(23)	82965	at	0	1	5½	....	*Ans.*	6049	10	8½
(24)	96872	at	0	1	5¾	....	*Ans.*	7164	9	10
(25)	59673	at	0	1	6	....	*Ans.*	4375	9	6
(26)	28784	at	0	1	6¼	....	*Ans.*	3188	15	8
(27)	49687	at	0	1	6½	....	*Ans.*	3830	0	9½
(28)	37968	at	0	1	6¾	....	*Ans.*	2966	5	0
(29)	42984	at	0	1	7	....	*Ans.*	3402	18	0
(30)	79824	at	0	1	7¼	....	*Ans.*	6402	11	0
(31)	39864	at	0	1	7½	....	*Ans.*	3238	19	0
(32)	96529	at	0	1	7¾	....	*Ans.*	7943	10	7¾
(33)	59867	at	0	1	8	....	*Ans.*	2163	18	4
(34)	25968	at	0	1	8¼	....	*Ans.*	2191	1	0
(35)	89257	at	0	1	8½	....	*Ans.*	7624	0	8½
(36)	62438	at	0	1	8¾	....	*Ans.*	5398	5	8½
(37)	92647	at	0	1	9	....	*Ans.*	8106	12	3
(38)	45293	at	0	1	9¼	....	*Ans.*	4010	6	4¼
(39)	78249	at	0	1	9½	....	*Ans.*	7009	16	1½
(40)	65982	at	0	1	9¾	....	*Ans.*	5979	12	4½
(41)	78256	at	0	1	10	....	*Ans.*	8173	9	4
(42)	49384	at	0	1	10¼	....	*Ans.*	4578	6	2
(43)	38625	at	0	1	10½	....	*Ans.*	3621	1	10½
(44)	20917	at	0	1	10¾	....	*Ans.*	1875	15	3¾
(45)	69782	at	0	1	11	....	*Ans.*	6687	18	10
(46)	52867	at	0	1	11¼	....	*Ans.*	5721	9	9¾
(47)	29783	at	0	1	11½	....	*Ans.*	2916	5	0½
(48)	59687	at	0	1	11¾	....	*Ans.*	5906	10	6¼
(49)	48692	at	0	2	6½	....	*Ans.*	6187	18	10
(50)	58965	at	0	7	3¾	....	*Ans.*	21559	1	6¾
(51)	46986	at	0	12	9½	....	*Ans.*	30051	9	3
(52)	486982	at	1	11	6½	....	*Ans.*	768011	3	10¾
(53)	59687	at	3	18	9½	....	*Ans.*	235141	18	2½

CASE 4.

	Required the value of		s.			£	s.
(1)	32596 pairs of braces..	at	2	each.	*Ans.*	3259	12
(2)	47985 pairs silk hose..	at	4	do.	*Ans.*	9597	0

	Required the value of			s.			£	s.
(3)	68957	pairs of shoes ..	at	6	each.	*Ans.*	20687	2
(4)	74329	parasols ..	at	8	do.	*Ans.*	29731	12
(5)	51687	lace habits ..	at	10	do.	*Ans.*	25843	10
(6)	73195	bonnets ..	at	12	do.	*Ans.*	43917	0
(7)	84927	satin stocks ..	at	14	do.	*Ans.*	59448	18
(8)	71396	silk waistcoats ..	at	16	do.	*Ans.*	57116	16
(9)	20498	beaver hats ..	at	18	do.	*Ans.*	18448	4

CASE 5.

	Required the value of			s.	d.			£	s.	d.
(1)	35698	cheeses ..	at	3	6	each.	*Ans.*	6247	3	0
(2)	67395	lbs. Castile soap	at	3	10	do.	*Ans.*	12917	7	6
(3)	82956	handkerchiefs	at	4	11½	do.	*Ans.*	20566	3	6
(4)	57967	pieces calico ..	at	6	8½	pr. pce.	*Ans.*	19276	0	8¼
(5)	82967	account books .	at	9	10½	each.	*Ans.*	40964	19	1½
(6)	57398	veils ..	at	12	8¼	do.	*Ans.*	36411	17	1½
(7)	69824	jackets ..	at	13	8½	do.	*Ans.*	47858	10	8
(8)	78579	lbs. of indigo..	at	15	9¼	do.	*Ans.*	61962	16	3¼
(9)	25718	lbs. of cochineal	at	16	7	do.	*Ans.*	21324	10	2
(10)	42951	barrels of butter	at	17	8½	do.	*Ans.*	38039	8	1½
(11)	32968	sides of bacon..	at	17	4½	do.	*Ans.*	28640	19	0
(12)	59632	ditto	at	15	9¼	do.	*Ans.*	47022	6	4
(13)	47389	bags of biscuit	at	14	11¼	do.	*Ans.*	35398	13	2¼
(14)	20983	hats ..	at	9	8¾	do.	*Ans.*	10207	7	1¼
(15)	76957	handkercfs. silk	at	3	7¼	do.	*Ans.*	60457	11	6¾
(16)	40969	silk hatbands..	at	8	11¾	do.	*Ans.*	18393	7	5¾
(17)	57396	muslin dresses	at	19	8½	do.	*Ans.*	56558	19	6
(18)	78965	caps ..	at	15	3¾	do.	*Ans.*	60457	11	6¾

CASE 6.

Req. the value of			s.	d.		£	s.	d.
(1)	46986	at	12	9½	*Ans.*	30051	9	3
(2)	53274	at	15	6¼.	*Ans.*	41342	16	10½
(3)	72949	at	14	9½.	*Ans.*	53951	17	3½
(4)	31572	at	13	6½.	*Ans.*	21376	17	6
(5)	61586	at	10	9¼.	*Ans.*	33166	12	6½
(6)	92918	at	11	8½.	*Ans.*	42687	8	3
(7)	31825	at	9	7¼.	*Ans.*	15282	12	7¼
(8)	92378	at	4	7¾.	*Ans.*	21458	12	9½
(9)	58965	at	7	3¾.	*Ans.*	21559	1	6¾
(10)	31572	at	13	6½.	*Ans.*	21376	17	6
(11)	61586	at	10	9¼.	*Ans.*	33166	12	6½

Req. the value of			s.	d.		£	s.	d.
(12)	72918	at	11	8½.	*Ans.*	42687	8	3
(13)	31825	at	9	7¼.	*Ans.*	15282	12	7¼
(14)	92378	at	4	7¾.	*Ans.*	21458	12	9½
(15)	27684	at	7	3¼.	*Ans.*	10064	5	9
(16)	91382	at	6	9¾.	*Ans.*	31126	19	10½
(17)	84749	at	3	2½.	*Ans.*	13595	3	0½
(18)	61537	at	13	8½.	*Ans.*	42178	9	8½
(19)	53274	at	15	6¼.	*Ans.*	41342	16	10½
(20)	78943	at	10	6¾.	*Ans.*	41691	15	5¼
(21)	87684	at	19	8¾.	*Ans.*	86496	12	3
(22)	76134	at	4	8½.	*Ans.*	17923	4	3
(23)	76543	at	9	6¾.	*Ans.*	36597	2	5¼
(24)	87693	at	5	1¾.	*Ans.*	22562	13	5¾

CASE 7.

Req. the value of			£	s.	d.		£	s.	d.
(1)	2968	at	2	18	6½.	*Ans.*	8687	11	8
(2)	7935	at	1	11	9¼.	*Ans.*	12605	1	6¾
(3)	2096	at	3	18	10½.	*Ans.*	8266	2	0
(4)	7569	at	4	15	8¼.	*Ans.*	36212	18	8¼
(5)	2997	at	1	3	6½.	*Ans.*	3527	14	4½
(6)	5789	at	2	5	6¼.	*Ans.*	13176	0	1¼
(7)	2593	at	2	16	7½.	*Ans.*	7341	8	7½
(8)	4932	at	3	14	6¾.	*Ans.*	18387	2	3
(9)	5987	at	3	15	8½.	*Ans.*	22663	5	9½
(10)	7965	at	7	18	10½.	*Ans.*	63271	19	4½
(11)	4329	at	8	14	9¼.	*Ans.*	37829	2	11¼
(12)	8469	at	9	17	11¼.	*Ans.*	83816	12	8¼

CASE 8.

Req. the value of			£	s.	d.		£	s.	d.
(1)	36789	at	3	3	4.	*Ans.*	116498	10	0
(2)	57325	at	2	18	6.	*Ans.*	167675	12	6
(3)	35198	at	3	15	9.	*Ans.*	133312	8	6
(4)	82917	at	5	12	3½.	*Ans.*	465544	8	1½
(5)	48276	at	3	2	9¾.	*Ans.*	151616	16	3
(6)	69574	at	2	18	7¾.	*Ans.*	204011	5	0½
(7)	40969	at	10	4	4½.	*Ans.*	418651	19	4½
(8)	58637	at	6	13	9½.	*Ans.*	392257	1	11½
(9)	51782	at	7	9	4¼.	*Ans.*	385829	16	9
(10)	83907	at	2	16	9¼.	*Ans.*	238173	10	3¾
(11)	58315	at	11	10	6¾.	*Ans.*	672262	12	2½
(12)	486982	at	1	11	6½.	*Ans.*	768011	3	10¾

Req. the value of			£	s.	d.		£	s.	d.
(13)	59687	at	3	18	9½.	*Ans.*	235141	18	2½
(14)	47926	at	2	14	7¼.	*Ans.*	130847	19	3½
(15)	24827	at	3	15	9½.	*Ans.*	94083	19	8½
(16)	74986	at	1	19	8.	*Ans.*	149222	4	8
(17)	46381	at	2	2	3¾.	*Ans.*	98123	4	9¾

CASE 9.

Required the value of the following quantities :—

	cwt.	qrs.	lbs.		£	s.	d.		£	s.	d.
(1)	36	3	19	at	2	15	6¼.	*Ans.*	102	9	9¼
(2)	57	2	18	at	5	16	8¼.	*Ans.*	336	8	3¼
(3)	48	2	16	at	4	19	7¼.	*Ans.*	242	5	0
(4)	53	3	14	at	2	19	6½.	*Ans.*	160	7	9½
(5)	88	3	12	at	1	19	8¼.	*Ans.*	176	17	6
(6)	73	2	11	at	3	9	7¼.	*Ans.*	256	2	8½
(7)	55	3	26	at	2	19	10½.	*Ans.*	167	11	10½
(8)	96	2	15	at	1	12	7½.	*Ans.*	833	19	4½
(9)	76	1	18	at	7	3	9½.	*Ans.*	549	7	2¼
(10)	68	0	17	at	3	15	6¼.	*Ans.*	257	6	10¼
(11)	98	2	16	at	5	16	9¼.	*Ans.*	575	18	7¼
(12)	84	3	19	at	4	18	6½.	*Ans.*	418	18	1¼
(13)	36	3	19	at	3	12	9.	*Ans.*	134	5	10½
(14)	49	2	14	at	4	9	6½.	*Ans.*	222	3	6
(15)	56	1	18	at	1	17	6¼.	*Ans.*	105	16	6¾
(16)	46	1	26	at	3	16	8½.	*Ans.*	155	0	8¼
(17)	67	3	15	at	2	19	6½.	*Ans.*	201	11	10¾
(18)	84	1	15	at	3	18	6½.	*Ans.*	331	7	7½
(19)	57	2	14	at	4	16	9½.	*Ans.*	278	17	7¼
(20)	97	1	14	at	5	11	6½.	*Ans.*	543	1	4¼
(21)	54	2	19	at	6	10	2¼.	*Ans.*	355	17	3
(22)	126	1	11	at	2	18	9½.	*Ans.*	371	8	2¼
(23)	272	3	2	at	5	13	6¾.	*Ans.*	1548	16	2
(24)	94	1	15	at	1	19	5¼.	*Ans.*	186	2	2¾
(25)	47	2	19	at	6	1	11½.	*Ans.*	290	13	8¼
(26)	54	1	17	at	4	13	6¾.	*Ans.*	254	9	11¼
(27)	68	3	5	at	2	17	5¼.	*Ans.*	197	11	4¼
(28)	126	2	19	at	1	16	11¼.	*Ans.*	233	18	9½
(29)	37	3	15	at	2	19	8½.	*Ans.*	113	2	11½
(30)	64	1	19	at	3	12	7¾.	*Ans.*	168	12	1¾
(31)	69	2	17	at	4	19	8½.	*Ans.*	347	4	10
(32)	73	1	18	at	9	10	3½.	*Ans.*	688	9	5
(33)	36	3	9	at	3	15	9½.	*Ans.*	139	10	3½

Required the value of the following goods :—

	cwt.	qrs.	lbs.		£	s.	d.		£	s.	d.
(34)	44	3	7	at	6	14	5½.	*Ans.*	301	5	4¾
(35)	14	2	14	at	8	3	4.	*Ans.*	119	8	9

(36) What is the value of 32 cwt. 2 qrs. 18 lbs., at 2*l*. 14*s*. per cwt.? *Ans*. 88*l*. 3*s*. 7*d*.

(37) Gave 2*l*. 6*s*. 8*d*. for 1 cwt. of sugar, what cost 36 cwt. 2 qrs. 19 lbs.? *Ans*. 85*l*. 11*s*. 3*d*.

(38) Sold 95 cwt. 1 qr. 18 lbs. of cheese, at 3*l*. 4*s*. 6*d*. per cwt.; what did I get for the cheese? *Ans*. 307*l*. 18*s*. 11¾*d*.

MENTAL ARITHMETIC.

CASE 1.

Given the price of one to find the price of a dozen.

RULE.—Call the pence shillings, and add 3*d*. for 1 farthing, 6*d*. for 2 farthings, and 9*d*. for 3 farthings.

Ex.—Gave 4½*d*. for 1 lb. of sugar, what cost 1 dozen lbs.?

We call the 4*d*. 4 shillings, and as there are 2 farthings in a halfpenny, we add 6*d*. to the 4 shillings, obtaining 4*s*. 6*d*. for the value of 1 dozen.

Ex.		*d*.		*s*.	*d*.	Ex.		*d*.		*s*.	*d*.
(1)	1 article at	5½	=	5	6	(7)	1 article at	5¾	=	5	9
(2)	”	6½	=	6	6	(8)	”	8½	=	8	6
(3)	”	9¾	=	9	9	(9)	”	3½	=	3	6
(4)	”	7½	=	7	6	(10)	”	4¾	=	4	9
(5)	”	19¾	=	19	9	(11)	”	10½	=	10	6
(6)	”	2½	=	2	6						

CASE 2.

Having the price of one article to find the price of any number of dozens.

RULE.—Find the value of 1 dozen by the preceding Rule, and then multiply the value of the one dozen by the number of dozens.

Ex.—If one lb. cost 19½*d*., what cost 4 dozen?

	s.	*d*.
1 dozen at 19½*d*. =	19	6
		4
£3	18	0

Required the value of 3 dozen at 9½*d.* per lb.

	s.	*d.*
1 dozen at 9½*d.* =	9	6
		3
£1	8	6

Find the value of 7 dozen of oranges at 1½*d.* each.

	s.	*d.*
1 dozen at 1½*d.* =	1	6
		7
	10	6

Required the value of any number of dozens, having the price of one article.

		d.		£	*s.*	*d*
(1)	2 dozen	at 8½	*Ans.*	0	17	0
(2)	4 „	at 6½	*Ans.*	1	6	0
(3)	3 „	at 9¼	*Ans.*	1	7	9
(4)	12 „	at 10¼	*Ans.*	6	3	0
(5)	8 „	at 4½	*Ans.*	1	16	0
(6)	5 „	at 5¼	*Ans.*	1	6	3
(7)	9 „	at 3¾	*Ans.*	1	13	9
(8)	5 „	at 2¾	*Ans.*	0	3	9

CASE 3.

NOTE.—If there are *any over* after taking out the greatest number of dozens, multiply the price by the remaining number.

Ex. Required the value of 75 at 3½*d.*

1 dozen =	0	3	6
6 dozen =	1	1	0
3 articles =	0	0	10½
∴ 75 at 3½*d.* =	1	1	10½

Here we find the value of 6 dozen as in the preceding case; then the value of 3, which we add to the 1*l.* 1*s.* 0*d.*, and obtain 1*l.* 1*s.* 10½*d.*

Ex.		*d.*	£	*s.*	*d.*
(1)	91 art. at	6¼ =	2	7	4¾
(2)	74 „	11½ =	3	10	11
(3)	105 „	9¼ =	4	0	11¼
(4)	63 „	5¾ =	1	10	2¼
(5)	87 „	10½ =	3	16	1½

Ex.		*d.*	£	*s.*	*d.*
(6)	91 art. at	11¾ =	4	9	1¼
(7)	68 „	4¼ =	1	4	1
(8)	55 „	8½ =	1	18	11½
(9)	97 „	11¼ =	4	11	10¼

CASE 4.

To find the price of any number of lbs.

I. When the price is *pence*.

RULE.—Call the lbs. pence, and convert the pence into shillings (which may generally be done by inspection); then multiply by the given number of pence per lb. in the price.

Ex. 1. Required the value of 129¼ lbs. of raisins at 4*d.* per lb.

	£	*s.*	*d.*
129¼ lbs. called pence =	0	10	9¼
			4
	2	3	1

Ex. 2. Required the value of 320¼ lbs. at 3*d.* per lb.

	£	*s.*	*d.*
320¼ lbs. called pence =	1	6	8¼
			3
	4	0	0¾

NOTE.—If there be a farthing, halfpenny, or three farthings in the price, it is better to obtain the value of this farthing, halfpenny, &c. first, and to add it in, whilst performing the multiplication.

Ex. 136¼ at 5½.

	£	*s.*	*d.*		*s.*	*d.*
136¼ lbs. called pence =	0	11	4¼	its ½ =	5	8
			5			
	3	2	5¼			

Here the price is 5½*d.*; we therefore take ½ the 11*s.* 4¼*d.* (the lbs. reckoned as pence) which is 5*s.* 8*d.*, which we add to the 5 times 11*s.* 4¼*d.*, getting for product 3*l.* 2*s.* 4¼*d.*

Ex.	*d.*	£	*s.*	*d.*	Ex.	*d.*	£	*s.*	*d.*
(1) 14½ art. at	10¾=	0	12	11¾	(6) 125 art. at	4¼=	2	4	3¼
(2) 209¼ ,,	7¾=	6	15	0½	(7) 753 ,,	6½=	20	7	10½
(3) 209 ,,	5¾=	5	0	0¾	(8) 729½ ,,	9¼=	28	1	11¼
(4) 496 ,,	11½=	23	15	4	(9) 173 ,,	2¾=	1	19	7¾
(5) 249¼ ,,	3½=	3	12	7½	(10) 215 ,,	8¼=	3	19	0¾

CASE 5.

To find the price of any number of lbs. when the price is shillings.

Call the lbs. weight, shillings, and convert these shillings into pounds sterling, then multiply this converted number by the given number of shillings in the price, taking aliquot parts of a shilling for the remaining pence.

Ex.—What is the value of 412½ lbs. at 10*s*. 6*d*. per lb.?

	£	*s*.	*d*.		£	*s*.	*d*.
412½ lbs. called shillings =	20	12	6	its ½ =	10	6	3
			10½				
	216	11	3				

Here we reckon 412½ as shillings, which = 20*l*. 12*s*. 3*d*.; we then take ½ of this, and obtain 10*l*. 6*s*. 1½*d*.; then multiply 20*l*. 12*s*. 3*d*. by 10, taking in the value of ½, and obtain 216*l*. 11*s*. 3*d*.

Ex.			*s*.	*d*.		£	*s*.	*d*.
(1)	725½	articles at	12	2	=	441	6	11
(2)	6597	„	3	4	=	1096	3	4
(3)	299	„	1	8	=	24	18	4
(4)	517¼	„	13	6	=	349	2	10½
(5)	428½	„	7	10	=	167	8	7
(6)	257½	„	13	2	=	168	8	9

CASE 6.

If both quantity and price contain a fraction; find the value of the dozens as before, and then find the value of the remaining parts of the price, which add in whilst multiplying by the number of dozens.

Ex.—79½ lbs. at 3*s*. 6¼*d*.

d.		*d*.		£	*s*.	*d*.	
6¼	=	42¼,	and when called shillings =	2	2	3	value of 1 doz.
						6	
				13	19	10¾	value of 79½ at 3*s*. 6¼*d*.

£	*s*.	*d*.		
1	1	1½	= value of	6
	5	3¼	= „	1½
1	6	4¾	= „	7½

Ex.			*s*.	*d*.		£	*s*.	*d*.
(1)	55¼	lbs. at	4	2½	=	11	12	6
(2)	71¾	„	5	6¼	=	19	16	1¼
(3)	92½	„	3	7½	=	16	15	3¾
(4)	87¼	„	6	11¼	=	30	5	3½

CASE 7.

To calculate the value by Scores.

Reckon the shillings of the price of one article as pounds; this gives the value of 20 articles, which multiply by the number of scores; the last product is the answer.

Ex.—50 hams at 7*s*. each.

7	0	0	= value of 20
		2	
14	0	0	= „ 40
3	10	0	= „ 10
17	10	0	

Acres, roods, and perches can be calculated in a similar manner.

Call the acres pounds; let the *roods*, multiplied by 5, be called shillings, and let the perches, multiplied by 1½, be called pence.

Ex.—Required the annual rent of 64 acres, 3 roods, 22 poles, at 2*l*. 5*s*. per acre.

64	3	22
	5	1½
64	17	9
		2¼
129	15	6
16	4	5¼
£145	19	11¼

Here we put down the multipliers 5 and 1½ underneath the terms by which they are to be multiplied. We begin with the multiplier 1½, saying 1½ times 22 is 33, *i. e.* 33*d.* = 2*s*. 9*d.*; we put down the 9*d*. and carry the 2; we then multiply the 3 roods by the 5, adding in the 2 carried, and obtain 17, which we call shillings. We have thus found the value of 64 acres, 3 roods, 22 poles, at 1*l*. per acre, viz. 64*l*. 17*s*. 9*d*.; finally, we multiply the value of one acre by the 2¼*l*., or by 2*l*. 5*s*.

Required the value of

	acres.	roods.	poles.		£	*s*.	*d*.			£	*s*.	*d*.
(1)	44	3	39	at	3	4	6	per acre.	*Ans.*	145	2	0¾
(2)	17	2	27	at	2	9	6	„	*Ans.*	43	14	7
(3)	28	3	38	at	1	7	6	„	*Ans.*	39	17	1¾
(4)	19	2	19	at	3	18	6	„	*Ans.*	77	0	0¼

The proof of the Rule arises, from considering first the quantity of land given at 1*l*. per acre; thus 64 acres at 1*l*. will be 64*l*., and 3 roods must necessarily be 3 multiplied by 5*s*. (the value of 1 rood); and as 1 rood is 5*s*., it follows that the perch is $\frac{1}{40}$th part of 5*s*., or $\frac{60}{40}$ = 1½ for multiplier.

Ounces, dwts., and grains Troy, can be calculated in a similar manner.

Call the ounces pounds, the dwts. shillings, and add half the number of grains for pence.

Ex.—Required the value of 15 oz. 11 dwts. 17 grs., at 2*s*. 6*d*. per ounce.

		£	*s*.	*d*.	
2*s*. 6*d*.	$\frac{1}{8}$	15	11	8½	
		1	18	11½	value at 2*s*. 6*d*. per oz.

Here we call 15 oz. 15*l*., the 11 dwts. 11*s*., the 17 grs. 8½*d*.

Required the value of

	oz.	dwts.	grs.		*s*.	*d*.			£	*s*.	*d*.
(1)	17	15	19	at	3	8	per oz.	*Ans*.	3	5	2½
(2)	11	17	13	at	2	7	„	*Ans*.	1	10	8
(3)	19	8	12	at	4	3	„	*Ans*.	4	2	6½
(4)	14	17	19	at	6	2	„	*Ans*.	4	11	9½
(5)	16	18	18	at	7	6	„	*Ans*.	6	7	0¼

Proof.—At 1*l*. per ounce, for 15 oz. we obtain 15*l*.; now as 20 dwts. make an ounce, and 20 shillings a pound, each dwt. is reckoned as 1*s*. For 11 dwts. × 20 ÷ 20 = 11 shillings; and as 12*d*. make a shilling, it follows that $\frac{17\times\cancel{12}}{\underset{2}{\cancel{24}}}=\frac{17}{2}=8\frac{1}{2}$.

CASE 8.

To find the Interest of any Sum of Money for any number of Months at 5 per cent. (The Rule for any number of Years at 5 per cent. is in Interest.)

The product of the pounds of the principal by the months, called pence, is the interest required. Or reckon 1*d*. for every 1*l*. sterling, and multiply by the months.

Find the interest on 75*l*. for 7 months at 5 per cent.

75 × 7 = 525*d*. = 43*s*. 9*d*.

	s.	*d*.	
By second Rule 75*l*. at 1*d*. for each =	6	3	
		7	
	43	9	as above.

Here 75*l*. at 1*d*. for each is 6*s*. 3*d*. per month for interest.

Required the interest on

	£	*s*.	*d*.					£	*s*.	*d*.
(1)	84	12	0	for 8	months at 5 per cent.		*Ans*.	2	16	4
(2)	37	15	0	for 7	„	„	*Ans*.	1	2	0¼
(3)	65	13	0	for 4	„	„	*Ans*.	1	1	10
(4)	82	19	0	for 5	„	„	*Ans*.	1	14	6½

Proof.—If 100*l*. gain 5*l*. in one year, 1*l*. must gain the $\frac{1}{20}$th of a pound or 1*s*. Thus 100 : 5 :: 1 : $\frac{1}{20}$*l*. or 1*s*.; ∴ as 1*l*. gains 1*s*. in a year or 12 months, it follows that 1*l*. must gain $\frac{1}{12}$th part of 1*s*. in 1 month, *i. e.*, 1*l*. must gain 1*d*. per month; hence the Rule.

CASE 9.

Interest for years at 5 per cent.

This may be calculated by the Rule in Interest, or by the preceding Rule.

RULE.—Multiply the principal by the years, and reckon 1*s*. for every 1*l*. in the product, 3*d*. for 5*s*., 6*d*. for 10*s*., and 9*d*. for 15*s*.

Required the interest on 4295*l*. 15*s*. for 7 years at 5 per cent.

£	s.		£	s.	d.
4295	15				
	7				
30070	5	=	1503	10	3

Here we multiply the principal by 7, and obtain 30070*l*. 5*s*.; we then take these pounds as shillings, and obtain 1503*l*. 10*s*., to which we add 3*d*. for the 5*s*. of the 30070*l*. 5*s*.

The proof is considered unnecessary; the Rule being the preceding one enlarged.

Ex.—Required the interest on the following sums at 5 per cent.

	£	s.	d.				£	s.	d.
(1)	429	12	0	for 3	years.	*Ans.*	64	8	9
(2)	72	15	0	for 2½	„	*Ans.*	9	1	10½
(3)	841	10	0	for 7	„	*Ans.*	284	10	6
(4)	785	15	0	for 6	„	*Ans.*	235	14	6
(5)	208	5	0	for 4	„	*Ans.*	40	13	0

CASE 10.

For years and months at 5 per cent.

If the principal be divided by the years reckoned as shillings, and the months be reckoned as aliquot parts of a shilling, the sum of these will be the interest required.

Ex.—Required the interest on 419*l*. 16*s*. 8*d*. for 5 years and 8 months.

		£	s.	d.
5	$\frac{1}{4}$	419	16	8
6	$\frac{1}{10}$	104	19	2
2	$\frac{1}{3}$	10	9	11
		3	9	11½
		118	19	0½

Questions in this Rule can be taken from Interest.

CASE 11.

Having the price per ounce, to find it per pound.

RULE.—Call the price in farthings, shillings, and divide by 3.

Ex.—Required the value of 1 lb. of tobacco, at 3¾*d*. per oz.

Here 3¾=15 farthings, which being called shillings, and divided by 3=5*s*.

Exs.—At 6¼*d*., 8½*d*., 4¼*d*., 6¾*d*., 7¼*d*., 8¼*d*., 12¾*d*., 18½*d*., 15¼*d*., 22¼*d*.

Ans. 8*s*. 3*d*., 11*s*. 4*d*., 5*s*. 8*d*., 9*s*., 9*s*. 8*d*., 11*s*., 13*s*. 9*d*., 24*s*. 8*d*., 20*s*. 4*d*., 29*s*. 8*d*.

Proof.—Because if the price of an ounce be reduced to farthings, and multiplied by 16, and then divided by 48, (the number of farthings in a shilling,) the answer will be in shillings; thus,

$\frac{\overset{5}{\cancel{15}}\times\cancel{16}}{\underset{\cancel{3}}{\cancel{48}}} = 5s.$

Or reduce the price to pence and add ⅓, which call shillings.

Ex.—At 6*d*. per oz., what is it per lb.?

$$\begin{array}{rl} & 6 \\ \frac{1}{3}= & \underline{2} \\ & 8 \text{ which, called shillings, is } 8s. \end{array}$$

Proof.—As 1 lb. at 1*d*. per oz. is 1⅓ shilling (*i. e*. 16*d*.) per lb., therefore at any number of pence per lb., if we add ⅓ of the pence to the pence, it is equivalent to multiplying the pence by 1⅓ shillings: hence the rule.

CASE 12.

Having the price of a pound, to find it per ounce.

RULE.—Call the shillings farthings, and multiply by 3.

Ex.—At 7*s*. per lb., what is it per oz.?

7*s*. called 7 farthings, and multiplied by 3=21 farthings=5¼d. per oz.

Exs.—At 8*s*., 18*s*., 12*s*., 13*s*., 29*s*., 14*s*., 15*s*., 6*s*., 2*s*., 17*s*., 16*s*.

Ans. 6*d*., 13½*d*., 9*d*., 9¾*d*., 20¼*d*., 10½*d*., 11¼*d*., 4½*d*., 1½*d*., 12¾*d*., 12*d*.

Proof.—Because the shillings multiplied by 48 farthings, and divided by 16, (the ozs. in a lb.,) will give the answer in farthings; thus, $\frac{7\times\overset{3}{\cancel{48}}}{\cancel{16}} = 21\text{ f.} = 5\frac{1}{4}d.$

Or reduce the shillings to pence; subtract the fourth part, and call it pence.

Ex.—At 8*s*. per lb., what is the price per oz.?

Here 8
subtract $\frac{1}{4}$=2
$\overline{6}$ called pence=6*d*.

Case 13.

Having the value of a lb., to find it per cwt.

Rule.—9*s*. 4*d*. multiplied by the price in pence per lb. is the value sought.

Ex.—If I give 2*d*. per lb., what is that per cwt.?

Here we multiply 9*s*. 4*d*. by 2, and obtain 18*s*. 8*d*. for the value of 1 cwt.

Exs.—At 3*d*., 4*d*., 5*d*., 6*d*., 7*d*., 8*d*., 9*d*., 10*d*., 12*d*., 14*d*., 15*d*., 16*d*., 18*d*., 19*d*.

Ans. 1*l*. 8*s*., 37*s*. 4*d*., 46*s*. 8*d*., 56*s*., 65*s*. 4*d*., 74*s*. 8*d*., 84*s*., 93*s*. 4*d*., 112*s*., 130*s*. 8*d*., 7*l*., 149*s*. 4*d*., 8*l*. 8*s*., 177*s*. 4*d*.

Proof.—Because 112 lbs. at 1*d*. per lb. is 9*s*. 4*d*., ∴ if 9*s*. 4*d*. be multiplied by the number of pence, it must give the answer required.

Case 14.

To find the value of a lb., having given the price per cwt.

Rule.—The value of the article in shillings, multiplied by 3, and divided by 7, is the value sought in farthings.

Ex.—At 22*s*. 4*d*. per cwt., what is that per lb.?

Here 22*s*. 4*d*. multiplied by 3, and divided by 7 $= \frac{67}{7} = 9$ farthings = $2\frac{1}{4}$*d*.

Proof.—Because 22*s*. 4*d*. × 48 (the number of farthings in a shilling), and divided by 112, gives the rule; thus, $\frac{22{\cdot}4\times\overset{3}{\cancel{48}}}{\underset{7}{\cancel{112}}} = \frac{(22\times4)\times3}{7}$.

Case 15.

Given the value of a dozen, to find the value of one.

Rule.—Reckon every shilling as 1*d*.; for 3*d*. add 1 farthing, for 6*d*. add 2 farthings, and for 9*d*. add 3 farthings.

Ex.—At 2*s* 3*d*. per dozen, what is the value of 1?

Here 2s. reckoned as 2d., and ¼d. for the 3d., make 2¼d. for one.

Exs.—At 3s. 3d., 6s. 4d., 7s. 8d., 9s., 8s. 6d., 4s. 6d.

Ans. 3¼d., 6¼d., 7½d., 9d., 8½d., 4½d.

Proof.—Because if the price of one dozen be reduced to farthings, and divided by 12, the quotient is in farthings: $\frac{\overset{9}{17}\times 4}{\underset{3}{12}} = 2\frac{1}{4}d.$

Case 16.

Given the value of a score, to find the value of one.

Rule.—Reckon each pound as a shilling, 15s. as 9d., 10s. as 6d., 5s. as 3d.

Ex.—At 17l. 10s. per score, what is that for one?

Here 17l. 10s., reckoned as 17½d.=17s. 6d.

Exs.—At 25l. 12s. 6d., 9l. 10s., 6l. 18s. 6d.

Proof.—Because if the pounds be multiplied by 20, and divided by 20, we obtain $\frac{\text{£17 10s.} \times \cancel{20} \text{ shillings}}{\cancel{20} \text{ or the score}} = \text{17s. 6d.}$

Case 17.

To find the value of one hundred, the price of one being given.

Rule.—Multiply the price in shillings by 5, the product is pounds.

Ex.—At 12s., 29s., 13s., 21s., 17s., 25s. 6d., 29s. 6d.

Ans. 61l. 5s., 145l., 65l., 105l., 85l., 126l. 10s., 147l. 10s.

If the product give any pence, reckon 15s. for 9d., 10s. for 6d., 5s. for 3d., and in proportion for other intermediate pence.

Case 18.

To find the value of one, having the price of a hundred.

Rule.—Multiply the value of 100 by 8, and add ⅛ of the product to the product; the sum of the pounds, reckoned as farthings, is the price.

Ex.—At 3l. 9s. 6d.×8+⅛ of 3l. 9s. 6d.=33l. 3s. 2¼d.

Exs.—At 6l. 18s. 10d., 4l. 11s. 6d.
Ans. 16½d., 11½d.

Here 3l. 9s. 6d. multiplied by 8, and ⅛ added=33l. 3s. 2¼d., which, reckoning ¼d. for every pound, gives 8¼d. for one.

CASE 19.

Given the value of one, to find the price of a thousand.

RULE.—Reckon 25*l.* for every 6*d.* in the price.

Ex.—At 4*s.*—Here there are 8 sixpences in 4*s.*; consequently 8 × 25 = 200*l.* value of 1000 articles.

At 4*s.*, 6*s.*, 8*s.*, 10*s.*, 5*s.* 9*d.*, 4*s.* 7*d.*, 16*s.* 2*d.*

Ans. 100*l.*, 300*l.*, 400*l.*, 500*l.*, 282*l.* 10*s.*, 229*l.* 3*s.* 4*d.*

Proof.—Because 1000 articles at 6*d.* each is 25*l.*; hence the rule.

CASE 20.

To find the price of one, having the price of a thousand.

RULE.—Deduct $\frac{1}{25}$ of the value as pounds, which will give the value of one in farthings.

Ex.—At 250*l.*,

$$\begin{array}{ll} \text{Here} & 250 \\ \div 25 & \underline{\;\;10\;\;} = \text{number of farthings in price of one} = 60d. = 5s. \\ & 240 \end{array}$$

At 780*l.*, 232*l.*, 380*l.*, 724*l.*, 1440*l.*, 642*l.*

Proof.—Because 1000 articles at $\frac{1}{4}$*d.* each is 1000 farthings, and the $\frac{1}{25}$ part of 1000 is 40, which, subtracted from 1000, leaves 960, which is the number of farthings in a pound; hence the rule.

CASE 21.

To find the amount per year, the rate per day being given.

If 1*l.* 10*s.* 5*d.* be multiplied by the number of pence per day, it will give the amount per year.

Ex.—At 3*d.*,

Here 1*l.* 10*s.* 5*d.* × 3 = 4*l.* 11*s.* 3*d.*

At 8*d.*, 4*d.*, 2*d.*, 6*d.*, 14*d.*, 2*s.* 2$\frac{1}{2}$*d.*, 7*s.* 8$\frac{1}{2}$*d.*

Proof.—Because in a year there are 365 days, and these at 1*d.* per day, are 365 pence = 1*l.* 10*s.* 5*d.*; hence the rule.

CASE 22.

To find the value of cwts., quarters, and pounds.

RULE 1.—Call the cwts. £s., multiply the qrs. by 5 for shillings, and the lbs. by 2$\frac{1}{7}$ for pence, which will give the value of cwts., qrs., and lbs., at 1*l.* per cwt.

2.—Multiply this value by the pounds of the price, and take liquot parts for the shillings and pence; the sum of all these is he answer to the question.

Ex.—Find the value of 69 cwts. 3 qrs. 14 lbs., at 2*l.* 10*s.* per wt.

69	3	14	
	5	2$\frac{1}{7}$	
£69	17	6	the cwts. called £*s.*
		2$\frac{1}{2}$	the price per cwt.
139	15	0	
34	18	9	
£174	13	9	price of 69 cwts. 3 qrs. 14 lbs.

Proof.—Any number of cwts., at 1*l.* per cwt., must be the same number of £*s.* as there are cwts.; and as 1 qr. is the $\frac{1}{4}$ part of 1 cwt., its value must be the $\frac{1}{4}$ part of a cwt., or 5*s.*; and as 1 lb. is the $\frac{1}{28}$ part of 1 qr., its value must be $\frac{1}{28}$ part of the 1 qr., or $\frac{60}{28}$*d.*=2$\frac{1}{7}$*d.*: hence the rule.

CASE 23.

Having the value of one, to find the value of twelve hundred.

RULE.—Multiply the price in pence by 5, and call it £*s.* If there be a farthing, call the $\frac{1}{4}$*d.* 5*s.*; if $\frac{1}{2}$*d.*, call it 10*s.*; if $\frac{3}{4}$*d.*, call it 15*s.*

Or add $\frac{1}{4}$ of the price in farthings to the farthings, and call the sum £*s.*

Ex.—At 6$\frac{3}{4}$*d.* for one, what is that for 1200?

6$\frac{3}{4}$=27 farthings.
6$\frac{3}{4}$=$\frac{1}{4}$ of 27.

33$\frac{3}{4}$=£33 15*s.* Or, 6$\frac{3}{4}$*d.* × 5=33$\frac{3}{4}$, called £*s.*=33*l.* 15*s.*

At 9$\frac{1}{4}$*d.*, 3$\frac{3}{4}$*d.*, 4$\frac{1}{2}$*d.*, 10$\frac{1}{4}$*d.*, 5$\frac{3}{4}$*d.*, 8$\frac{3}{4}$*d.*

Ans. 46*l.* 5*s.*, 18*l.* 15*s.*, 22*l.* 10*s.*, 51*l.* 5*s.*, 28*l.* 15*s.*, 43*l.* 15*s.*

CASE 24.

To find the value of any number of grosses.

RULE.—The value of one dozen taken as the value of another, will give the value required.

Ex.—Required the value of 7 grosses, at 9$\frac{1}{4}$d. for one article.

	£.	s.	d.
One dozen, at 9¼d. each	0	9	3
This taken as value of a dozen	5	11	0
Which multiplied by number of grosses, 7 =	38	17	0 = value of 7 grosses.

To reduce cwts., qrs., and lbs. to lbs.

RULE.—Place the cwts., with 2 ciphers affixed, on the left hand, and the lbs. in the qrs. and lbs. underneath, then multiply the given cwts. by 12; the sum of all these is the number of lbs.

Ex.—Required the number of lbs. in 44 cwt. 3 qrs. 18 lbs.

44 3 18

4400 = the cwts., with 2 ciphers affixed.
102 = lbs. in 3 qrs. 18 lbs.
528 = 12 times the 44 cwt.
5030

A line might be saved by adding in the pounds in the qrs. and lbs., with 12 times the cwts.

The common method.

44 3 18
4
179
28
1450
358
5030

TARE AND TRET

Is a rule by which merchants make allowances to buyers.

These allowances have various names.

1. *Tare* is an allowance for the weight of the cask, bag, box, &c.
2. *Tret* is an allowance made of 4 lbs. in every 104 lbs., or 1 in 26, for dust, &c.
3. *Cloff* is an allowance of 2 lbs. in every 3 cwt., or 1 lb. in every 168 lbs.; now seldom allowed.
4. *Suttle* is the weight that remains after part of the allowances have been deducted.
5. *Gross* weight is the whole weight of the goods, without any deductions.

The cases are—

(1) When tare only is allowed.
(2) When tare and tret are allowed.
(3) When tare, tret, and cloff are allowed.

The 1st divides into two sections—

1. When the tare is in one sum.
2. When the tare is at a certain allowance per bag, box, or barrel.

CASE 1.

SECTION 1.—*When the Tare is in one Sum.*

Rule.—Subtract the tare from the gross, the remainder is neat weight.

SECTION 2.—*When the Tare is at per Box, Bag, &c.*

Rule.—Multiply the tare by the number of boxes, bags, &c., and subtract the product from the gross, the remainder is neat. If the tare per box, &c., is given, subtract the tare from the weight, and multiply the suttle by the number of boxes, &c. for the neat.

CASE 2.

When Tare and Tret are allowed.

Rule.—Subtract the tare from the gross, as in the last rule, and the remainder is suttle; then divide the suttle by 26, which gives tret, and subtract this from the suttle, the remainder is neat.

CASE 3.

When Tare, Tret, and Cloff are allowed.

Rule.—Subtract the tare and tret as in the former rule, and the remainder is suttle; then multiply the cwts. suttle by 2, and divide by 3; the quotient gives lbs. cloff, which subtract from the suttle, and the remainder is neat.

CASE 1.

What is the neat weight of

	cwts.	*qrs.*	*lbs.*				*cwts.*	*qr.*	*lbs.*
(1)	85	3	19	tare	374 lbs.?	*Ans.*	82	2	9
(2)	162	3	12	tare	278 lbs.?	*Ans.*	160	1	14
(3)	264	1	18	tare	576 lbs.?	*Ans.*	259	1	2
(4)	1296	1	13	tare	2094 lbs.?	*Ans.*	1277	2	19
(5)	373	2	17	tare	1568 lbs.?	*Ans.*	359	2	17
(6)	563	1	14	tare	3296 lbs.?	*Ans.*	533	3	22
(7)	829	1	19	tare	6132 lbs.?	*Ans.*	774	2	19
(8)	784	3	15	tare	6973 lbs?	*Ans.*	722	2	14
(9)	176	1	13	tare	2017 lbs.?	*Ans.*	158	1	12
(10)	653	2	14	tare	5173 lbs.?	*Ans.*	607	1	21

SECOND DIVISION.

	What is the nt. wt. of	each weighing cwts.	qrs.	lbs.	tare		Ans.		
(1)	6 barrels of figs	0	3	27	tare 13	lbs. per barrel?	5	1	0
(2)	6 barrels of pork	2	1	24	tare 16½	lbs. per barrel?	13	3	17
(3)	14 barrels of pimento	1	2	19	tare 13¼	lbs. per barrel?	21	2	21
(4)	54 barrels of currants	0	2	19	tare 12	lbs. per barrel?	30	1	14
(5)	7 barrels of currants	0	3	18	tare 16	lbs. per barrel?	5	1	14
(6)	23 barrels of beef	2	3	14	tare 19	lbs. per barrel?	62	0	25
(7)	19 barrels of pimento	2	3	18	tare 16	lbs. per barrel?	52	2	10
(8)	9 barrels of hams	2	1	13	tare 13	lbs. per barrel?	20	1	0
(9)	13 hhds. of sugar	155	2	16	tare 23	lbs. per hhd.	152	3	25
(10)	12 barrels of beef	3	3	11	tare 10	lbs. per barrel?	45	0	12

CASE 2.

(1) Required neat weight of 12 hhds. sugar, weight 159 cwt., 2 qr. 8 lbs., tare 10 lbs. per hhd., tret as usual.

Ans. 252 cwt., 1 qr., 18 lbs.

(2) If 44 bags of hops weigh 250 lbs. each, and the tare be 11 lbs, and tret as usual; what will be the neat weight?

Ans. 10112 lbs.

(3) If 33 bags of rice weigh 4072 lbs., tare 19 lbs. per bag, tret as usual; required neat weight? *Ans.* 3313 lbs.

(4) Required neat weight of 12 hhds. sugar, weight 148 cwt., 3 qrs., 19 lbs., tare 14 lbs. per hhd., tret as usual.

Ans. 141 cwt., 3 qrs., 0 lb.

(5) In 35 barrels of figs, each weighing 3 qrs., 18 lbs., tare 10½ lbs. per barrel, tret as usual? *Ans.* 27 cwt., 1 qr., 27¼ lbs.

(6) In 84 boxes of indigo, weight 78 cwt., 1 qr., 19 lbs., tare 11 lbs. per box, tret as usual; required neat weight?

Ans. 67 cwt., 1 qr., 25 lbs.

(7) In 3 puncheons of prunes, each 1120 lbs. tare 33 lbs. per puncheon, tret as usual; required neat weight in cwts.?

Ans. 28 cwt.

(8) If 71 chests of tea, each 84 lbs., tare 7½ lbs. per chest, tret as usual; required neat weight in cwts.?
Ans. 11 cwt., 2 qrs., 14 lbs.

Case 3.

(1) Required the neat weight of 12 hhds. of sugar, weighing 86 cwt., 3 qrs., 12 lbs., tare 14 lbs. per hhd., tret and cloff as usual. *Ans.* 81 cwt., 2 qrs., 11 lbs.

(2) What is the neat weight of 7 casks of currants, each 1 cwt., 2 qrs., 5 lbs., tare 8 lbs., per cask, tret and cloff as usual?
Ans. 9 cwt., 3 qrs., 13 lbs.

(3) Required the neat weight of 3½ hhds. of prunes, weight 12 cwt. each, tare 11 lbs., tret and cloff as usual; required the neat weight. *Ans.* 39 cwt., 3 qrs., 9 lbs.

(4) 7 boxes of indigo, weight each 2 cwt., 1 qr., 19 lbs., tare 9½ lbs. per barrel, tret and cloff as usual; required the neat weight. *Ans.* 15 cwt., 2 qrs., 15 lbs.

(5) 14 bags of cochineal, each weighing 3 qrs., 18 lbs., tare 7 lbs. per bag, tret and cloff as usual; required the neat weight. *Ans.* 11 cwt, 1 qr. 12 lbs.

(6) 14 chests of tea, each weighing 90 lbs., tare 5 lbs. per chest, tret and cloff as usual; required the neat weight.
Ans. 10 cwt., 0 qr., 16 lbs.

(7) 26 bags of pimento, each weighing 1 cwt., 1 qr., 8 lbs., tare 7 lbs. per bag, tret and cloff as usual; required the neat weight. *Ans.* 31 cwt., 1 qr., 5 lbs.

(8) If the tare be 14 lbs. per hhd., tret and cloff as usual; what would be the nt. wt. on 5 hhds. of prunes, each weighing 10 cwt., 1 qr., 14 lbs.? *Ans.* 49 cwt. 1 lb.

(9) Required the weight of the tare, tret, and cloff of 7 hhds. of sugar, each weighing 12 cwt., 2 qrs., tare 29 lbs. per hhd., tret and cloff as usual. *Ans.* 5 cwt., 3 qrs., 21 lbs.

(10) In 7½ hhds. of sugar, each weighing 11 cwt., 3 qrs., 19 lbs., tare 23 lbs. per hhd, tret and cloff as usual: required neat weight? *Ans.* 83 cwt., 3 qrs., 26 lbs.

(11) The weight of 44 bags of indigo was 5968 lbs., tare 12 lbs. per 2 bags, tret and cloff as usual; required nt. wt.
Ans. 48 cwt., 2 qrs., 21 lb.

(12) Required the neat weight of 5 barrels of sugar, each weighing 7½ cwt., tare 13 lb. per cwt., tret and cloff as usual.
Ans. 31 cwt., 2 qrs., 23 lbs.

(13) In 16 hhds. of sugar, each 14 cwt., 2 qrs., 19 lbs., tare 14 lbs. per cwt., tret and cloff as usual; required the neat weight.
Ans. 196 cwt., 1 qr., 7 lbs.

(14) What is the neat weight of 48 barrels of figs, each 2 qrs. 18 lbs., tare 14½ lbs. per barrel, tret and cloff as usual? *Ans.* 25 cwt., 3 qrs., 11 lbs.

(15) Required the neat weight of 15 hhds. sugar, each 17 cwt., 2 qrs., 18 lbs., tare 16 lbs. per cwt., tret and cloff as usual. *Ans.* 312 cwt., 2 qrs., 3 lbs.

SIMPLE INTEREST.

INTEREST is a rule, by which we discover what we are to pay for the use of a certain sum of money for a certain time, at a certain rate per cent.

DEFINITION.

1. The *principal* is the money lent.
2. The *rate* is a certain sum paid by the borrower to the lender for the loan of 100*l.* for any stated time, generally for 12 months.
3. The *interest* is the rate when the principal is 100*l.*; but in every other instance it means the *sum paid* by the borrower to the lender for the loan of the principal.
4. The *amount* is the sum of the principal and interest.

CASE 1.

RULE 1.—*To find the Interest for One Year.*

Rule.—Multiply the principal by the rate per cent., and divide by 100; the quotient is the interest. The following is shorter in particular cases. When the rate per cent. is any aliquot part of 100, divide by this aliquot part, and the quotient is the interest.

Ex. 1.—Required interest on 480*l.* for 1 year, at 6 per cent.

$$\begin{array}{r} \pounds480 \\ 6 \\ \hline 28.80 \\ 20 \\ \hline 16.00 \end{array}$$

Here we multiply the principal by the rate per cent., and divide by 100, and obtain for interest 28*l.* 16*s.*

Ex. 2.—Required interest on 566*l.* 13*s.* 4*d.*, for one year, at 5 per cent.

$$\begin{array}{r|r|rrr} & & \pounds & s. & d. \\ 5 & \frac{1}{20} & 566 & 13 & 4 \\ \hline & & 28 & 6 & 8 \end{array}$$

Here, we perceive, that 5 is an aliquot part of 100, viz. $\frac{1}{20}$. We therefore divide by 20, and obtain 28*l.* 6*s.* 8*d.* for interest.

RULE 2.—*To find the Interest of any Sum of Money for any Number of Years.*

Find the interest for 1 year, and multiply it by the number of years; the product is the interest. Or, which is generally much shorter, multiply the rate and time together, and divide the principal by the aliquot parts of the product, taken of 100; the sum of these quotients is the interest.

Ex.—Required the simple interest on 500*l.* for 12½ years, at 4 per cent.?

$$12\tfrac{1}{2}\times4=50=\left|\,\tfrac{1}{2}\,\right|\tfrac{500}{250}\ \textit{Ans.}$$

RULE 3.—*To find the Interest for Months.*

Rule.—Find the interest for 1 year; then say, as 12 months is to the given number of months, so is the interest for 12 months to the interest for the given number of months.

Ex.—Find the simple interest on 382*l.* 12*s.* 6*d.* for 8 months, at 5 per cent.

$$\begin{array}{rrrrr}
 & 5\,|\tfrac{1}{20}| & 382 & 12 & 6 \\ \cline{3-5}
\cancel{12} : \cancel{8} :: & & 19 & 2 & 7\tfrac{1}{2} \\
3 \quad 2 & & & & 2 \\ \cline{3-5}
 & 3) & 38 & 5 & 3 \\ \cline{3-5}
 & & 12 & 15 & 1 \\ \hline
\end{array}$$

RULE 4.—*To find the Interest for Days.*

Rule.—Find the interest for one year; then as 365 : number of days, :: interest for a year, : the interest for the given days.

Note.—A shorter rule is as follows—multiply the principal by the days, and that product by double the rate, and divide this product by 73000; the quotient is the interest required; ex. the last rule.

Find the interest on 680*l.* for 50 days, at 6 per cent. The old rule is,

$\frac{680\times6\times50}{365\times100}$, which, when multiplied by $\frac{2}{2}$, is = to $\frac{680\times12\times50}{730\times100}$ $=\frac{480}{73}\Big|\frac{000}{000}=5l.\ 11s.\ 9\tfrac{1}{4}d.+35$. Hence is seen the reason of doubling the rate, and dividing by 73000.

What is the Simple Interest on

£ *s.* *d.*

(1) 2198 12 6 for 6 years and 10 months, at 2¼ per cent.? *Ans.* £338 0 8¾

(2) 42915 13 8½ for 3½ yrs., at 6¼ per ct.? *Ans.* £9387 16 1¼

(3) 2968 13 8½ for 1 yr., at 7½ per ct.? *Ans.* £222 13 0

	£	s.	d.				
(4)	7869	12	6	for 1 yr., at 4½ per ct.?	*Ans.* £354	2	7¾
(5)	4298	19	8	for 12 yrs. and 9 months, at 5 per ct.?	*Ans.* £2740	12	0
(6)	9826	13	8¼	for 1 yr., at 2½ per ct.?	*Ans.* £245	13	4
(7)	14794	18	10½	6¾ yrs., at 8½ per ct.?	*Ans.* £8488	11	11½
(8)	7964	12	8	for 1 yr., at 4¾ per ct.?	*Ans.* £378	6	4¾
(9)	5279	14	8¼	for 1 yr., at 3 per ct.?	*Ans.* £158	7	10
(10)	2479	16	8½	for 4 yrs., at 1¾ per ct.?	*Ans.* £173	11	8¾
(11)	49637	14	8	for 9 yrs. and 126 days, at 5 per ct.?	*Ans.* £23194	12	5¾
(12)	7319	18	10¼	for 3 yrs. 10 months, at 6½ per ct.?	*Ans.* £1824	1	0
(13)	2968	19	6½	for 11 yrs. and 120 days, at 4½ per ct.?	*Ans.* £1513	13	2
(14)	7865	18	11	for 8 yrs., 2 months, and 14 days, at 6 per ct.?	*Ans.* £3873	19	5¾
(15)	79643	16	8½	for 7½ yrs., at 6¼ per ct.?	*Ans.* £37333	0	11½
(16)	219	19	11½	for 4 yrs. and 6 months, at 3¾ per ct.?	*Ans.* £36	4	4¾
(17)	2190	19	11½	for 5 yrs., at 8 per ct.?	*Ans.* £876	7	11½
(18)	42178	13	11½	for 2 yrs., 3 months, at 7 per ct.?	*Ans.* £6643	2	10½
(19)	496	19	6½	for 3 yrs., and 5 months, at 4¼ per ct.?	*Ans.* £72	3	2¾
(20)	9469	17	11½	for 7 yrs., at 2 per ct.?	*Ans.* £1325	15	8½
(21)	7864	13	6¼	for 1 yr., at 2¼ per ct.?	*Ans.* £176	19	1¼
(22)	7963	15	9½	for 10 yrs., at 6¾ per ct.?	*Ans.* £526	12	6
(23)	41596	13	10	for 12½ yrs., at 6 per ct.?	*Ans.* £31197	10	4½
(24)	248	13	9	for 5 yrs., at 12 per ct.?	*Ans.* £149	4	3
(25)	968	14	11½	for 3 yrs., at 10 per ct.?	*Ans.* 291	12	5½
(26)	2098	16	11¼	for 5 yrs., at 7 per ct.?	*Ans.* 734	11	11
(27)	7298	13	7½	for 9 yrs., at 14 per ct.?	*Ans.* 9196	6	8¾
(28)	6092	17	3¼	for 7 yrs., at 7 per ct.?	*Ans.* 2985	10	0¼
(29)	9329	16	8½	for 5 yrs., at 16 per ct.?	*Ans.* 7463	17	4½
(30)	5329	12	9½	for 2 yrs., at 18½ per ct.?	*Ans.* 1971	19	3¾
(31)	7916	13	8¾	for 4½ yrs., at 15 per ct.?	*Ans.* 5343	15	2½
(32)	3729	10	7½	for 6 yrs., at 12 per ct.?	*Ans.* 2685	5	3
(33)	5863	2	9¼	for 6 yrs., at 7 per ct.?	*Ans.* 2462	10	4
(34)	1728	3	4¾	for 2 yrs., at 12 per ct.?	*Ans.* 414	15	2¼
(35)	9172	19	8¾	for 4 yrs., at 6½ per ct.?	*Ans.* 2384	19	6
(36)	2498	13	11¼	for 3½ yrs., at 10 per ct.?	*Ans.* 874	10	10¼
(37)	6172	14	8½	for 5 yrs., at 7½ per ct.?	*Ans.* 2314	15	5¾

A Table for finding the number of Days from any Day in any Month, to the same Day in any other Month.

To	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
January . . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June.	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	61	92	123	153
August. . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . . .	92	123	151	182	212	243	273	304	334	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December. .	31	62	90	121	151	182	212	243	274	304	335	365

If the end of the month of February be included in the number, in a Leap Year one day more must be added.

Simple Interest for Days and Months.

Required the simple interest on the following sums, for the respective times and rates:—

	£	s.	d.			£	s.	d.
(1)	326	12	0	for 48 days at 5 per cent.	*Ans.* £	2	2	11¼+6348.
(2)	648	17	6	for 14 days at 4½ per cent.	*Ans.*	1	2	4¾
(3)	267	19	0	for 250 days at 3½ per cent.	*Ans.*	£6	8	4+29000 rem.
(4)	473	16	8	for 32 days at 3 per cent.	*Ans.*	1	4	11+7240 rem.
(5)	382	12	6	for 8 months at 5 per cent.	*Ans.*	12	15	1
(6)	557	19	6	for 4 months at 6 per cent.	*Ans.*	11	3	2¼
(7)	596	14	8½	for 7 months at 4 per cent.	*Ans.*	13	18	5½
(8)	865	19	10½	for 7 months at 3 per cent.	*Ans.*	15	3	1
(9)	739	14	6¼	for 15 months at 2½ per ct.	*Ans.*	23	2	3¾
(10)	586	15	11¾	for 16 months at 7 per ct.	*Ans.*	54	15	4
(11)	687	9	11¾	for 10 months at 5⅓ per ct.	*Ans.*	30	11	1¼
(12)	729	19	8	for 135 days at 4 per cent.	*Ans.*	10	15	11¾
(13)	209	11	8	for 73 days at 5 per cent.	*Ans.*	2	1	11
(14)	938	12	10	for 116 days at 7 per cent.	*Ans.*	20	17	7½
(15)	869	14	11	for 37 days at 6 per cent.	*Ans.*	5	5	9½
(16)	257	19	6½	for 176 days at 5 per cent.	*Ans.*	3	7	3½

To calculate the Interest on Accounts Current.

RULE 1.—Find the interest on all the sums on the debtor side of the account. 2. Then find the interest on all the sums on the credit side.

NOTE.—The interest is to be calculated from the time the sums become due to the day of making up the account.

The difference of the sums of the interest on each side is the interest to be carried to the proper side of the account current.

Ex.—Required the interest at 5 per cent. on the following account current till August 25, 1842.

Messrs. Bell, Mills & Co.'s Account Current with R. Jackson.

DR.	£	*s.*	*d.*	CR.	£	*s.*	*d.*
Jan. 17, To Balance.	256	10	0	Feb. 7, By Cash..	173	10	0
Feb. 5, To Merchandize.....	196	15	0	Mar. 16, By Indigo, due......	248	12	0
Mar. 14, To Bill *due*.	330	14	0	Apr. 10, By Brandy, due	341	7	0
Apr. 16, To Cotton .	450	0	0	June 15, By Cochineal, due .	252	6	0

Calculation of the Interest.

DEBTOR SIDE.

		£	*s.*	*d.*		£	*s.*	*d.*	£	*s.*	*d.*
Jan. 17, Aug. 25,	is 220 days, on	256	10	0	is	7	14	7			
Feb. 5, Aug. 25,	is 201 ,,	196	15	0	is	5	8	4¼			
Mar. 14, Aug. 25,	is 164 ,,	330	14	0	is	7	8	7			
April 16, Aug. 25,	is 131 ,,	450	0	0	is	3	4	7¼			
									23	16	1½

CREDIT SIDE.

		£	*s.*	*d.*		£	*s.*	*d.*	£	*s.*	*d.*
Feb. 7, Aug. 25,	is 197 days, on	173	10	0	is	4	13	7¾			
Mar. 16, Aug. 25,	is 162 ,,	248	12	0	is	5	10	4¼			
April 10, Aug. 25,	is 137 ,,	341	7	0	is	6	8	1¼			
June 15, Aug. 25,	is 71 ,,	252	6	0	is	2	9	0¾			
									19	1	2

Balance due to Messrs. Bell & Co.... £ 4 14 11½

In all accounts current, interest tables are used in *real* business, which materially facilitate the calculation of interest.

CASE 2.

Given the amount, principal, and rate, to find the time.

RULE.—From the amount subtract the principal, divide the difference by the product of the principal and the rate. The quotient is the time.

Ex.—How long will 400*l.* be in amounting to 520*l.* at 5 per cent.?

Here 520 is the amount.
400 the principal.
$400\times\frac{5}{100}=20)\overline{120}$
6 years.

2. How long will 700*l.* be in amounting to 1093*l.* 15*s.* at 4½ per cent.?

£.
1093·75
700
$700\times\cdot045=31\cdot5)\overline{393\cdot75}(12\cdot5=12\tfrac{1}{2}$ years.
315
787
630
1575
1575

Any of the preceding questions can be taken for examples to this Rule.

CASE 3.

Given the amount, principal, and time to find the rate.

RULE.—From the amount subtract the principal, and divide the difference by the product of the principal and the time. The quotient is the rate.

3. What rate per cent. must be given for 6 years, that 300*l.* may amount to 363*l.*?

363
300
$300\times 6=1800)\overline{63\cdot000}(\cdot035=3\tfrac{1}{2}$ per cent.
5400
9000
9000

Given amount, rate, and time, to find principal.

Rule.—Divide the amount by the product of the rate and time, plus 1; the quotient is the principal.

Ex.—What principal put out to interest at $3\frac{1}{2}$ per cent., for 6 years, will amount to 363*l.*

$$\text{Here } \frac{363}{1+rt.}=\frac{363}{1+(\cdot035\times6)}=\frac{363}{1\cdot21}=\pounds300$$

The formulæ for Simple Interest are obtained from the following proofs:—

Let $r=$ rate per cent.
„ $p=$ principal.
„ $a=$ amount.
„ $t=$ time.

Then $100 : r :: p : pr$. which may be read, if 100*l.* gain r (any rate per cent.), what will the principal gain? The answer is pr. But if the time be for more than 1 year, the statement will be, if 100 gain r, what will $p\times t$ gain?

$$\begin{aligned}
&\text{If } 100 : r :: p\times t : prt, \text{ for interest.}\\
&\text{Conseq. } p+prt=a \text{ (def. 4)}\\
&\therefore p\,(1+rt)=a\\
&p=\frac{a}{1\times rt} \text{ (formula 1).}
\end{aligned}$$

$$\begin{aligned}
\text{Again } p+prt&=a\\
prt&=a-p\\
t&=\frac{a-p}{pr} \text{ (formula 2).}
\end{aligned}$$

$$\begin{aligned}
\text{Again } p+prt&=a\\
prt&=a-p\\
r&=\frac{a-p}{pt} \text{ (formula 3.)}
\end{aligned}$$

COMPOUND INTEREST.

Definition.

Compound Interest arises from letting the simple interest remain, and be reckoned as principal, on which sum interest is allowed, which interest is added to the preceding principal, and the sum again considered as principal; and the interest allowed thereon is again added to the last principal, and so on for any number of years.

RULE.—Find the interest of the given sum for one year, and add it to the given sum: reckoning this sum as principal. Then find the interest on this sum, which add to the preceding principal, and so on; performing the like operation as many times as there are years mentioned in the question. Finally, subtract the original principal from the last amount, and the difference is the *compound interest.*

When the interest is to be found for years and months, or years and days, find the interest for one year more than is mentioned in the question, then by Proportion, say as 365 days : the number of days in the question, :: interest for 1 year : interest for given days : if for months, say : 12 months : number of months :: interest for 1 year : interest for the number of months, which added to the last principal will give the amount. See the preceding remark in the last paragraph.

Find the compound interest on the following sums, at their given rates, and for their given times:—

					Ans.		
	£	*s.*	*d.*		£	*s.*	*d.*
(1)	420	0	0	for 5 years, at 5 per cent.	126	0	8¾
(2)	568	0	0	for 3 years, at 5 per cent.	89	10	7¼
(3)	242	12	0	for 4 years, at 5 per cent.	294	17	7
(4)	328	16	0	for 2½ years, at 5 per cent.	42	15	3¼
(5)	698	14	8	for 3 years, at 5 per cent.	110	2	8¾
(6)	546	15	6	for 4 years, at 5 per cent.	117	16	7¾
(7)	298	16	0	for 3¾ years, at 4 per cent.	47	7	9½
(8)	749	10	0	for 2 years, at 3½ per cent.	52	6	11½
(9)	847	12	0	for 3 years, at 4 per cent.	105	16	7¾
(10)	473	19	0	for 2 years, at 3¾ per cent.	36	4	3
(11)	314	0	0	for 4 years, at 5 per cent.	67	13	3¾
(12)	328	16	0	for 4 years, at 5 per cent.	52	5	2
(13)	239	6	9	for 3 years, at 4½ per cent.	33	15	8¼
(14)	493	16	4½	for 5 years, at 4 per cent.	106	13	10
(15)	1289	14	5¾	for 3½ years, at 5¼ per cent.	253	6	8¾
(16)	2175	15	11¼	for 2½ years, at 3 per cent.	167	2	6½
(17)	5196	3	2¼	for 1½ year, at 7½ per cent.	597	2	7
(18)	325	11	4¾	for 2½ years, at 6 per cent.	51	4	2¼
(19)	173	15	6½	for 3 years, at 5 per cent.	27	7	9¾

Required the compound interest on

£ *s.* *d.*

(1) 698 14 0 for 3 years and 173 days, at 4 per cent.

Ans. 102*l.* 2*s.* 10*d.*

	£.	s.	d.	
(2)	424	9	6	for 1 year and 49 days, at 3 per cent. *Ans.* 14*l.* 9*l.* 10¼*d.*
(3)	64	12	8	for 3 years and 76 days, at 6 per cent. *Ans.* 22*l.* 8*s.* 10¼*d.*
(4)	69	12	8½	for 3 years and 154 days, at 2½ per cent. *Ans.* 6*l.* 12*s.* 10½*d.*
(5)	548	12	0	for 5 years and 10 months, at 5 per cent. *Ans.* 180*l.* 14*s.* 8¾*d.*
(6)	244	16	0	for 1 year and 73 days, at 4 per cent. *Ans.* 11*l.* 16*s.* 6*d.*
(7)	965	17	0	for 3 years and 271 days, at 5 per cent. *Ans.* 194*l.* 12*s.* 7¾*d.*
(8)	204	10	0	for 2 years and 300 days, at 4 per cent. *Ans.* 23*l.* 19*s.* 1½*d.*
(9)	446	13	0	for 1 year and 37 days, at 3 per cent. *Ans.* 14*l.* 15*s.* 11½*d.*

COMMISSION, INSURANCE, AND BROKERAGE.

Commission is an allowance made by a merchant to his agent, or correspondent, for the buying or selling goods; it is a percentage, and is calculated in the same manner as interest.

Brokerage is also an allowance made to a broker for purchasing or selling goods, or for obtaining a buyer; the percentage is much smaller in amount than in commission, as the trouble is generally less.

Insurance is an allowance paid by one merchant to another, or to an insurance office (generally Lloyd's). The party receiving it engages to "*make good*" the value of the articles insured, whether ship, house, merchandise, furniture, &c., &c. It assumes various names, according to the element or agent insured against: thus, sea-insurances, fire-insurances. Parties also insure against the risk of capture by the enemy in war-time. The party who takes the risk is called the *insurer* or *underwriter*, and the party receiving protection is called the *insured*. The *premium* is the money paid, and the paper or parchment on which the contract is drawn, is called the *policy*.

Questions in these rules are solved the same as simple interest.

Ex.—What is the commission on 3968*l.* 12*s.*, at 3½ per cent.?

2½	$\frac{1}{40}$	3698	12	0
1	$\frac{1}{100}$	92	9	3½
		36	19	8¾
		£129	9	0¼

Here we take parts of the interest of 100*l.*, viz., 2½ is the $\frac{1}{40}$, and 1 is the $\frac{1}{100}$; the results from these parts added together=129*l.* 9*s.* 0¼*d.*

3698·6
·035
184930
110958
129·4510=129*l.* 9*s.* 0¼*d.*

But the shortest way to solve questions of this kind is by decimals, as in the annexed example.

Find the commission on

	£	s.	d.			£.	s.	d.
(1)	4864	15	6	at 2½ per cent.	*Ans.*	121	12	4½
(2)	3826	12	4	at 2 per cent.	*Ans.*	76	15	7¾
(3)	6832	15	9	at 3 per cent.	*Ans.*	204	14	8
(4)	2197	7	8	at 2½ per cent.	*Ans.*	54	18	6
(5)	9654	5	6	at 4¼ per cent.	*Ans.*	410	6	1½
(6)	4832	9	7½	at 3 per cent.	*Ans.*	144	19	5¾

INSURANCE.

Find the insurance on

(1) „ 36941*l.* 12*s.* 8*d.* at 6⅝ per cent.; policy, 4*l.* 5*s.* 6*d.*; brokerage, ⅜ per cent. *Ans.* 2590*l.* 3*s.* 9¼*d.*

(2) „ 24729*l.* 12*s.* 8½*d.* at 12⅜ per cent.; policy, 7*l.* 8*s.* 9*d.*; brokerage, ⅛ per cent. *Ans.* 3098*l.* 12*s.* 10½*d.*

(3) „ 62987*l.* 11*s.* 9*d.* at 7⅝ per cent.; policy, 5*l.* 4*s.* 7*d.*; brokerage, ⅜ per cent. *Ans.* 5044*l.* 4*s.* 8½*d.*

(4) „ 57319*l.* 19*s.* 10½*d.* at 6⅜ per cent.; policy, 13*l.* 8*s.* 2*d.*; brokerage, ¼ per cent. *Ans.* 3810*l.* 17*s.* 1¾*d.*

(5) „ 63172*l.* 19*s.* 10*d.* at 11⅝ per cent.; policy, 6*l.* 12*s.* 8*d.*; brokerage, ⅜ per cent. *Ans.* 7587*l.* 7*s.* 10*d.*

The marine insurances during the war were,

	£	s.		Now. £	Now. s.
To East Indies........	12	12	..	2	10
West Indies	15	15 and 18*l.* 18*s.*		1	5
Constantinople, 1797	42	0	..	1	10
——— 1799	31	10			
Oporto	15	15	..	1	10
Hamburgh	7	7	..	1	10
Dublin	5	5	..	0	15

EXPLANATION OF THE STOCKS, OR PUBLIC FUNDS.

Stocks had their origin in the necessities of Government, in the reign of William III., when it was found that the exigencies of the state were of too large a nature to be supplied by means of taxation. Money, therefore, was borrowed from the people, for which interest was allowed. The money for the payment of the interest on loans is raised by taxation, and this is called the *Funding System;* the loans themselves being called the *National Debt.*

When Government wishes to obtain a loan, notice is given by the Chancellor of the Exchequer that it is required, and a day is appointed to receive the tenders or *bidding of capitalists.* On these occasions, various parties unite not only *on* the Stock Exchange, but also *off,* "The highest bidder being the buyer." The different funds or stock in which the loan is taken are called *omnium,* when sold together, and when sold separately, before the instalments are paid up, are called *scrip.* They generally consist of a portion of 3 per cent. Consols, and of a higher interest stock, as New Fours or Navy Fives, and a portion of *Long Annuities.* The loan usually bears, on its first introduction into the market, a small premium, which affords to the contractors a large profit. It is generally paid by instalments of ¼ at each payment, a few months intervening each instalment.

It will be observed that no loan contracted for by Government is promised to be repaid on demand, but this purpose is effected by other means; there are parties always to be found who are willing to purchase any stock offered in the market; or if by any chance none of the stock is required on that day, then the broker himself generally purchases it, *i. e.,* if it be marketable. Thus the holder of any quantity of stock obtains cash for it as easily as if Government were to pay it on demand. In effecting a sale of his capital, the holder will receive more or less than he gave for it, according as the market price of the stock on the day of sale is *greater* or *less* than what he originally gave.

Ex.—If a person purchases 3 per cent. Consols, at 89⅜, on Monday, and sells them at 90⅛ on Friday, it is evident he will gain by the sale, because he has obtained ¾ of a pound more on each 100 of stock than he gave; but if he had sold the stock for 89, he would have lost ⅜ of a pound by the sale.

All loans are effected by making one portion of them *terminable,* and another portion *perpetual,* annuities. Thus, the *Long Annuities* are called *terminable,* because in a few years they will terminate; but the 3 per cent. Consols, 3½ per cent., New 4's, &c., are *perpetual*—interest annually being paid on these stocks, till Government redeem them by "paying them off" at *par, i.e.,* 100*l.* money for 100*l.* stock.

When we buy stock, we give a *specific* sum for a *nominal* one. Thus if we wish to buy 500*l.*, 3 per cent. Consols, and they are at 89⅜, we shall give a *specific* sum, viz., 89*l.* 7*s.* 6*d.*, for a *nominal* sum, viz., 100*l.* 3 per cent. Consols. We should in this instance receive 3*l.* per annum for our 89*l.* 7*s.* 6d. Other stocks afford a larger percentage in interest, but these require a larger sum to purchase. The 3 per cent. Consols, and 3 per cent. Reduced, being larger stocks than a few others, are consequently always a shade higher in price, because Government will always find more difficulty "in paying off" these stocks than others of smaller amount.

There are other loans besides the above, that Government are compelled to effect, and these are called the *unfunded* debt, as the above are called the *funded* debt. These consist of Exchequer, Navy, Ordnance, and Victualling Bills.

There are also the stocks of public companies, as Bank Stock, India Stock. These latter are usually quoted in the prices of public funds. The others are quoted in Foreign Stock List or Share List.

PRICES FOR THE WEEK.

Wednesday, 8th March, to Tuesday, 15th, 1842.

THE FUNDS.	Wed.	Thurs.	Friday.	Sat.	Monday.	Tuesday.
3 per cent. Consols .	89⅛	89	89	89⅜	89½	89¾
Consols for Acct. . .	89¼	89⅜	89¼	89⅝	89⅝	9
3 per cent. Reduced .	. .	. .	. .	. .	. .	.
3½ per cent. Reduced .	99	. .	99⅜	. .	. .	. .
New 3½ per cent. . .	98¾	98⅝	98⅞	99	99	99¼
India Stock	242	. .	. .	. .	. .	. .
Bank Stock	. .	. .	. .	171	170½	. .
Exchequer Bills . .	30p.	32p.	30p.	30p.	31p.	31p.
India Bonds . . - .	10p.	10p.	8p.	30p.	10p.	8p.
Bank Long Annuities.	16⅝	16	16⅝	. .	16	. .

Explanation.—3 per cent. Consols.

The price of this is invariably a shade higher than the 3 per cent. Reduced (except a short time before the interest on the latter is paid), for the reason already given.

On Wednesday, 8th, it was quoted at 89⅛, though probably during the day it had fluctuated; but the closing price is the one quoted in the list.

It will be understood that 89⅛ means 89*l.* 2*s.* 6*d.*, and that this sum was given for 100*l.* stock; this is exclusive of the brokerage, which is ⅛ per cent. on this stock.

The next entry is "Consols for Account." This means that some of this stock taken on time, called "time bargains," was agreed to be taken at 89¼. If the price on the future day be *above* or *below* the quoted price, the parties called ***bulls*** or ***bears*** gain or lose. The difference of price between this and the 3 per cent. Consols, probably arose from some supposed improvement in the aspect of political affairs.

3 per cent. Reduced. None sold.

3½ per cent. Reduced: 99 means that 99*l.* were given for 100*l.* stock; this is also exclusive of the brokerage, which is never quoted.

New 3½ per cent.: 98¾ means that 98*l.* 15*s.* was the price of 100*l.* stock, for which the holder will receive 3*l.* 10*s.* per cent. as long as he holds the stock.

India Stock: 242 means that 242*l.* was the price for 100*l.* stock, for which the holder will receive 8 or 10 per cent., and have the privilege of voting at elections of directors, should his stock amount to 2000*l.*

Bank Stock. None sold.

Exchequer Bills, 30 p., means, that for a 100*l.* Exchequer Bill, 30*s.* premium must be given, *i. e.* 101*l.* 10*s.* These bills bear an interest of 2*d.* a day.

India Bonds, 10 p., means the same as the preceding.

Bank Long Annuities (these bonds pay brokerage 1*s.* per 100*l.*): 16⅝ means, that these annuities are purchaseable at 16⅝ year's purchase; *i. e.*, if I want to buy an annuity of 50*l.* a-year in these annuities, I must pay 16⅝ × 50 = 831*l.* 5*s.* for it; and this annuity will be paid annually till they terminate in the year 1859?

The cases into which this rule are divisible are as follow:—

I. To find the value of any quantity of stock.

Rule 1.—Add the brokerage to the market-price, then turn the fraction into a decimal, or seek it in the accompanying table, and affix it to the integer; then multiply the quantity of stock by the price so increased, and divide by 100; the quotient will give the value required.

II. To find what quantity of stock can be received for any sum of money.

Rule.—Add the brokerage to the quoted price, turn the fraction (if any) of the price into a decimal, and the shillings and pence (if any) of the sum given, and affix them to their respective integers; then divide the sum given by the price of the stock, with its brokerage added; the quotient will give the quantity of stock that can be obtained for the given sum.

III. To *equate* stocks.

Say, as the price at *par* of the given stock is to its market-price, so is the price at par of any other stock to the equivalent *price.*

Ex.—If 3 per cent. consols be at $80\frac{1}{2}$ (with brokerage), what ought to be given for New Fours to obtain the same interest.

$$\begin{array}{ccccc}\cancel{80} & : & 80\frac{1}{2} & :: & \cancel{80}\\ 3 & & 4 & & 4\end{array}$$

3)322

$107\frac{1}{3} = 107l.\ 6s.\ 8d.$

The following decimals of a pound should be learnt:—

$\frac{1}{16}$ of a £	= ·0625	$\frac{3}{8}$ of a £	= ·375	$\frac{11}{16}$ of a £	= ·6875
$\frac{1}{8}$,,	= ·125	$\frac{7}{16}$,,	= ·4375	$\frac{3}{4}$,,	= ·75
$\frac{3}{16}$,,	= ·1875	$\frac{1}{2}$,,	= ·5	$\frac{13}{16}$,,	= ·8125
$\frac{1}{4}$,,	= ·25	$\frac{9}{16}$,,	= ·5625	$\frac{7}{8}$,,	= ·875
$\frac{5}{16}$,,	= ·3125	$\frac{5}{8}$,,	= ·625	$\frac{15}{16}$,,	= ·9375

The brokerage paid for buying or selling stock is 2*s*. 6*d*. per ct.
And for India bonds, 1*s*. ,,

Ex. to Rule 1.

Here £3978 19 4 at $89\frac{5}{8}$

3978·9666
89·625

198948330
79579332
238737996
358106994
318317328

356614·8815250 = when divided by 100, to 3566·1488 = $3566l.\ 2s.\ 11\frac{1}{2}d.$

Ex. to Rule 2.

Here £5968 12 6 at $80\frac{1}{2}$

5968·625 at 80·5

5968·625, when multiplied by 100 = 596862·5 at 80·5

Therefore 80·5) 596862·5 (7414·440 = $7414l.\ 8s.\ 9\frac{1}{2}d.$ stock.

5635
3336
3220
1162
805
3575
3220
3550
3220
3300
3220
80

PURCHASING OF STOCKS.

What is the price of the following quantities of stock, brokerage $\frac{1}{8}$ per cent.?

					Ans.		
	£.	s.	d.		£.	s.	d.
(1)	3978	19	4	New 4 per ct., at $89\frac{1}{2}$?	3566	2	$11\frac{1}{2}$
(2)	23172	12	8	3 per ct. Consols, at $79\frac{3}{8}$?	18422	4	$10\frac{1}{4}$
(3)	12984	12	9	3 per ct. Red., at $76\frac{1}{4}$?	9917	0	4
(4)	49682	19	8	Bank Stock, at $195\frac{5}{8}$?	97254	8	$9\frac{3}{4}$
(5)	2793	13	6	$3\frac{1}{2}$ per ct., at $84\frac{3}{8}$?	2360	13	$1\frac{1}{4}$
(6)	17297	14	$10\frac{1}{2}$	India Stock, at $212\frac{1}{4}$?	36736	1	$7\frac{3}{4}$
(7)	9764	15	6	Bank Stock, at $185\frac{1}{4}$?	18101	9	$0\frac{1}{4}$
(8)	7391	14	9	3 per ct. Consols, at $86\frac{1}{4}$?	6384	12	3
(9)	69	0	0	per ann. of L. Ann., at $16\frac{1}{2}$?	1147	2	6
(10)	765	0	0	per ann. of L. Ann., at $17\frac{1}{4}$?	13291	17	6
(11)	8968	14	6	3 per ct. Consols, at $84\frac{5}{8}$?	7600	19	$10\frac{1}{2}$
(12)	2347	11	$6\frac{1}{2}$	3 per ct. Reduced, at $83\frac{3}{4}$?	1969	0	$7\frac{1}{4}$
(13)	9692	9	$9\frac{1}{4}$	4 per cents., at $92\frac{1}{8}$?	8941	6	5

(14) Sold 2007*l.* 11*s.* 0*d.* Navy 5 per cents., at $105\frac{3}{4}$; what did I get for the stock? *Ans.* 2125*l.* 9*s.* $10\frac{3}{4}$*d.*

(15) Bought 2569*l.* 12*s.* 6*d.* $3\frac{1}{2}$ per cents., at $87\frac{1}{2}$? *Ans.* 2251*l.* 12*s.* 8d.

(16) I wish to invest 4900*l.* in 3 per cent Consols, at $79\frac{7}{8}$; how much stock shall I get for my money? *Ans.* 6125*l.*

(17) A gentleman wishes to lay out cash to the amount of 5968*l.* 12*s*, 6*d.* in 3 per cent. Consols, at $80\frac{3}{8}$; what quantity can he buy? *Ans.* 7414*l.* 8*s.* $9\frac{1}{2}$.d

DISCOUNT.

DEFINITION.

Discount is an allowance made by the holder of a bill to a person paying the bill before it is due; or it is the sum which, when added to the present worth (or money paid), will amount to the same sum for which this bill was drawn. Thus,

If at 5 per cent. 100*l.* would amount to 105*l.* in one year, it follows that the present worth of a bill drawn for 105*l.*, due 12 months hence, must be 100*l.* We therefore obtain the following rule:—

: 100*l.*, with its interest for the given time,
: That interest,
:: The given sum
: The discount.

Or : 100*l.*, with its interest for the given time,
: That interest,
:: The given sum
: The present worth.

Ex.—Required the discount of 210*l.*, due 12 months hence, at 5 per cent.

$$\begin{array}{lllllll} \cancel{105} & : & \cancel{5} & : & \cancel{210} & : & 10 \\ \cancel{21} & & & & 10 & & \end{array}$$

Here we find the interest of 100*l.* for the given time is 5*l.* We then add this interest to 100*l.*, and obtain 105*l.* We then state the question as in the rule, and, after cancelling, obtain 10 as the discount. The present worth might be found by subtracting the discount from the sum given; or it could be found by the second part of the rule.

Ex.—What is the present worth of 210*l.*, due 12 months hence, at 5 per cent.

$$\begin{array}{lllllll} \cancel{105} & : & 100 & :: & \cancel{210} & :: & 200 \\ & & & & 2 & & \end{array}$$

Here we cancel the first term into the third, obtaining for quotient 2, which we multiply by the second term, and obtain for product 200, which is the present worth.

Solution of question 2:

$$\begin{array}{c|c|r|l} 4 & \frac{1}{3} & 6 & \text{Interest of } 100l. \text{ for one year.} \\ & & 2 & \text{Ditto for four months.} \\ \cline{3-3} & & 8 & \text{Ditto for one year and 4 months.} \end{array}$$

$$\begin{array}{rcccrrr} 100 & & & & & & \\ \hline & & & & \pounds & s. & d. \\ \cancel{108} & : & \cancel{8} & :: & 372 & 10 & 9 \\ 27 & & 2 & & & & 2 \\ \cline{5-7} & & & & 3)745 & 1 & 6 \\ \cline{5-7} & & & & 9)248 & 7 & 2 \\ \cline{5-7} \text{Discount} & . & & . & \pounds 27 & 11 & 10\frac{3}{4} \\ \cline{5-7} \end{array}$$

Find the discount on

£ s. d.

(1) 372 10 9 for 1 year and 4 months, at 6 per cent.?
Ans. £27 11 $10\frac{3}{4}$

(2) 719 16 11 for 3 years and 2 months, at 5 per cent.?
Ans. 98 7 11+12

(3) 284 13 $6\frac{1}{2}$ for 2 years, at $1\frac{1}{2}$ per cent.?
Ans. 8 5 $9\frac{3}{4}$+93

(4) 179 5 $4\frac{1}{4}$ for 1 year and 11 months, at $3\frac{3}{4}$ per cent.?
Ans. 12 0 5+11

(5) 549 13 8 for 3 months, at $9\frac{1}{2}$ per cent.?
Ans. 12 17 $5\frac{3}{4}$+203

(6) 269 17 9 for $2\frac{1}{4}$ months, at $4\frac{1}{4}$ per cent.?
Ans. 2 7 $4\frac{1}{4}$+1763

(7) 796 15 $8\frac{1}{4}$ for 2 years and 173 days, at $4\frac{1}{2}$ per cent.?
Ans. 79 16 $3\frac{1}{2}$+53917

	Find the discount on				Ans.		
	£.	s.	d.		£.	s.	d.
(8)	973	14	$9\frac{3}{4}$	for 9 months, at $1\frac{1}{4}$ per ct.?	9	0	$10\frac{1}{2}+87$
(9)	265	9	4	for $10\frac{1}{2}$ months, at $2\frac{1}{4}$ per ct.?	5	3	$6\frac{1}{4}+$
(10)	373	1	$4\frac{1}{2}$	for 11 months, at 10 per ct.?	31	6	$6\frac{1}{4}+43$
(11)	639	11	2	for $4\frac{1}{2}$ months, at $3\frac{3}{4}$ per ct.?	8	17	$4\frac{1}{2}$
(12)	298	13	7	for 13 months, at $6\frac{1}{2}$ per ct.?	*Ans.* £19	12	$11\frac{1}{2}+1230$
(13)	265	10	9	for $7\frac{1}{2}$ months, at $2\frac{3}{4}$ per ct.?	4	9	$8\frac{3}{4}$
(14)	92	18	2	for 6 months, at 7 per ct.?	3	2	10
(15)	543	18	10	for 8 months, at $5\frac{1}{2}$ per ct.?	19	4	$9\frac{1}{4}+165$
(16)	739	18	9	for $2\frac{1}{2}$ years, at 4 per ct.?	67	5	$4\frac{4}{11}$
(17)	268	10	$4\frac{1}{2}$	for 7 months, at 5 per ct.?	7	2	$2\frac{1}{4}+111$

(18) What is the discount on 479*l*. 13*s*. $9\frac{3}{4}$ for five months, at $2\frac{1}{4}$ per cent.? *Ans.* 4 9 $1\frac{1}{4}+114$

(19) What is the discount on 62*l*. 12*s*. 8*d*. for 3 months, at $6\frac{1}{4}$ per cent.? *Ans.* 0 19 $3\frac{1}{4}+3$

(20) Find the discount and present worth of 48*l*. 12*s*. 6*d*. for 3 months. at 6 per cent.? *Ans.* 0 14 $4\frac{1}{4}$

(21) Find the discount and present worth of 73*l*. 19*s*. 4*d*. for $2\frac{1}{2}$ months, at 5 per cent.?

Ans. Present worth, 73*l*. 4*s*. 1*d*. +1; discount, 15*s*. 3*d*.

(22) Find the discount and present worth of 694*l*. 11*s*. $2\frac{1}{4}$*d*. for 2 years 8 months, at $3\frac{1}{4}$ per ct.? *Ans.* 60 0 $5\frac{1}{4}+51$

(23) Discount on 729*l*. 12*s*. 6*d*. for 3 years and 9 months, at 4 per cent.? *Ans.* 95 3 $4\frac{1}{4}+\frac{17}{23}$

(24) A gentleman wishes to discount a bill which has 48 days to run; the amount of the bill is 63*l*. 10*s*., and the rate to be allowed $4\frac{1}{2}$ per cent.? *Ans.* 7*s*. $5\frac{1}{2}$*d*. + 6742

BARTER.

Barter is the method employed by merchants to exchange one commodity for another, so that neither party may sustain loss. Sometimes one commodity, and a certain sum in cash, are *given* for another, sometimes the same are *received* for another.

The principal of solution depends on finding the value of the commodity whose price and quantity are given.

An example will illustrate it.

Ex.—I want to exchange 150 cwt. of soap at 2*l*. 10*s*. per cwt., for 5 pieces of cloth, each containing 25 yards, at 19*s*. 4*d*. per yard; what cash must I give or receive in the exchange?

150
2½
300
75
375 value of soap.
120 16 8 value of cloth.
£254 3 4 I must receive.

5
25
125
19⅓
2375
41 8 0
20)2416 8 0
£120 16 8

Here we find the value of both articles, and perceive the difference to be 254*l.* 3*s.* 4*d.*, which (as the soap is worth more than the cloth) I am to receive.

Ex.—Gave 25*l.* 12*s.*, and 14 cwt. of sugar, valued at 2*l.* 14*s.* per cwt. for 84 lbs. of indigo; what is the indigo valued at per lb.?

14 0
2
| 10 | ½ | 28 0
| 4 | ⅕ | 7 0
2 16
37 16
25 12
63 8
20
84)1268(15
84
428
420
8
12
84)96)1
84
$\frac{12}{84}=\frac{1}{7}$

Here we find the value of the sugar to be 37*l.* 16*s.*, to which we add 25*l.* 12*s.*, and obtain = 63*l.* 8*s.*; we then divide by 84, and obtain for quotient 15*s.* 1$\frac{1}{7}$*d.* as the price of 1 lb. of indigo.

Ans. 15*s.* 1$\frac{1}{7}$*d.*

(1) How much cotton, at 4½*d.* per lb., must be given in barter for 896 hhds. of sugar, each weighing 15 cwt., at 2*l.* 16*s.* 8½*d.* per cwt.? *Ans.* 2032426, ⅔ lbs. over.

(2) Gave 98 cwt. of cheese for 73 cwt., 2 qrs., 18 lbs. of butter, at 6*l.* 2*s.* per cwt., and 20*l.*; what did I give per lb. for the cheese? *Ans.* 10¼*d.* +541.

(3) How much sugar, at 10$\frac{1}{2}$*d.* per lb., must be given in barter for 58 cwt., 3 qrs., 18 lbs. of indigo, at 15*s.* 6*d.* per lb.? *Ans.* 1043 cwt., 2 qrs., 6 lbs.+36.

(4) Gave 72 hhds. of sugar, each weighing 7 cwt., for 59 cwt., 2 qrs., 14 lbs. of indigo, at 4*l.* 18s. 6*d.* per cwt.; what was the sugar valued at per 18 cwt.? *Ans.* 10*l.* 10*s.* 2*d.*

(5) Gave 20*l.* 16*s.* 8*d.*, and 57 cwt. of pork, valued at 4$\frac{1}{2}$*d.* per lb., for 62 cwt. of cheese; what did I value the cheese at per lb.? *Ans.* 4$\frac{3}{4}$*d.*+2976.

(6) If I give 21 cwt., 3 qrs., 18 lbs. of cheese for 71 cwt. of raisins, valued at 122*l.* 18*s.* 9*d.*, what did I give per lb. for the cheese? *Ans.* 1*s.* 0*d.*+228.

(7) Gave 29*l.* 12*s.*, and 48 tons of coals for 68 cwt., 2 qrs., 19 lbs. of sugar, at 1*l.* 1*s.* 6*d.* per cwt.; what were the coals per ton? *Ans.* 18*s.* 5*d*$\frac{1}{12}$.

(8) Gave 69 cwt., 1 qr., 8 lbs. of butter for 29 cwt., 3 qrs. 19 lbs. of coffee, at 2*s.* 2*d.* per lb., and 4 cwt., 1 qr., 12 lbs. of cocoa, at 2*s.* 6*d.* per lb.; what is the value of the butter per lb.? *Ans.* 1*s.* 1*d.*+834.

(9) A has nutmegs worth 2*s.* 9*d.* per lb., which he is willing to exchange with B for 33 cwt., 2 qrs, 19 lbs. of tea, at 3*s.* 9$\frac{1}{2}$*d.* per lb.; what quantity of nutmegs must B receive from A? *Ans.* 46 cwt., 1 qr., 19 lbs.+$\frac{19}{28}$.

(10) A merchant gave 23*l.* 10*s.*, and 7 cwt., 2 qrs., 19 lbs. of sugar, valued at 6$\frac{1}{4}$*d.* per lb., for 12 cwt., 1 qr., 18 lbs. of soap; what was the price of the soap per lb.? *Ans.* 8*d.*+$\frac{41}{139}$.

(11) C gave 32*l.* 10*s.*, and 14 barrels of figs, weight 73 lbs. per barrel, tare 7 lbs. per barrel, for 24 cwt. 1 qr. of cheese; what did I value the cheese at per lb.? *Ans.* 4$\frac{3}{4}$*d.*+772 rem.

(12) Gave 9$\frac{1}{2}$*d.* per lb. for butter; how much soap, at 3$\frac{3}{4}$*d.*, can I get for 126 cwt., 2 qrs., 14 lbs. of butter? *Ans.* 35927$\frac{11}{15}$ lbs.

(13) A gives B 35 cwt. of indigo, at 17*s.* 2*d.* per lb., for 50 cwt. of pimento, and 28*l.* 12*s.*; what did B value his pimento per lb.? *Ans.* 11*s.* 10$\frac{3}{4}$*d.*+$\frac{157}{175}$.

(14) Sold 1 lb. of cheese for 4$\frac{3}{4}$*d.*; what quantity must I give for 22 cwt. 3 qrs. of cochineal, at 13*s.* 4*d.* per lb.? *Ans.* 766 cwt., 1 qr., 7 lbs.+$\frac{7}{19}$.

(15) Bought 68 cwt., 1 qr., 10 lbs. of cocoa, at 1*s.* 2$\frac{1}{2}$*d.* per lb.; what quantity of coffee must I give for the cocoa, the coffee being valued at 2*s.* 2*d.* per lb.? *Ans.* 38 cwt., 0 qr., 14 lbs.

EQUATION OF PAYMENTS.

By this rule an equated time is discovered for the payment of any number of bills by *one* bill.

Rule.—Multiply each bill by the time it has to run, then add all these products together for a dividend, and all the amounts of the bills for a divisor, the quotient is the equated time.

N. B.—Cancel before you divide.

Ex. 1.

$$\begin{array}{rl}
 & 300\times 6=1800 \\
 & 500\times 7=3500 \\
 & 900\times 8=7200 \\
 & \underline{630\times 5=3150} \\
233\cancel{0} &)\,1565\cancel{0}\,(6\tfrac{167}{233} \text{ months.} \\
 & \underline{1398} \\
 & \underline{167} \\
 & \underline{233}
\end{array}$$

Required equated time of payment of the following bills:—

(1) 300*l.* at 6 months, 500*l.* at 7 months, 900*l.* at 8 months, and 630*l.* at 5 months. *Ans.* $6\frac{167}{233}$ months.

(2) 249*l.* at 3 months, 375*l.* at 7 months, 697*l.* at 8 months, and 592*l.* at 9 months. *Ans.* $7\frac{885}{1913}$ months.

(3) 439*l.* at 8 months, 375*l.* at 6 months, 493*l.* at 11 months, and 189*l.* at 14 months. *Ans.* $9\frac{367}{1496}$ months.

(4) 265*l.* at 8 months, 982*l.* at 3 months, 576*l.* at 3 months, and 982*l.* at 4 months. *Ans.* $3\frac{2207}{2805}$ months.

(5) 271*l.* 10*s.* at 5 months, 397*l.* at 11 months, 595*l.* at 6 months, and 813*l.* at 7 months. *Ans.* $7\frac{900}{4153}$ months.

(6) A for 932*l.* at 73 days, B for 671*l.* at 118 days, C for 273*l.* at 67 days, D for 595*l.* at 253 days, and E for 714*l.* at 57 days. *Ans.* $167\frac{921}{1539}$ days.

(7) 496*l.* at 7 months, 637*l.* at 9 months, 473*l.* at 6 months, 873*l.* at 12 months, and 459*l.* at 3 months. *Ans.* $8\frac{196}{1469}$ days.

(8) On June 14th I have three bills, as follows:—A for 296*l.* 19*s.*, due August 17th, B for 315*l.* 9*s.*, due September 9th, and C for 298*l.* 14*s.*, due December 5th. Required the time of a bill that shall be paid in one payment in lieu of the above bills. *Ans.* $106\frac{479}{18222}$ days.

(9) This is September 12th. I have four bills due, as below: 75*l.*, due October 14th, 933*l.*, due December 19th, 673*l.*, due November 23rd, and 578*l.*, due October 29th. Required the equated time of payment of these bills. *Ans.* $75\frac{31}{2259}$ days.

(10) A merchant, January 1st, has the following bills, for which he wishes to have one bill:—One for 290*l.*, due February 16th, one for 373*l.*, due March 19th, one for 273*l.* 12*s.*, due April 3rd, one for 796*l.*, due May 3rd. Required the equated time of payment. *Ans.* $94\frac{7399}{8663}$ days.

FELLOWSHIP WITHOUT TIME.

Fellowship Without Time is a rule to discover the gains or losses of parties who join their stocks together, and agree to participate equally in the profits or losses of the adventure. It may be applied to the distribution of bankrupts' effects, to property left by will where the assets are less than the amount of legacies, to prize-money, &c.

Rule.—As the sum of all the shares of the parties (or if dividing a bankrupt's estates, as the sum of all his creditors' accounts): the money gained, lost, (or money left per will,) :: each particular share : to the *true* share of the gain, loss, or legacy.

The proof is found by adding together all the shares of gain, loss, or money left, which must of necessity equal the sum divided or sum lost.

Ex.—A, B, and C trade together; A put in 300*l*., B 400*l*., and C 600*l*.; they gain 240*l*. What is each man's share?

$$\begin{array}{r} 300 \\ 400 \\ 600 \\ \hline 13\cancel{0}\cancel{0} : 240 :: 3\cancel{0}\cancel{0} \end{array}$$

$$\begin{array}{r} 3 \\ \hline 13)720 \\ \hline £55\ \ 7\ \ 8\frac{1}{4}+\frac{3}{13} \\ \hline \end{array}$$

$$\begin{array}{r} 13\cancel{0}\cancel{0} : 240 : 4\cancel{0}\cancel{0} \\ 4 \\ \hline 13)960 \\ \hline £73\ \ 16\ \ 11+\frac{4}{13} \\ \hline \end{array}$$

£	s.	d.
55	7	$8\frac{1}{4}+\frac{3}{13}$
73	16	$11\ +\frac{4}{13}$
129	4	$7\frac{1}{4}+\frac{7}{13}$
240		
110	15	$4\frac{1}{2}+\frac{6}{13}$

Here we add the shares together, and proceed as the rule directs; taking care to cancel. In questions of this kind, *i. e.*, where the shares are the multiples of any unit, as 400 is of 100, also 600 of 100, it is better to proceed as follows. It will be perceived that 400 is $\frac{1}{3}$ more than 300, therefore if we add $\frac{1}{3}$ of A's share to A we obtain B's share. Again, it will be perceived that C's share is twice A's; if we therefore multiply A's share by 2 we obtain the same result as above.

FELLOWSHIP WITH TIME.

Rule.—Multiply each person's money by the time it is in partnership, then add up all the resulting products and reserve the sum. Proceed in this manner with every person's money and time, and reserve the sums; then add all the reserved sums together, and say, As the sum of all these reserved sums : any one of he reserved sums :: the money gained or lost : the person's hare of it.

Ex.—A and B trade together for 12 months; A put in 800*l.*, nd 5 months after 300*l.* more; two months after this he took out 00*l.*, but put in 450*l.* at the end of 3 months. B put in 3280*l.*, nd at the end of 7 months took out 1280*l.* They gained 540*l.* equired each man's share.

A	B
800 × 5 = 4000	3280 × 7 = 22960
300	1280
1100 × 2 = 2200	2000 × 5 = 10000
700	32960
400 × 3 = 1200	
450	£ *s.* *d.*
850 × 2 = 1700	540 0 0
9100	116 16 $7\frac{3}{4}+\frac{541}{701}$
	423 3 4 $+\frac{160}{701}$

$$\begin{array}{l}
32960 \\
9100 \\
\hline
\cancel{4}\cancel{2}\cancel{0}\cancel{6}\cancel{0} : 9100 :: \cancel{5}\cancel{4}\cancel{0} \\
701 \qquad\quad 9 \quad\; 9 \\
\quad 701\overline{)81900} \\
\qquad \underline{£116\ \ 16\ \ 7\frac{3}{4}+701}
\end{array}$$

Here we multiply 800 by 5, because the 800 was put into the tock and remained unaltered for 5 months. He then put in 300, hich made his stock 1100, and this was in for two months; at he end of this time he took out 700, leaving 400, which remained r 3 months, when he put in 450, making his stock 800, which emained for $12-5-2-3=2$ months. We then multiply each um by the time opposite to it, and obtain for sum of product 100. We then proceed with B's in the same manner, and obtain 2960 as sum of products of money and time. We then add these wo, viz., 9100+32960 together, and obtain for sum 42060. We en state as 42060 : 9100 :: 540 : 116*l.* 16*s.* $7\frac{3}{4}$*d.*+$\frac{541}{701}$. This ubtracted from 540 leaves 423*l.* 3*s.* 4*d.*+$\frac{160}{701}$ for B's share.

FELLOWSHIP.

(1) A, B, and C trade together; A put in 450*l.*, B 690*l.*, 720*l.*; they gain 425*l.* Required each man's share.

Ans. A's share 102*l.* 16*s.* $5\frac{1}{4}$*d.*+42, B's 157*l.* 13*s.* $2\frac{1}{2}$*d.*+52,
C's 164*l.* 10*s.* $4\frac{1}{4}$*d.*

(2) A, B, C, and D trade together; A put in 320*l.*, B 730*l.*, C 966*l.*, and D 544*l.*; they gain 370*l.* What is each man's share? *Ans.* A 46*l.* 5*s.*, B 105*l.* 10*s.* 1$\frac{3}{4}$*d.*+$\frac{1}{2}$, C 139*l.* 12*s.* 4*d.*+$\frac{1}{2}$,
D 78*l.* 12*s.* 6*d.*

(3) A, B, and C trade; A put in 640*l.*, B 725*l*, and C 325*l.*; they gain 500*l.* What is each man's share?
Ans. A 189*l.* 6*s.* 11$\frac{3}{4}$*d.*, B 214*l.* 9*s.* 11$\frac{1}{4}$*d.*, C 96*l.* 3*s.* 1*d.*

(4) A, B, C, and D trade together; A put in 300*l.*, B 625*l.*, C 825*l.*, and D 920*l.*; they gain 735*l.* What is each man's share? *Ans.* A 82*l.* 11*s.* 8*d.*, B 172*l.* 1*s.*, C 227*l.* 2*s.* 1$\frac{1}{2}$*d.*,
D 253*l.* 5*s.* 2$\frac{1}{2}$*d.*

(5) A, B, C, D, and E trade; A put in 1420*l.*, B 3200*l.*, C 7204*l.*, D 5460*l.*, and E 2480*l.*; they gain 1720*l.* Required each man's share. *Ans.* A 123*l.* 11*s.* 6$\frac{1}{4}$*d.*, B 278*l.* 9*s.* 8$\frac{1}{2}$*d.*,
C 626*l.* 18*s.* 10*d.*, D 475*l.* 3*s.* 4*d.*, E 215*l.* 16*s.* 6$\frac{1}{4}$*d.*

(6) A, B, and C trade; A put in 432*l.*, B 728*l.*, and C 540*l.*; they gain 296*l.* What is each man's share?
Ans. A 75*l.* 4*s.* 4$\frac{1}{2}$*d.*, B 126*l.* 15*s.* 1$\frac{3}{4}$*d.*, C 94*l.* 0*s.* 5$\frac{1}{2}$*d.*

(7) A, B, and C trade; A put in 1200*l.*, B 7280*l.*, and C 520*l.*; they lose 240*l.* What is each man's share of the loss?
Ans. A 32*l.*, B 194*l.* 2*s.* 8*d.*, C 13*l.* 17*s.* 4*d.*

(8) Divide 12960 acres amongst A, B, C, and D, so that their shares may be in proportion of 3, 5, 7, 9.
Ans. A 1620, B 2700, C 3780, D 4860.

(9) Divide 568 acres amongst A, B, and C, so that their shares may be in proportion of 36, 70, and 94.
Ans. A 102$\frac{6}{25}$, B 198$\frac{4}{5}$, C 266$\frac{24}{25}$.

(10) Five men, whose stocks are in proportion of 5, 7, 9, 12, and 15, trade together; their gain was 12493*l.* 19*s.* 8*d.* Required each man's share. *Ans.* 1301*l.* 9*s.* 1$\frac{1}{2}$*d.*+$\frac{1}{3}$, 1822*l.* 0*s.* 9$\frac{1}{4}$*d.*+$\frac{2}{3}$,
2342*l.* 12*s.* 5$\frac{1}{4}$*d.*, 3123*l.* 9*s.* 11*d.*, and 3904*l.* 7*s.* 4$\frac{3}{4}$*d.*

(11) A merchantman is taken by a ship of war; the captain has $\frac{3}{8}$ of the prize, which was valued at 26687*l.* 10*s.* 10*d.*, after deducting 156*l.* 18*s.* 10*d.* for some expenses; the other officers, of which there are 14, have $\frac{1}{7}$ of the remainder, and the men have the residue. Required each man's share.
Ans. The captain had 10007*l.* 16*s.* 6$\frac{3}{4}$*d.*, each officer
157*l.* 8*s.* 11*d.*+$\frac{3}{14}$, the men 14474*l.* 9*s.* 4$\frac{1}{2}$*d.*
to be divided amongst them.

Fellowship with Time.

(1) A, B, and C trade together for 15 months; A put in 300*l.*, and 5 months after 390*l.* more; but at 9 months he took out 220*l.* B put in 500*l.*, and at 7 months after took out 240*l.*, but

ut in that and 300l. more at the end of 14 months from their rst trading. C put in 960l., and 4 months after 300l. more, and months after they began to trade took out 500l., but put in at 2 months 300l.; they gain 600l. What is each man's share?

Ans. A 152l. 16s. 1$\frac{1}{4}$d.+73., B 122l. 1s. 8$\frac{1}{2}$d.+122., C 315l. 2s. 1$\frac{3}{4}$d.+83.

(2) A and B trade together for 14 months; A put in 2060l., ut in 4 months took out 500l., and at 11 months 300l. more, but ut in at 12 months 600l. B put in 700l., and in 9 months 960l. nore, but at 10 months took out 250l., and at 11 months put in 00l; they gain 800l. What is each man's share?

Ans. A 495l. 18s. 10d.$-\frac{378}{1947}$, B 304l. 1s. 1$\frac{3}{4}$d+435.

(3) A and B trade together for one year; A put in 600l., and t the end of 10 months 700l. more, but at 11 months took out 50l. B put in 1560l., but at the end of 5 months took out 800l.; hey gain 640l. What is each man's share?

Ans. A 247l. 1s. 6d.+ 696, B 329l. 18s. 5$\frac{3}{4}$d.+ 1441.

(4) A and B trade together for 16 months; A put in 6000l., t the end of 7 months took out 690l., but put in 2490l. at the nd of 11 months. B put in 4900l., and at the end of 11 months 80l.; they gain 800l. What is each man's share?

Ans. A 444l. 8s. 6d.+ 2208, B 355l. 11. 5$\frac{3}{4}$d.+ 16196.

(5) A, B, and C, trade together for 15$\frac{1}{2}$ months; A put in 00l., and at the end of 6 months took out 250l. B put in 965l., nd at 3 months 700l. more, but at 5 months took out 1005l. C put in 1500l., but at 10 months took out 850l.; they gain 688l. 12s. What was each man's share? *Ans.* A 138l. 6s. 9d$\frac{1}{4}$.+2605, B 179l. 3s. 10d.+126, C 371l. 1s. 4d.+716.

(6) A, B, and C trade together for 10 months; A put in 1600l., and at 5 months 300l., but took out that and 750l. at the end of 7 months. B put in 2450l., and took out 890l. at the end of 6 months, but put in 980l. at the end of 9 months. C put in 3500l., and at 5 months took out 780l., but put in at 9 months, 1500l., and at the end of 9$\frac{1}{2}$ months took out all he had in the stock; they gain 1490l.. What is each man's share?

Ans. A 315l. 15s. 7$\frac{1}{2}$d.+ 276, B 489l. 4s. 6$\frac{1}{4}$d., C 684l. 14s. 10$\frac{1}{4}$d.

(7) A and B trade together for 10 months; A put in 360l., and at 5 months 600l. more, but took out 250l. 3 months after. B put in 635l., and in 3 months after took out 248l., but in 4 months after put in 740l.; they gained 120l. Required each man's share. *Ans.* A 56l. 11s. 10$\frac{3}{4}$d.+ $\frac{1623}{6467}$, B 63l. 8s. 1d.+ $\frac{5044}{6467}$.

PROFIT AND LOSS

Is a rule by which merchants and other traders discover the gain or loss per cent. on any article sold; it also teaches them to rise or fall their prices so as to gain or lose at a certain rate per cent.

The *prime* cost of any article is the *price given* for it.

The *selling* price is the *price received* for it.

The *prime* cost, with the gain added, or loss subtracted, gives the *selling price*.

The *selling* price, with the *prime* cost subtracted, gives *the gain;* with the *gain* subtracted gives the *prime* cost; with the *loss* added, gives the prime cost.

Gain or *loss* per cent. means a certain increase or diminution on 100*l.* purchase money or prime cost. Thus, when 20 per cent. is gained, 120*l.* is the received price; but when 20 per cent. is lost, 80*l.* is the received price.

CASE 1.

Given the prime cost, and the selling price of an integer of any article, to find the gain or loss per cent.

RULE.—As the prime cost : 100*l.* : : the gain or loss given to the gain or loss per cent.

CASE 2.

Given the prime cost. with a certain proposed gain or loss per cent., to find the selling price.

RULE.—As 100*l.* : prime cost, : : 100*l.* with the gain added, or 100*l.* with the loss subtracted : the selling price.

CASE 3.

Given the selling price of an integer of any article, and the gain or loss per cent. on its sale, to find the prime cost.

RULE.—As 100*l.* with the gain added, or 100*l.* with its loss subtracted, : selling price : : 100*l.* : the prime cost.

CASE 4.

Given the selling price of an integer of any article, and the gain per cent., to find the gain per cent. at any other proposed price.

RULE 1.—As the selling price given : the proposed price : : 100*l.* with the gain per cent. added : to the selling price required.

RULE 2.—To find, at a different gain per cent., another selling price.

Rule.—As 100*l.* with the given gain added, : 100*l.* with the proposed gain added : the given selling price : to the required selling price.

Examples to the different Cases.

Case 1.—See question 1 in the following questions.

$$
\begin{array}{rrcrcrr}
s. & d. & & & & 2 & 4 \\
1 & 10 & : & 100 & :: & \underline{1} & \underline{10} \\
\cancel{22} & & & \underline{3} & & & \cancel{6} \\
11 & & & 11)300 & & & 3 \\
 & & & \multicolumn{4}{r}{\underline{\pounds 27\ \ 5\ \ 5\frac{1}{4}+\frac{9}{11}}}
\end{array}
$$

Here we state as directed. We then ÷ the 6 and 22 by 2 (2 being a factor of 6 and 22). We then multiply the 100 by 3, and divide by 11, obtaining 27*l.* 5*s.* $5\frac{1}{4}$*d.*+$\frac{1}{11}$ for answer.

Case 2.—See question 3, in the following questions.

$$
\begin{array}{rcrrcrr}
 & & s. & d. & & & \\
100 & : & 2 & 9 & :: & 125\frac{1}{2} & : \\
\underline{2} & & & & & \underline{2} & \\
200 & & & & & 251 & \\
 & & & & & \underline{2\frac{3}{4}} & \\
 & & & & & 502 & \\
 & & & & & 125\ \ 6 & \\
 & & & & & \underline{62\ \ 9} & \\
 & & & & 200) & \underline{600\ \ 3} & \\
 & & & & & \underline{s.3\ \ 5\frac{1}{4}+\frac{3}{5}} &
\end{array}
$$

Case 3.—See question.

(1) Sold tobacco at 2*s.* 4*d.* per lb. which cost 1*s.* 10*d.*, what o I gain or lose per cent.? *Ans.* 27*l.* 5*s.* $5\frac{1}{4}$*d.*+$\frac{9}{11}$.

(2) Bought 3 cwt., 2 qrs., 14 lbs. of coffee for 17*l.* 10*s.*, what ust I sell it at per lb. to gain 15 per cent.? *Ans.* $11\frac{3}{4}$*d.*+238.

(3) Gave 2*s.* 9*d.* per lb. for tea, I wish to gain $25\frac{1}{2}$ per cent., what rate must I sell it at per lb.? *Ans.* 3*s* $5\frac{1}{4}$*d.*+$\frac{3}{5}$.

(4) Bought 2 cwt., 0 qr., 18 lbs of chocolate for 5*l.* 16*s.* 8*d.*, what rate per cwt. must I sell it to lose $10\frac{1}{2}$ per cent.? *Ans.* 2*l.* 19*s.* $7\frac{3}{4}$*d.*

(5) Sold sugar, which cost $6\frac{3}{4}$*d.* per lb., for $8\frac{1}{2}$*d.* per lb., what id I gain per cent.? *Ans.* 25*l.* 18*s.* $6\frac{2}{9}$*d.*

(6) If I give 1*s.* 4*d.* per lb. for 3 cwt., 2 qrs., 19 lbs. of chocolate, and sold it for 19*l.* 18*s.*, what did I gain or lose, and how uch per cent.? *Ans.* Lost 7*l.* 10*s*, loss per cent. 27*l.* 7*s.* $5\frac{1}{4}$*d.*

(7) If I lose 3*s*. 4*d*. on the sale of 7 yards of silk, which cost 21*s*. 8*d*., what do I lose per cent.? *Ans*. 15*l*. 7*s*. 8¼*d*.+$\frac{3}{13}$.

(8) What is the gain or loss per cent. on the sale of 56 cwt., 2 qrs., 14 lb. for 968*l*. 12*s*., which cost 12*l*. 18*s*. per cwt.?
Ans. 35*l*. 1*s*. 3½*d*.

(9) Bought 68 cwt. of cheese at 49*s*. per cwt., which I sold for 37*s*. 6*d*. per cwt., what was the loss per cent.?
Ans. 23*l*. 9*s*. 4½*d*.+30.

(10) What is the gain or loss per cent. on the sale of 16 hhds. of sugar, each weighing 12 cwt. 2 qrs., which cost 350*l*. 12*s*., for 1*l*. 8*s*. 6*d*. per cwt. *Ans*. 18*l*. 14*s*. 2½*d*—2456 loss per cent.

(11) What is the gain or loss per cent. on the sale of 68 cwt. of cheese, at 3*l*. 14*s*. 6*d*. per cwt., which cost 226*l*. 18*s*.?
Ans. 11*l*. 12*s*. 8¼*d*+ 3078.

(12) Bought 7 hhds. of sugar, each containing 14 cwt., 3 qrs., 14 lbs., for 96*l*. 18*s*., what ought I to sell it at per lb. to gain 5 per cent.? *Ans*. 2*d*. per lb.+4339.

(13) Sold lead at 6¾*d*. per lb., how much ought I to sell 49 cwt. 2 qrs. 16 lb. for to gain 42 per cent.?
Ans. 222*l*. 1*s*. 0½*d*.

(14) Sold 58 cwt. 2 qrs. of sugar for 296*l*. 18*s*., what ought I to sell it at per lb. to gain 22½ per cent.?
Ans. 1*s*. 1¼*d*. + 1896.

(15) Gave 38*l*. 16*s*. 9*d*. for 20 cwt. of cheese, what must I sell it at per cwt. to gain 30 per ceut.? *Ans*. 2*l*. 10*s*. 5¾*d*.+⅖.

(16) Sold 45 cwt. of bacon, at 42*s*. 6*d*. per cwt., what ought I to have sold it at to gain 14 per cent.? *Ans*. 2*l*. 9*s*. 3⅖*d*.

(17) If I buy 22 barrels of herrings, at 27*s*. 6*d*. per barrel, and sell them for 18*s*. 9*d*., what do I lose on the whole, and at what rate per cent.?
Ans. 9*l*. 12*s*. 6*d*. on the whole; 12*l*. 1*s*. 4½*d* per cent.

(18) What must I sell cloth at per yard which cost 9*s*. 2*d*., in order that I may lose 17½ per cent.? *Ans*. 6*s*. 3*d*$\frac{1}{20}$.

(19) Bought 84 yards of cloth for 176*l*. 8*s*., sold half for 95*l*., and the remaining half for 1*l*. 17*s*. 6*d*. per yard, what do I gain or lose per cent.? *Ans*. 1*l*. 10*s*. 0¾*d*. $\frac{277}{441}$ per cent. lost.

(20) What must I have sold the preceding cloth at per yard to have lost 16 per cent.? *Ans*. 1*l*. 15*s*. 3¾*d*.

EXCHANGE.

EXCHANGE is a rule by which we find how much money of one country is equal to a given sum of another.

The *par of exchange* is a *fixed* price; thus, London gives 1*l*. sterling for 25 franes 22 centimes at Paris.

The *course of exchange* is the market price, and consists of a *ertain* price and an *uncertain* price, as in the above two counries.

Here the francs and centimes are the *uncertain* price, and the ound sterling the *certain* price.

The course of exchange fluctuates from various causes; someimes from a scarcity or superabundance of bills; sometimes from arge shipments, importation of corn, &c. In some cases London gives a *certain* price for an *uncertain* price of other countries.

Ex.—London gives 1*l.* sterling for an *uncertain* price at the following places:—Amsterdam, Hamburgh, Paris, Frankfort, Genoa; and receives a *certain* price from the following places: Madrid, Gibraltar, Leghorn, Venice, Naples, Palermo, Lisbon.

The *uncertain* or *variable* price is the one in which the course of exchange is always reckoned.

The lists circulated amongst the trading community are called *quotations*.

It is easy to perceive, by comparing the *course of exchange* with he *par*, whether or not the exchange is *for* or *against* a country.

Thus, if London receives the uncertain price *above* par, the exchange is in favour of London; if *below* par, *against* London.

Ex.—The par of exchange between London and Paris is 25f. 22c.; that is, 25 francs and 22 centimes are given by Paris for 1*l.* sterling; now, if the price should be 25f. 35c., the exchange s in favour of London, because London receives more for 1*l.* sterling in the latter instance than in the former. If it had been 24f. 90c. the exchange would have been *against* London, because the variable price is below par; and so of any other country.

FRANCE.

Accounts used to be kept in livres, sous, and deniers, but are now kept in francs and centimes.

Old Table.

			English Value.
12 deniers	=	1 sou or sol . . .	½ par.
20 sous	=	1 livre tournois. . .	10*d.*
3 livres	=	1 ecu.	

New Table.

20 centimes = 1 decime.
10 decimes = 1 franc.
20 francs = 1 napoleon, or louis-d'or.
And as 243 deniers = 1 franc, or new livre.
240 ditto = 1 old livre tournois.

Hence to find the livres contained in any number of francs, state

As 80 : 81 :: francs : livres.

		In English Money.
A centime	=	·38 of a farthing.
A decime	=	$\frac{3}{4}$ · 8.
A franc	=	$9\frac{1}{2}d.$
A napoleon	=	$15s.\ 10\frac{1}{2}d.$

Par of exchange 25 francs 22 cent. = 1*l.* sterling.

Ex. (1) How many pounds sterling can be received for 36984 French francs, exchange 24f. 15c. per pound sterling? *Ans.* $1531l.\ 8s.\ 6\frac{3}{4}d. + \frac{69}{161}$.

(2) How many francs can be received for 361*l.* 8*s.* 9*d.*, exchange 24f. 15c. per pound sterling? *Ans.* 8963·65 fr.

(3) A gentleman wishes to exchange 8584 livres for 355*l.* 4*s.*, what was the course of exchange? *Ans.* 24·16+

(4) How many pounds sterling can be received for 3247fr. exchange 24fr. 20c., per pound sterling? *Ans.* $134l.\ 3s.\ 5\frac{1}{2}d. + \frac{74}{121}$.

(5) How many sovereigns and half guineas, and of each an equal number, must be given in exchange for 78496 francs, exchange 24f. 50c.? *Ans.* 2533 sovereigns and half guineas + $19s.\ 6\frac{3}{4}d. \frac{36}{49}$.

AMSTERDAM, ANTWERP, AND ROTTERDAM.

They keep their acounts in florins, guilders, stivers, phennings, and frequently in pounds, shillings, pence, Flemish.

					s.	*d.*	
16	phennings	=	1 stiver	=	0	1	English.
20	stivers	=	1 guilder or florin	=	1	$8\frac{1}{2}$	ditto.
$2\frac{1}{2}$	guilders	=	1 rix dollar	=	4	4	ditto.
6	guilders	=	1 pound Fl.	=	0	7	ditto.

Also,

8 phennings = 1 groat, or penny Flemish.
12 pence = 1 shilling.
20 shillings = 1 pound.
And 100 cents. = 5 stivers; 20 stivers = 1 florin.

Exchange with Britain as above.
Ditto Hamburgh, 37 flor. for 40 marks Banco.
Ditto Paris, $57\frac{1}{2}$ fl. for 120 francs.

A pheuning	=	$\frac{1}{2}d.$	English.
Stiver	=	$1\frac{1}{20}d.$	ditto.
Florin	=	$1s.\ 8\frac{1}{2}d.$	ditto.
Pound Flemish	=	$10s.$	ditto.

To reduce Dutch Money to English.

State as the rate of Exchange : 1*l.* sterling :: the Dutch noney : to sterling required. And Dutch money is obtained rom sterling by reversing the preceding statement.

Par of Exchange 11 fl. 95 cents. for 1*l.* sterling.

Usance 1 month's date: no days of grace.

Ex. 1.—In 450*l.*, how many florins, exchange 12 fl. 9½ cents er 1*l.* sterling. *Ans.* 5410 fl. 275 cents.

2. Reduce 37596 fl. 19 cents. into pounds sterling, each at 12 fl. 68 cents. *Ans.* 3089*l.* 4*s.* 9½*d*+$\frac{170}{1217}$.

3. In 42731 fl. 18 cents., how much sterling, exchange 12 fl. 25 cents.? *Ans.* 3542*l.* 9*s.* 4¾*d.*+$\frac{241}{965}$.

HAMBURGH AND ALTONA.

There are five different kinds of money at Hamburgh.

1. Bank money, which is similar to our Public Funds. Banco bears a variable premium, fluctuating between 17 and 26 per cent.
2. Specie, which is worth ½ per cent. more than Banco.
3. The Gold Ducat fluctuates according to the rate of Exchange; sometimes worth more, sometimes less than Banco.
4. Light Coin (foreign coin) which, is estimated at 38 per cent. less than Banco.
5. Currency, which consists of the silver money coined subsequently to 1726.

		s.	*d.*	
12 Pennings	= 1 Schilling Banco ..	0	6¾	English.
16 Schillings	= 1 Mark	1	6	„
3 Marks	= 1 Rix Dollar	4	6	„

Ancient Money.

		s.	*d.*	
6 Pennings	= 1 Groat, or Penny ..	0	$\frac{9}{10}$	English.
12 Pence	= 1 Shilling	0	6¾	„
20 Shillings	= 1 Pound	10	0	„

Accounts are kept in marks and schillings, and in pounds, shillings, and pence Flemish; but this is nearly obsolete.

Present exchange of Hambro' in marks of 16 shillings Banco.

Hence 6 shillings Hambro', Banco = 1 shilling Flemish.

„ 3 marks Banco .. .. = 8 shillings „

Ex.—Reduce 4739 marks, 9 shillings Banco into sterling; exchange at 35*s.* 4*d.* Flemish per 1*l.* sterling.

```
35  4 : 1 :: 4739  9
12                16
---               -----
424              75833
                     2
           -----------
      424)151666(357 149/212
          1272
          ----
           2446
           2120
           ----
            3266
            2968
            ----
             298   149
             --- = ---.
             424   212
```

RUSSIA AND ST. PETERSBURGH.

100 copecks = 1 ruble.

The money consists of metallic and Bank currency, or paper.

The silver ruble is in value = 3*s*. 7½*d*. English.

The paper ruble „ = about 10*d*. (the exchange of it being variable).

Par of exchange 10½*d*. per ruble on bills of 3 months date.

There are 10 days of grace for bills "after date;" but for bills after sight 3 days are allowed. It must be remembered that Russia still reckons by the old style, which is 12 days later than the style in England. Thus, March 29th in Russia, would be in England April 10.

Paper rubles are reduced to silver ones by dividing 3*s*. 7½*d*. by the rate of exchange, which will give the equivalent number of paper rubles in one silver ruble.

(1) In 14973 rubles 87 copecks, how much sterling, exchange at 10¼*d*.? *Ans*. 639*l*. 10*s*. 2*d*.+67.

(2) In 275*l*. 14*s*. sterling, required the number of paper rubles, exchange at 10⅝*d*.? *Ans*. 6227 rubles + 49 rem.

(3) How many silver rubles can be received for 568*l*. 12*s*., exchange at 37¼*d*.? *Ans*. 3663 rubles + 69 rem.

Gold Coins.	In English Money. £	*s*.	*d*.
Ducat	0	9	2¼
Imperial of 10 rubles of 1763 ..	2	1	6¼
„ 9 „ ..	1	12	9
Silver Coins.			
Ruble of 100 copecks, 1807 ..	0	3	2

MADRID.

There are various modes of keeping accounts in Spain.

The following are the names of monies in which accounts are kept at Madrid:—

			s.	*d.*
$8\frac{1}{2}$	maravedies =	1 quarto	0	$1\frac{5}{16}$
4	quartos .. =	1 real	0	$5\frac{1}{4}$
8	reals .. =	1 piastre, or dollar of exchange	3	6
4	piastres .. =	1 pistole of exchange	14	0
375	maravedies =	1 ducat of exchange.		

A maravedi is $\frac{42}{272}$ of a penny sterling.

The piastre is the mode of exchange with England.

Par of exchange 36*d.* for 1 piastre.

At Madrid the usance of bills from England is			60 days' date.
At Cadiz	ditto	ditto	2 months.

English value of Coins.

	£	*s.*	*d.*
Doubloon, gold, since 1786	3	4	8
Half pistole, or crown	0	8	1

(1) Reduce 8172 piastres, 3 reals, 19 marav. into sterling, exchange at $35\frac{1}{4}$*d.* per dollar of exchange. *Ans.* 1200*l.* 6*s.* $6\frac{1}{2}$*d.* $\frac{27}{34}$.

(2) Reduce 8295 ducats of exchange into piastres.
Ans. 11436 pia. $\frac{33}{34}$.

(3) In 478*l.* 12*s.* 4*d.*, how many piastres, exchange at $35\frac{1}{4}$*d.* per piastre. *Ans.* 3258 p. 5 r. 11 m. $\frac{47}{141}$.

(4) A merchant has a bill for 7968 piastres, 5 reals, how much sterling will he receive, exchange at $35\frac{1}{2}$*d.*?
Ans. 1178*l.* 3*s.* 10*d.*

LISBON.

There are three kinds of money in Portugal.

1. *Effective*, i. e., specie.
2. *Paper Money*, which is always at a discount; particularly in war time, when it is frequently at 30 per cent. discount.
3. *Legal Money*, i. e., half specie, half paper.

Accounts are kept in Portugal in milrees and rees.

1000 rees = 1 milree. 400 rees = 1 crusado.

The mark ⊕ is used to separate milrees and rees.

Par of exchange 49*d.* for 1 milree.

Usance 60 days' after date, and 6 days' grace.

Value of Coins in English Money.

GOLD.	£	s.	d.
Lisbonine, or moidore	1	6	$11\frac{1}{4}$
Portuguese	1	15	11
Piece of 16 testons, or 1600 rees	0	8	$11\frac{1}{4}$
Crusada	0	2	$7\frac{1}{4}$
SILVER.			
New crusada	0	4	11

(1) How many crusadas in 473*l.* 12*s.*? *Ans.* 3637 cru. $99\frac{1}{5}$. rees.

(2) How many milrees in 55*l.* 7*s.* 6*d.*, exchange at par? *Ans.* 271 mil., $224\frac{22}{29}$ rees.

(3) How many milrees can be exchanged for 85*l.* 12*s.* 9p., exchange at $61\frac{1}{2}$*d.* per milree? *Ans.* 334 mil. 195 rees, 5 rem.

(4) How many pounds sterling can be received for 31594 milrees, exchange at $59\frac{1}{4}$*d.*? *Ans.* 7888*l.* 12*s.* $10\frac{1}{2}$*d.*

(5) How many milrees, each 63*d.* per mil., must be received in exchange for 72590 francs, exchange at 24 fr. 19 c. per 1*l.* sterling? *Ans.* $11431\frac{5}{7}+\frac{1039}{2418}$.

(6) How many rees must be exchanged for a bill of 824*l.* 12*s.* value, exchange $61\frac{1}{2}$*d.* per milree? *Ans.* $3217951\frac{27}{123}$ rees.

LEGHORN.

Accounts are kept at Leghorn in denari, soldi, and piastres.

12 denari = 1 soldo.
20 soldi = 1 piastre or dollar.

Par of exchange 49*d.* for 1 piastre.

Usance 3 months after date.

(1) In 560*l.* how many piastres, exchange $48\frac{1}{4}$*d.*? *Ans.* $2785\frac{95}{193}$ piastres.

(2) In 7960 soldi, how much sterling, exchange $49\frac{1}{4}$*d.*? *Ans.* 1633*l.* 9*s.* 2*d.*

(3) Reduce 2579 piastres, 11 soldi, 4 denari, into sterling, exchange at $50\frac{3}{4}$*d.* *Ans.* 136*l.* 7*s.* $4\frac{1}{4}$d. + $\frac{1}{120}$.

GENOA.

At Genoa, and all places in the kingdom of Sardinia, an assimilation of French money took place in 1820.

The calculation of the exchanges is the same as that of France.

100 centessima = 1 lira nuova, or livre, and this is equal to the French franc.

Par of exchange 25 lire 16 centimes for 1*l.* sterling.
Usance 3 months after date.

It is presumed sufficient examples have already been given for he pupil to obtain a knowledge of exchange. A *thorough* know-edge is only to be acquired from such a Work as Dr. KELLY'S *ambist;* to which, for the arbitration of exchange, and the mode f negotiating bills, the learner is referred. From that work, he ill also be enabled to understand *remittances* and *returns*, which, ith the preceding articles, would occupy more space than could e devoted to them in this work. We shall, therefore, conclude his subject by introducing to the learner's notice the exchange etween London and several places not mentioned in Lloyd's List.

Berlin gives 8 rix-dollars of 24 groschen,
Vienna and Trieste give 11 florins of 60 cruetzers,
Gottenburgh gives 4½ rix-dollars of 48 shillings,
Copenhagen „ 6 „ 96 „
Dantzic .. „ 25 florins of 30 groschen,
Konigsburg „ 19 „ „
Constantinople and Smyrna give 18 piastres of 80 aspers,
For the 1*l.* sterl.
Malta receives 48½*d.* for 1 pezza.
Rome gives 48⅕ paoli for 1*l.*
Dresden and Leipzig give 6 rix-dollars, 15¾ groschen, for 1*l.*
Canada gives 111⅑*l.* currency for 100*l.* sterling.
Jamaica gives 140*l.* „ „
United States gives a dollar for 4*s.* 6*d.* sterling

ALLIGATION

s a Rule which teaches the method of mixing simple quantities. t is usually divided into four Cases.

ALLIGATION MEDIAL.

CASE 1.

iven certain quantities and their rates to find the mean rate of the mixture.

RULE.—Multiply each quantity by its value for a unit of the uantity, and divide the sum of the products by the sum of the antities.

(1) 10 galls. of brandy at 16*s.* 4*d.*, were mixed with 36 at ., 45 at 6*s.*, and 18 at 10*s.* Required the price of 1 gallon of is mixture. *Ans.* 8*s.* 3*d.*+25.

(2) 20 galls. of wine at 9*s*. 6*d*. were mixed with 14 at 13*s*. 6*d*., and 20 at 12*s*. 6*d*. Required the mean rate. *Ans*. 11*s*. 7¾*d*.+6.

(3) If 14 lbs. of sugar at 6*d*. per lb. be mixed with 8 lbs. at 7*d*., 10 lbs. at 8*d*., and 12 lbs. at 6½*d*., what is the value of 2 lbs. of this compound? *Ans*. 1*s*. 1½*d*.+4.

(4) A refiner melts 14 lbs. silver bullion of 8 oz. fine, with 10 lbs. of 6 oz. fine, and 14 lbs. of 9 oz. fine. Required the fineness of 1 lb. of the metal. *Ans*. 7 oz. 16 dwts.+32.

(5) A farmer has 30 bushels of wheat at 9*s*. per bushel, 49 at 3*s*. 6*d*., and 60 at 2*s*. 6*d*. per bushel. Required the mean rate. *Ans*. 4*s*. $3\frac{19}{139}$*d*.

(6) A wine merchant has 16 gallons of Cape at 5*s*. 6*d*., 27 of Canary at 8*s*. 6*d*., 59 of Sherry at 7*s*. 9*d*., and 73 of Bucellas at 5*s*. 9*d*. Required the price of a gallon of this mixture. *Ans*. 6*s*. 9¾*d*.+$\frac{24}{35}$.

ALLIGATION ALTERNATE.

CASE 2.

Given the rates of the simple quantities to find the quantity of each to be sold at a mean rate.

RULE.—1. Place the quantities one under another; the average rate being placed on the left of them.

2. Link a greater and less rate than the *average* rate together.

3. Find the difference between each rate and the average, and place it opposite to the rate to which it is linked. The differences so found will be the answers required.

Ex.—What quantity of tea at 6*s*., 8*s*., and 10*s*., must be mixed together to make a composition worth 7*s*. per lb.?

$$7\left\{\begin{array}{r}6\\8\\10\end{array}\right. \quad \left.\begin{array}{rrr} 1+3= & 4 \text{ of } 6s. = & 24\\ & 1 \text{ of } 8s. = & 8\\ & \underline{1} \text{ of } 10s. = & \underline{10}\\ & 6 \qquad & 6)\overline{42}\\ & & \underline{7} \end{array}\right\}\text{Proof.}$$

Here we link the 6, being the term less than the mean rate, to each of the other prices; placing the difference of the 6 and 7 opposite each of the other prices (viz., 8 and 10). We then place the difference of the 8 and 10 opposite to the 6, because they are linked to it. Then we prove the question by the preceding rule, *i. e.*, we multiply each quantity by its rate and divide the sum of the products by the sum of the quantities, and obtain 7*s*. as the mean rate.

(1) A vintner mixes wine at 10*s*. per gallon with others at 4*s*., 8*s*., and 9*s*., per gallon; what quantity of each must be taken sell the mixture at 10*s*. 6*d*.?

Ans. 3½ at 8*s*., 3½ at 9*s*., 3½ at 10*s*., 4½ at 14*s*.

(2) A grocer mixes sugars at 8*d*., 7*d*., 5*d*., and 9*d*., per lb. equired the quantity of each, that the mixture may be sold for *d*. per lb. *Ans*. 6 at 5*d*., 1 at 7*d*., 1 at 8*d*., 1 at 9*d*.

(3) A maltster has malt at 5*s*. 6*d*., 4*s*. 6*d*., and 8*s*., per bushel; hat quantity of each must be taken that the price of 2 bushels ay be 11*s*. 4*d*.? *Ans*. 29 at 4*s*. 6*d*., 29 at 5*s*. 6*d*., 14 at 8*s*,

(4) What quantity of corn at 62*s*., 80*s*., and 95*s*., per quarter. ust a farmer mix together that the compound may be worth 84*s*. er quarter? *Ans*. 11 at 62*s*., 11 at 80*s*., 26 at 95*s*.

The last two rules are chiefly employed in business, but there re two others occasionally used, called Alligation Partial and lligation Total.

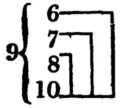
Alligation Partial is when *one* of the quantities and the prices f the others are given to find the rest.

Alligation Total is when the whole quantity wanted is given, he mean rate and the prices of the quantities of which the compound is to consist, to find the quantities at each price.

ALLIGATION PARTIAL.

RULE.—Place, link, and find the differences of the quantities s in the last rule. Then state; as the difference opposite the uantity given is to the difference so is the given quantity to each f the quantities required.

EX.—(1) A farmer has 32 bushels of wheat at 6*s*. per bushel, hich he mixes with other quantities at 7*s*., 8*s*., and 10*s*., per ushel. What quantity of each sort must he take to make a composition worth 9*s*. per bushel? *Ans*. 32 at 6*s*., &c.



3+2+1=6

1	32 at 6 = 192	1 : 1 : 32 : 32
1	32 at 7 = 224	1 : 1 : 32 : 32
1	32 at 8 = 256	1 : 6 : 32 : 192
6	192 at 10 = 1920	
	288)2592(9	
	2592	

(2) A tobacconist having 30 lbs. of tobacco at 2*s*. 4*d*. per lb., ishes to mix it with other sorts at 2*s*. 6*d*., 1*s*. 10*d*., 2*s*. 3*d*., and *s*. per lb. What quantity of each must he take so as to be nabled to sell the mixture at 2*s*. 8*d*. per lb.?

Ans. 30 at 2*s*. 6*d*., 30 at 1*s*. 10*d*., 30 at 2*s*. 3*d*., and 21⅔ at 4*s*.

(3) A distiller has 56 gallons of brandy at 10*s*., which he intends mixing with other sorts at 9*s*., 11*s*., and 14*s*., per gallon. Required the quantity of each that will admit the composition to be sold for 12*s*. 6*d*. per gallon?

Ans. 56 at 10*s*., 56 at 9*s*., 56 at 11*s*., 280 at 14*s*.

(4) A grocer has 43 lbs. of tea worth 10*s*. per lb., which he mixes with others at 8*s*., 9*s*., and 12*s*., per lb.; he wishes to sell the mixture at 10*s*. 6*d*. per lb. How many pounds of each must he take? *Ans*. 43 at 8*s*., 43 at 9*s*., 43 at 10*s*., 129 at 12*s*.

(5) What quantity of coffee at 3*s*. 4*d*., 3*s*. 8*d*., and 4*s*. 2*d*., can be mixed with 10 lbs., at 3*s*. 10*d*., to make a mixture worth 3*s*. 9*d*. per lb.? *Ans*. 1 at 3*s*. 4*d*., 5 at 3*s*. 8*d*., 1 at 4*s*. 2*d*.

ALLIGATION TOTAL.

RULE.—Place, link, and find the difference as before. Then the sum of the differences is to each difference, as the quantity given is to the quantity required of each.

Ex.—A grocer has four sorts of sugar, at 10*d*., 8*d*., 6*d*., and 5*d*., per lb., and wishes to make a composition of 128 lbs. worth 7*d*. per lb.

Proof.

$$7\left\{\begin{array}{l}10\\8\\6\\5\end{array}\right.\quad\begin{array}{c}2\\1\\1\\\underline{3}\\7\end{array}\quad\begin{array}{rcl}36\frac{4}{7}\text{ at }10 &=& 365\frac{5}{7}\\18\frac{2}{7}\text{ at }8 &=& 146\frac{2}{7}\\18\frac{2}{7}\text{ at }6 &=& 109\frac{5}{7}\\54\frac{6}{7}\text{ at }5 &=& \underline{274\frac{2}{7}}\\ && 7\overline{)896}\\ && \underline{128}\end{array}$$

$$7 : 2 :: 128 : 36\tfrac{4}{7}$$
$$7 : 1 :: 128 : 18\tfrac{2}{7}$$
$$7 : 3 :: 128 : 54\tfrac{6}{7}$$

Here we take the differences as before, and having found the sum (7), we state as in the rule, and obtain $36\frac{4}{7}$ at 10*d*., $18\frac{2}{7}$ at 8*d*., &c.; we then prove the question by the 1st Case, or Alligation Medial.

(2) How much rum, at 6*s*., 8*s*., and 15*s*., per gallon, must be mixed together to make a mixture of 38 gallons, at 14*s*. 6*d*. per gallon? *Ans*. $1\frac{3}{16}$ at 6*s*., $1\frac{3}{16}$ at 8*s*., $35\frac{5}{8}$ at 15*s*.

(3) A grocer has 3 sorts of tea, at 6*s*., 8*s*., and 9*s*., per lb., wishes to make a composition of 26 lbs., to be sold at 6*s*. 6*d*. per lb. *Ans*. $36\frac{4}{5}$ at 6*s*., $4\frac{3}{5}$ at 8*s*., $4\frac{3}{5}$ at 9*s*.

(4) What quantity of brandy, at 7*s*., 8*s*., 12*s*., and 14*s*., per gallon, must be mixed to make a compound of 46 gallons worth 10*s*. 6*d*. per gallon? *Ans*. $7\frac{2}{3}$ at 3*s*., $7\frac{2}{3}$ at 4*s*., $30\frac{2}{3}$ at 5*s*.

PART IV.

ARITHMETICAL AND GEOMETRICAL PROGRESSION, ROOTS, AND SURDS.

Arithmetical Progression.

(1) An Arithmetical Progression is a series of numbers in-
asing or decreasing by the addition or subtraction of a certain
mber called the Ratio, or common difference, as 2, 4, 6, 8, &c.,
1 14, 11, 8, 5, &c., where in the first series the ratio is 2, and
the second the ratio is 3.

(2) A Geometrical Progression is a series of numbers in-
asing or decreasing by multiplication or division; the *constant*
mber which causes the increase or decrease being called the
tio, as before.

The first and last terms of the series are called the *extremes*,
1 the terms between them are called the *means*.

The numbers themselves are called the *terms* of the series,
1 the number by which the increase or decrease arises is
ratio termed.

Thus 14, 16, 18, 20, 22, &c., is called an *increasing* series.

The 14, 16, &c., being called the *terms*, and the number pro-
cing the increase, (viz. 2,) is called the ratio.

Again, 20, 15, 10, 5, is called a *decreasing* series, the ratio
ng 5.

In Arithmetical Progression there are five terms, viz., the
nmon difference, the two extremes, the number of terms, and
sum of the series; any three of which being given, the others
y be found by the following formulæ:—

Let $a =$ the least term, $z =$ greatest term, $n =$ number of
ms, $s =$ sum of all the terms, $d =$ difference.

ıen 1. $z = a + d(n-1)$; $\quad d = \frac{z - a}{n-1}$

2. $a = z - d(n-1)$; $\quad n = \frac{z-a}{d} + 1 = \frac{z-a+d}{d}$

3. $s = (a+z)\frac{n}{2}$;

4. $s = \{z - \frac{1}{2}d(n-1)\}n$ $\quad$ or $n = \frac{2s}{a+z}$

5. $s = \{a + \frac{1}{2}d(n-1)\}n$ $\quad$ or $n = \frac{2(z-s)+d}{d}$

EXPLANATION OF FORMULÆ.

1. $z=a+d(n-1)$ } z or the greatest term $=a$ or the first term, added to the product of the ratio and the number of terms less one.

Ex.—If $z=20$, $a=5$, $n=4$, $d=5$,

Then $\left\{\begin{array}{l} z=a+d\,(n-1) \\ z=5+5\,(3)=5+15=20 \end{array}\right.$

2. $a=z-d(n-1)$ } a or the least term $=z$ or the greatest term, minus the product of the ratio and the number of terms less one.

3. $s=\left(a+z\right)\frac{n}{2}$ } $s=$ the sum of the first and last terms multiplied by $\frac{1}{2}$ the number of terms.

It is not deemed necessary to pursue the explanation further. The learner will easily obtain any term by substituting for the values of the other terms.

(1) A man increases his first year's income by 500*l.*, the second by 1000*l.*, the third by 1500*l.*, and so on. He began business with 2700*l.*, and traded for 17 years; what was he worth at the end of that time? *Ans.* 113900*l.*

(2) 220 stones are placed in a straight line exactly 2¼ yds. asunder, and the first 2½ from the basket; how far will a person go whilst picking up the stones, returning with one at a time and putting it in the basket? *Ans.* 31$\frac{42}{176}$ miles.

(3) A snail crawls up a pole, 180 ft. high, 3 feet the first night, 5 feet the second, and so on. What distance will it have gone the tenth night, and how far will it be down the pole on the fifteenth night, supposing it reaches the top and then descends? *Ans.* 70 ft. and 232¼.

(4) A boy flying a kite had let out 108 yds. of string, he then began to let it out by starts; the first one let out 3½ yds., the second 7, the third 10½, and so on. After letting out 50 separate times he came to the end of his string. How many yds. had he? *Ans.* 1908 yds.

(5) A sets out from London, and B from Carlisle, distant 302 miles. A goes 1 mile the first day, 3 the second, 5 the third, and so on; B went 2 miles the first day, 6 the second, and 10 the third. How far will they be apart on the 7th day after starting? *Ans.* 156 miles.

(6) A snail crawls up a maypole 132 ft. high; the first day it goes 2 ft., the second 4 ft., the third 6 ft., and so on. Required the time of its going to the top and descending to 36 ft. from the bottom. *Ans.* 14 . 60794045.

(7) A grasshopper leaps 1 ft. the first leap, 14 inches the second, 16 the third, and so on, for 12 leaps. What was the length of the last leap, and how far did it go in all?

Ans. Whole distance 23 ft.; last leap 34 inches.

GEOMETRICAL PROGRESSION.

FORMULÆ.

(1) $ar^{n-1}=z$

(2) $r^{n-1}=\frac{z}{a}$ or $\frac{\log. z-\log. a}{\log. r}+1=n$

(3) $a=\frac{z}{r^{n-1}}$

(4) $\frac{rz-a}{r-1}=s$ or $z+\frac{z-a}{r-1}=s$

(5) $\left(\frac{z}{a}\right)^{\frac{1}{n-1}}=r$ or $(\log. z-\log. a)\div(n-1)=\log. r$

(6) $\frac{s-a}{s-z}=r$

(7) $\left(\frac{s-a}{s-z}\right)^{n-1}=\frac{z}{a}$ or $\frac{\log. z-\log. a}{\log. (s-a)-\log. (s-z)}+1=n$

(8) $\frac{a\left(r^n-1\right)}{r-1}=s$

(9) $r^n=\frac{(r-1)s}{a}+1$ or $\frac{(\log. (r-1)s+a)-\log. a}{\log. r}=n$

(10) $\frac{sr}{a}-r^n=\frac{s-a}{a}$

(11) $\frac{z\left(r^n-1\right)}{r^n-r^{n-1}}=s$

(12) $rz-(r-1)s=a$

(13) $\frac{r^{n-1}s}{s-z}-r^n=\frac{z}{s-z}$

(14) $\frac{(r-1)s}{r^n-1}=a$

(15) $\dfrac{r^n - r^{n-1}}{r^n - 1} \times s = r$

If the series be an infinite decreasing series, $a = o$ and n is infinite.

(1) $s = \dfrac{rz}{r-1} = z + \dfrac{z}{r-1} = \dfrac{z^2}{z - \frac{z}{r}}$

(2) $\dfrac{s(r-1)}{r} = z$

(3) $r = \dfrac{s}{s-z}$

(1) A gentleman dying left five sons, to whom, and to his executors, he bequeathed his property as follows:—His youngest son was to have 4 times as much as his executors, to whom he bequeathed 60*l.* each; his next son was to have 3 times as much as the youngest, and so on for each son. What did the father die worth? *Ans.* 28920*l.*

(2) Given the extremes 3 and 58, and the number of terms 12, to find the common difference and the sum of the series.
Ans. C D 5, and sum of terms 336.

(3) A number of trees were planted in the form of an isosceles triangle. The number at the vertex was 1; and the second row 5; and so on to the last. There were 20 rows; required the number of all the trees, and the number in the last row.
Ans. Number of trees 780, and last row 77.

(4) A gentleman gave as a wedding portion to his daughter 20*l.*, and doubled it every fourth year. The daughter lived 20 years. What did her husband receive the last year, and all the time she lived? *Ans.* Last year 640*l.* Total 1140*l.*

(5) A blacksmith agreed with a groom to shoe his master's 2 horses at the following rate:—To receive the three-thousandth part of the sum arising from the following agreement: viz., that as each shoe was to have 8 nails, he should have 2 grains of barley for the first, and should double it to the last; that for every quart he should be allowed 16000 grs., and that the barley should be valued at 30*s.* per quarter. Required the number of grains of barley, and the total value of it, and the sum the blacksmith received. *Ans.* Number 8589934590. Price 3145*l.* 4*s.* 6¼*d.*

PERMUTATIONS AND COMBINATIONS.

DEFINITION.

Permutations of any number of quantities are the different ways in which these quantities can be taken; thus in the 5 quantities 5, 4, 3, 2, 1, the various ways in which these numbers can be taken 2 and 2, are as below.

5 (4 . 3 . 2 . 1)
4 (5 . 3 . 2 . 1)
3 (5 . 4 . 2 . 1)
2 (5 . 4 . 3 . 1)
1 (5 . 4 . 3 . 2)

From this arrangement we perceive that 5 can be taken with every one of the numbers; viz., 5 . 4; 5 . 3; 5 . 2; 5 . 1, making 4 changes in this instance. Secondly, that 4 can be taken with every one of the numbers, viz. 4 . 5; 4 . 3; 4 . 2; 4 . 1; making also 4 permutations; and so on of all the other arranged numbers. Hence 4 permutations can be formed by taking each of the quantities first with all the other quantities, and therefore there will be as many sets as there are letters. And as there are 5 quantities, and each begins a set of four permutations, we thence perceive that the number of permutations which can be formed out of 5 numbers, taken 2 at a time, is 20, or $n\ (n-1)$.

The following formulæ will, by mere substitution, always solve all cases that occur in permutations:—

If n be the No. of things to be taken...	2 and 2,	The No. of permutations is .	$n\ (n-1)$
”	3 and 3,	”	$n\ (n-1)\ (n-2)$
”	4 and 4,	”	$n\ (n-1)\ (n-2)\ (n-3)$

” and so on, where it is easy to perceive that there will be as many factors in the formula as the *number* of things to be taken; thus if the number of things be taken 2 and 2, there are 2 factors, viz. n and $(n-1)$. If taken 4 and 4, then there are 4 factors, viz. $n\ (n-1)\ (n-2)\ (n-3)$, and so on. And as we have before stated that if $n =$ the number of things on which we are to perform the permutations, we shall find that in all permutations the first term will be equal to the number of things; the second term will be $n-1$, or one less than the number of things; the third term will be $n-2$, or two less than the number of things, and so on.

The number of permutations that can be formed out of n things taken one at a time is $n\ (n-1)\ (n-2) \ldots . (n-n+1)$, or the terms decreasing from n, to unity, multiplied into themselves.

Ex. (1) How many ways can 4 persons be placed about a table at dinner?

$n\ (n-1)\ (n-2)\ (n-3)$ is the formula.
Hence $4\ .\ 3\ .\ 2\ .\ 1 = 24$ different ways.

(2) From a company of soldiers, consisting of 48 men, are taken 5 at a time to guard a certain station. Required the number of different selections that can be formed. *Ans.* 215476480.

(3) Required the number of selections that can be formed from a pack of cards, 52 in number, if 3 be taken at a time.
Ans. 132600.

(4) How many changes can be rung upon 9 bells?
Ans. 362880.

ON SERIES.

To sum a Geometrical Decreasing Infinite Series.

Rule.—Square the first term, and divide the square by the difference between the first and second.

Ex. (1)—Sum the geometrical decreasing infinite series 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c., &c.

$$\text{Here } \frac{8^2}{8-4} = \frac{64}{4} = 16$$

(2) Required the sum of the decreasing geometrical infinite series, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, &c.

$$\text{Here } \frac{9^2}{9-3} = \frac{81}{6} = 13\tfrac{1}{2}$$

Sum the following geometrical decreasing infinite series:—

(3) 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, &c. ad infinitum $= 12$.
(4) 5, $\frac{5}{3}$, $\frac{5}{9}$, $\frac{5}{27}$, &c. " $= 7\frac{1}{2}$.
(5) 6, 2, $\frac{2}{3}$, $\frac{2}{9}$, &c. " $= 9$
(6) 8, $\frac{8}{3}$, $\frac{8}{9}$, $\frac{8}{27}$, &c. " $= 12$.
(7) 12, 6, 3, $\frac{3}{2}$, &c. " $= 24$.
(8) 14, $\frac{14}{5}$, $\frac{14}{25}$, $\frac{14}{125}$, &c. " $= 17\frac{1}{2}$.
(9) 19, $\frac{19}{8}$, $\frac{19}{64}$, $\frac{19}{512}$, &c. " $= 21\frac{5}{7}$.
(10) 13, $\frac{13}{12}$, $\frac{13}{144}$, $\frac{13}{1728}$, &c. " $= 14\frac{2}{11}$.
(11) 5, $\frac{5}{2}$, $\frac{5}{4}$, $\frac{5}{8}$, &c. " $= 10$.

Series of the following form, where the factors in the denominators increase in arithmetical ratio, and the numerator is 1, may be summed by the following rule:—

RULE.—Take the difference between the first and last factor in the denominator of the first term, then multiply the product of all the remaining factors (omitting the last) by this difference, and it will give the sum of the series; unity being its numerator.

Ex.—Sum the series $\frac{1}{2.3.4}$, + $\frac{1}{3.4.5}$, + $\frac{1}{4.5.6}$ + &c., ad infinitum.

Here 4 is the last factor and $4 - 2 = 2$ the difference; then we multiply the remaining factors 2 and 3, by this difference, and obtain 12; therefore the sum of the series is $\frac{1}{12}$.

(2) Sum $\frac{1}{2.4.6}$, $\frac{1}{4.6.8}$, $\frac{1}{6.8.10}$, &c. *Ans.* $\frac{1}{32}$.

(3) Sum $\frac{1}{5.7.9}$, $\frac{1}{7.9.11}$, $\frac{1}{9.11.13}$, &c. *Ans.* $\frac{1}{140}$.

INVOLUTION.

Involution is the raising of any quantity to any power. The quantity to be raised is called the *root*, and the number resulting from the involution, is called the *power*. Thus—

$5 = 5$ is called the 1st power of 5.
$5 \times 5 = 25$ „ 2nd power of 5, or the square.
$5 \times 5 \times 5 = 125$ „ 3rd power of 5, or the cube.
$5 \times 5 \times 5 \times 5 = 625$ „ 4th power of 5, or biquadrate.
$5 \times 5 \times 5 \times 5 \times 5 = 3125$ „ 5th power of 5.

And so on.

Now, by adopting a process in algebra, we can express these quantities in a much briefer manner, and thereby shorten the lengthy process which the above means (if often repeated) would ntail. Thus, instead of saying 5 × 5, we express our meaning by adding a number, called an *index*, to the right hand side of the root; thus, the latter expression would be represented by $5^2 = 25$.

$5 \times 5 \times 5 = 125$ by $5^3 = 125$
$5 \times 5 \times 5 \times 5 = 625$ by $5^4 = 625$
$5 \times 5 \times 5 \times 5 \times 5 = 3125$ by $5^5 = 3125$

Now this principle is used in fractional quantities as well as in integral, thus—

$(\frac{5}{9}) = (\frac{5}{9})^1$, the first power.
$(\frac{5}{9})^2 = \frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$, the second power.
$(\frac{5}{9})^3 = \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$, the third power.

here it will be observed there are as many multiplications, less 1, s the index denotes; thus in the second power we have *one* multiplication; in the third power 2 multiplications; in the fourth ower 3 multiplications. Also that

$$5\times5\times5=5^{3}$$
$$\text{And } 5\times5\times5\times5=5^{4}$$

Whence multiplying the t on the left hand side of the e tion, we obtain

$$5\times5\times5\times5\times5\times5\times5=5^{7}$$ which equals 5^{3+4}.

Also in Division $5\times5\times5\times5=5^{4}$.
And $5\times5=5^{2}$.

$$\text{Whence } \frac{5\times5\times5\times5}{5\times5}=5\times5=5^{2}$$

$$\text{Also } \frac{5^{4}}{5^{2}}=5^{4-2}=5,$$

(1) Involve 1·5 to the third power. *Ans.* 3·375.
(2) „ 2·31 to the fourth power. *Ans.* 28·47396321
(3) „ 1·3 to the seventh power. *Ans.* 6·2748517.
(4) „ ·4 to the ninth power. *Ans.* ·000252144.
(5) „ ·09 to the third power. *Ans.* ·000729.
(6) „ ·75 to the second power. *Ans.* ·5625.
(7) What is $(48)^{2}$? *Ans.* 2304.
(8) „ 5·1^{3} ? *Ans.* 132·651.
(9) „ ·073^{3} ? *Ans.* ·000389017.
(10) „ $(\frac{2}{5})^{2}$? *Ans.* $\frac{4}{25}$.
(11) „ $(\frac{7}{9})^{5}$? *Ans.* $\frac{16801}{59049}$.
(12) „ $(12)^{6}$? *Ans.* 2985984.
(13) „ $(1\frac{1}{4})^{5}$? *Ans.* $\frac{3125}{1024}$.
(14) „ $(5\frac{1}{6})^{3}$? *Ans.* $\frac{29791}{216}$.
(15) „ 1·03^{9} ? *Ans.* 1·1189999626139244
(16) „ ·009^{16} ? *Ans.* 0000000000000000185
(17) „ 1·09^{16} ? *Ans.* 1·677100110
(18) „ 87^{6} ? *Ans.* 4336251210
(19) „ $(2\frac{1}{9})^{12}$? *Ans.* 2·5198+.
(20) „ $(2\frac{1}{9})^{3}$? *Ans.* $\frac{6859}{729}$.

EVOLUTION

Is the method of finding the root of any number.

The notation used in this process is that by which we re sent the root to be extracted by a fractional number, called index.

$5^{\frac{1}{3}}$ means the cube root of 5.
$5^{\frac{1}{2}}$ means the square root of 5.
$5^{\frac{1}{7}}$ means the seventh root of 5.
$5^{\frac{1}{9}}$ means the ninth root of 5.

The fractional numbers, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{9}$, being called the indices of the root.

Sometimes another method is adopted, thus,

$\sqrt{5}$ means square root of 5.
$\sqrt[3]{5}$ means cube root of 5.
$\sqrt[4]{5}$ means biquadrate root of 5
$\sqrt[5]{5}$ means 5th root of 5, &c.

Extraction of the square root.

Preparatory to our giving a rule for the extraction of the square root, it will be proper to set before the learner the following operation. Thus,

Multiply 54 by 54.

Now, as the learner has already been taught in Simple Addition, that 54 consists of $50+4$: if we put the second 54 underneath the first, altered to the same form, we shall have

$$\left.\begin{matrix}50+4\\50+4\end{matrix}\right\}\text{ when }\times d=\left\{\begin{matrix}4\times 4\\4\times 50\\50\times 4\\50\times 50\end{matrix}\right\}\text{or }4^2+2(4\times 50)+50^2=2916.$$

We thus perceive that the square of any number consisting of 2 figures, is equal to the square of the tens (viz. 50), + the square of the units (viz. 4) + also twice the product of the unit and tens (viz. twice 4×50); we thence infer that the root of the number 2916 can be obtained by the reverse operation.

Ex.—Extract the root of 2916

$$\begin{array}{r}2500\\\hline 416\\\hline\end{array}$$

We know that the greatest number contained in our operation above is 50^2, we therefore subtract 50^2 from our power; we have then 416 remaining; and we have perceived before, that the square of 54 contained 50^2, and twice the product of 50, and *the other part* of the root to be found. Now, if we divide 416 by 100 (that is twice the 50), we shall obtain the quotient 4 (with a remainder 16), which is the *remaining root* sought.

The number 416 is therefore evidently $2(50\times 4), + 4^2$. Hence the rule hereafter.

Before the learner attempt to solve a question, he should learn the following table:—

The square of 1 = 1	The square of 6 = 36
" 2 = 4	" 7 = 49
" 3 = 9	" 8 = 64
" 4 = 16	" 9 = 81
" 5 = 25	&c. &c.

From this table we perceive, that in the square of any number consisting of any of the digits, the product never exceeds 2 figures; hence the reason in the following rule for *pointing* the given number into periods of 2 numbers.

Rule.—Point the figures alternately; commencing at the unit's place, and proceeding to the left hand; but if there be no unit figure, place a 0 for the place it should occupy, and point alternately the numbers to the right hand. Then find the greatest square in the left hand period, placing its root to the right hand of the numbers, a curve (as in division) separating them. 2ndly, Subtract the power of the root found from this period, and bring down the numbers in the next point. 3dly, Double the quotient numbers, and see how often the product is contained in the new dividend, omitting the unit of the dividend in the trial. 4thly, Place the root in the quotient also to the right of the divisor; then multiply the new divisor by the root figure *last* found, and place the product under the new dividend. 5thly, Subtract and bring down the next point; double the quotient numbers as before (or, which is more simple, add the last root figure to the last divisor figure); see how often they are contained in the new divisor; place the root in the quotient, and also on the left of the new divisor. 6thly, Multiply the new divisor by the root figure, subtract the product from the new dividend, and proceed in the same manner for every other root figure to be found.

N. B.—An easy method of proving the work is obtained from the following operation:—

Make a cross—

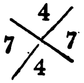
Rule.—Cast out the 9's from the root found, place this number on each side of the cross, multiply these numbers together, and divide by 9; place the remainder on the top of the cross. Cast out the 9's from the bottom remainder, and subtract the remainder, after so doing, from 9; add this last remainder to the power (i. e. the number whose root was to be found), cast out the

9's from the sum, the remainder will be the same as the number at the top of the cross, if the question is correctly solved.

Ex.—Extract the square root of 2775.

2775(52
25
102)275
204
71

Here we first point the numbers, beginning at the 5 (the unit's figure), then the 7. We then look for the greatest power contained in the first point, and find it 25; we therefore put the 25 under the 27, and the root of the 25 (5), in the quotient. We then subtract the 25 from the 27, obtaining 2 as a remainder; we then bring down the numbers in the next point, viz., 75, making for a new dividend 275. We then double the numbers in the quotient (viz. the 5), and obtain 10. We then try *this* into the new dividend, rejecting the unit's figure (the 5), and find that 10 is contained in 27, twice; we therefore place the 2 in the quotient, and also on the right hand of the divisor; we then multiply the 102 by the root figure (2), and obtain 204 for product; which we subtract from 275, and obtain 71 as remainder: having no more numbers to bring down, we infer hat 52 is the root of 2775, with a remainder of 71. To prove t, make a cross, and as 52 is the root found on attempting to ast out the 9's, we perceive that the numbers, when added together, do not exceed 9; but amount only to 7, which we put on each side of the cross. We then multiply hese numbers together, obtaining 49; which, when ÷ by 9, gives a quotient of 5, and a remainder 4, which we put on the top. We then proceed to the emainder 71, and find that no 9 (when the numbers are added ogether), is contained therein; we therefore subtract the 8 or 7+1) from 9, and obtain for remainder 1; which we carry to the um of the digits of the power (viz. 2+7 + 7+5), casting out the 's, and obtain for remainder 4: this number being the same as he one on the top of the cross, we infer the question is right. n this proof it is supposed no digit of the remainder is misplaced, otherwise the question will *prove*, and yet be wrong.

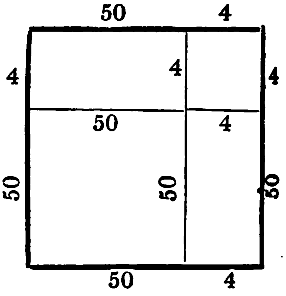


Ex. 2.—Extract the square root of ·00798.

·007980)·089
64
69)1580
1521
59

Here there is no integer; we therefore place a 0 for its place, and point the numbers proceeding to the right hand, we find we are compelled to add a 0 after the 8 (which neither increases nor diminishes the number ·00798) to make an even period. We then seek the greatest power to the first point (viz. the 79), and find it is 64. We put this under the 79, and its root (8) in the quotient. We proceed exactly, as in the last question, finding, at the final operation, 89 for quotient, and 59 for remainder; but, on looking at the periods in the power, we observe that there are 3, we therefore must prefix a 0 to the quotient, because there must be a root figure for each period, and obtain ·089, which is the root to three places of decimals.

N. B.—There will always be as many integers as there are *points over* the integers; and as many decimals as there are *points over* the decimals. This is evident; for every period must produce one figure as a root; therefore, there will always be as many figures in the root as there are periods pointed off—whether integers or decimals, or both. Perhaps the following diagram may render the rule for the square root more easily understood. Here it will be perceived the square, whose sides are respectively = to 54, is divided into portions, viz. 50 and 4 : and it will also be perceived, that the two squares, together with the 2 rectangles, make up the square, whose side is 54 (Euc. 4, 2). The sides of the 2 squares are respectively 50 and 4, which may be represented by 50^2 and 4^2; and the 2 rectangles have their sides 50 and 4 each; and as the 2 rectangles are =, it follows, that the double of one is = to the sum of the two, which double can be represented by 2 (50 × 4). We therefore perceive, by the proposition of Euclid, that the square of any line divided into 2 unequal parts is = to the square of each part, together with twice the rectangle of the 2 parts; and that the side of any square is = to the square root of the contents of that square. This proof is equally applicable to numbers. Hence the rule.



SQUARE ROOT.

Extract the square root of

(1) 7298432 to 8 decimals	*Ans.* 2701·56061868
(2) 629473 to 7 decimals	*Ans.* 793·3933737
(3) 49	*Ans.* 7
(4) 32169874 to 4 decimals	*Ans.* 5671·8490
(5) 41957319 to 4 decimals	*Ans.* 6473·2013
(6) 82913986 to 4 decimals	*Ans.* 9105·7117
(7) 473256 to 3 decimals	*Ans.* 687·946

Extract the square root of

(8) 532298 to 6 decimals *Ans.* 729·597225
(9) 4732194 to 5 decimals *Ans.* 2175·36845
(10) 21986273 to 4 decimals *Ans.* 4688·9522
(11) 895394398 to 4 decimals *Ans.* 29923·1415
(12) 57209869532 to 4 decimals *Ans.* 239185·8334
(13) 482174957391 to 4 decimals *Ans.* 694460·19
(14) 298673429 to 2 decimals *Ans.* 72821·76
(15) 38172965 to 4 decimals *Ans.* 6178·4273
(16) 20986573 to 4 decimals *Ans.* 4581·1104
(17) 49867398 to 4 decimals *Ans.* 7061·6852
(18) 209862714 to 4 decimals *Ans.* 14486·6391
(19) 598632479 to 3 decimals *Ans.* 24466·967

When there are decimals in the proposed power,

Rule.—Point the first whole number (or, if there be not one point *the place* of it); then point left and right. Extract the root as before: remembering that there will be as many whole numbers as there are points over the whole numbers, and as many decimals as there are points over the decimals.

Extract the square root of

(20) 909·4967 *Ans.* 30·1578
(21) ·0098209 *Ans.* ·31338
(22) 278·4963791 *Ans.* 16·6882
(23) 31·78215 to 4 places of decimals *Ans.* 5·6375
(24) 27829·32 *Ans.* 166·8212
(25) 4·9684957 *Ans.* 2·229
(26) 316·82947 *Ans.* 17·7997
(27) ·00000097 to 6 places of decimals *Ans.* ·000984
(28) ·0098 to 4 places ditto *Ans.* ·0989
(29) 2·179845 to 7 places ditto *Ans.* 1·4764298
(30) 6587·209 to 4 places ditto *Ans.* 81·1616
(31) 12·00000737 to 6 places ditto *Ans.* 3·464102
(32) 4·71 to 3 places ditto *Ans.* 2·1700
(33) 79·182 to 3 places ditto *Ans.* 8·898

(34) Required the side of a square whose superficial content is 3984·298 square yards. *Ans.* 63·1212 yards.

(35) A man has 5 fields: 1st contains 46 acres, 3 roods, 5 poles; 2nd, 69 acres, 2 roods, 17 poles; 3d, 109 acres, 1 rood, 7 poles: 4th, 398 acres, 1 rood, 9 poles; 5th, 62 acres, 1 rood, 1 pole. Required the side of a square in yards that shall be equal to the whole of the above fields. *Ans.* 1822·4762 yards.

(36) There is a field consisting of 48 acres, 2 roods, 4 poles. It is required to make it into a square field; required its side in yards. *Ans.* 484·5214.

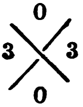
To find the square root of a fraction.

Rule.—If the numerator and denominator be rational quantities, extract the roots; but if surds, reduce the fraction to a decimal, and extract its root, as in the preceding rule.

Ex. 1.—Required the square root of $\frac{2500}{36}=\frac{50}{6}=8\frac{1}{3}$.
Here the numerator and denominator being rational, we take the root of each, and obtain $\frac{50}{6}$, which gives $8\frac{1}{3}$.

Ex. 2.—Required the square root of $\frac{24}{7}$.

```
24/7=3·42857142(1.8516
     1
28  |242
 8  |224
365 |1885
  5 |1825
3701 |6071
   1 |3701
37026|237042
     |222156
       14886
```



Here, perceiving that the fraction is a surd, we reduce it to a decimal, and extract its root; which we find to be 1·8516.

Find the Square Roots of the following Fractions.

(1) $\frac{5}{7}=$ ·845154
(2) $\frac{3}{5}=$ ·7745968
(3) $\frac{1}{121}=\frac{1}{11}$
(4) $\frac{2}{121}=$ ·1286648
(5) $\frac{36}{7}=$ 2·267766
(6) $\frac{7}{35}=$ ·7071068
(7) $\frac{6}{49}=$ ·349927
(8) $\frac{253}{267}=$
(9) $\frac{48}{137}=$ ·592052
(10) $\frac{217}{538}=$ ·6350948

To find a Mean Proportional between Two Numbers.

The square root of the product of the numbers is the mean sought.

(1) Required a mean proportional between 5 and 14.
Ans. 8·00374.

	Between	*Ans.*		Between
(2)	7 and 10	=8·366	(8)	12 and 13=12·4899960
(3)	9 and 12	=10·3923048	(9)	9 and 16=12·
(4)	8 and 16	=11·3137085	(10)	7 and 12= 9·1651514
(5)	7 and 15	=10·2469508	(11)	8 and 19=12·3288280
(6)	3 and 14	= 6·4807407	(12)	4 and 7= 5·2195026
(7)	8 and 3	= 4·8989795		

Given any two sides of a ⊥ ∠ *d* △ to find the other.

CASE 1.

When the Base and Perpendicular are given, to find the Hypothenuse.

RULE.—The square root of the sum of the squares of the base and perpendicular is the length of the hypothenuse.

Ex.—(1) The base of a $\perp \angle d \triangle$ is 12, and the perpendicular 4. Required the hypothenuse. *Ans.* 12·649.

(2) The base is 10, the perpendicular 11. Required the hypothenuse. *Ans.* 14·866.

(3) The base of a $\triangle = 10$ ft., and the perpendicular $= 7·5$ ft. Required the hypothenuse. *Ans.* 12·5.

CASE 2.

When the Perpendicular and Hypothenuse are given, to find the Base.

RULE.—The square root of the difference of the squares of the hypothenuse and perpendicular is the length of the base.

Ex.—1. The hypothenuse is 50, and the perpendicular 40. Required the base. *Ans.* 30.

(2) The hypothenuse is 15 and the perpendicular 9. Required the base. *Ans.* 12.

(3) Given the hypothenuse $= 50$ feet, and the perpendicular $= 40$. Required the base. *Ans.* 30.

CASE 3.

When the Hypothenuse and Base are given, to find the Perpendicular.

RULE.—The square root of the difference of the squares of the hypothenuse and base is the perpendicular.

Ex.—(1) The hypothenuse is 25 and the base 7. Required he perpendicular. *Ans.* 24.

(2) The hypothenuse is 61, the base 60. Required the perendicular. *Ans.* 11.

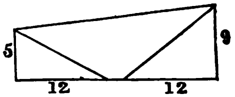
(3) A man has a ladder 36 ft. long, which, when put on the utside of a ditch 20 ft. wide, exactly reaches the top of the wall. Required the height of the wall. *Ans.* 29·933.

(4) A ladder 40 ft. long is placed against a wall 14 ft. high, nd just reaches the top; it is then turned over and touches the op of another wall 26 ft. high. Required the breadth of the treet. *Ans.* 30·3 yds.

(5) A castle has a wall of a certain height, and a ditch which

increases in depth towards the wall 5 ft. for every 6 yds. A ladder, 47 ft. high, is placed 9 yds. from the bottom of the wall, and exactly reaches the bottom of the parapet. Required the height of parapet from the ditch. *Ans.* 53·63 yds.

(6) A man whilst dancing on a tight rope 9 ft. from the ground on one side, and 5 on the other, the rope broke; each end touched the ground 12 ft. from the bottom of the pole to which it was attached. Required the breadth between the poles and the length of the rope, supposing the rope and pieces in each case to have been right lines. *Ans.* 28 ft. length, 27·712 dist.



(7) Given the base of a △ = 10 feet, and the perpendicular = 7·5. Required the area of another which shall contain 3 times as much. *Ans.* 112·5.

CUBE ROOT.

The extraction of the Cube Root of any number is the finding such a number, which, when multiplied twice into itself, shall produce the number whose root is to be found.

It is easily found, by Multiplication, that the cube of any of the digits is as follows:—

Digits 1	2	3	4	5	6	7	8	9	also 10	100
Cubes 1	8	27	64	125	216	343	512	729	1000	1000000

It will be perceived, on inspection, that the numbers called digits are the roots of the numbers called cubes, and that the cube of any one of the digits does not exceed three figures, and that the cube of 10 consists of four figures. Hence it follows that the cube root of every number between 1 and 1000 consists of one figure only, between 1000 and 1000000 of two figures, and so on. If we therefore point the given number, whose root is to be found, into periods of three figures each, the number of points will determine the number of the figures in the cube root. We therefore infer that there will be as many whole numbers as there are points *over* the whole numbers, and as many decimals as there are points *over* the decimals; agreeing with the remark made on the square root, page 224.

If we take a cube whose side is 27 inches, the solid content of the cube is 19683; therefore, as the cube of 27 is 19683, the root of 19683 is 27. And if 19683 be pointed, as directed in the preceding paragraph, we shall perceive, agreeably to what has already been remarked, that the root must consist of two numbers. For 19683 when pointed, as directed, will be $\dot{1}968\dot{3}$. We perceive that it is divided into two periods, 19000 and 683. We easily perceive, by comparing the second period, that its root must be 20, because the cube of 20 is 8000. If we were to take it at 30 we shall find its cube is 27000, this is greater than 19000: 30, therefore, cannot be its root. We therefore see that the cube root of 19683 is between 20 and 30. Having obtained the first figure of our root, viz., 2, let us represent our unknown, or second figure by x; our root will then be $(20+x)$. Now if this be cubed we shall obtain the power required, which must $= 19683$.

$(y+x)^3 = y^3+3y^2x+3yx^2+x^3$, and if we substitute for our quantity y, its equal, viz. 20, we shall obtain $3\cdot20^2\cdot x$ for our 2nd term, and $3\cdot20x^2$ for our 3rd, and x^3 for the fourth.

Or by substituting for the value y, its value 20.

$$\begin{array}{llll}
20+x & & & \\
20+x & & & \\
\hline
400+ & 40x+x^2 & & \\
20+ & x & & \\
\hline
8000+ & 800x+20x^2 & & \\
 & 400x+40x^2+x^3 & & \\
\hline
8000+ & 1200x+60x^2+x^3 & &
\end{array}$$

Now we have found the cube of our first number to be 8000; if we therefore take away 8000 from 19683, we obtain a remainder of 11683, which must equal $1200x+60x^2+x^3$. Now the coefficient of x is 1200, and if this be tried into 11683 we shall find that it will go 17 times; but this we immediately perceive cannot be the other part of the root, because we have omitted $(60x^2+x^3)$ in our divisor, and because we know it cannot exceed 9. By a process after given, we shall find that the root is 7.

In using our trial divisor 1200, we find that it consists of $20^2\times3$; hence the method of finding a trial divisor:—"Square your quotient, and multiply this square by 3 for a trial divisor:"

Our work, therefore, will stand thus:

$$\begin{array}{lr}
(y+x)^3 = y^3+3y^2x+3yx^2+x^3 \text{ and this} = \text{the No. } & \dot{1}968\dot{3}(20+7 \\
\text{Also, } (y+x)^3 = 8000+3\cdot20^2x & 8000 \\
\qquad +3.20x^2+x^3 \qquad 3.20^2.=1200) & 11682 \\
\therefore \text{ if we subtract } 3y^2x = 3.20^2.7 \text{ ———————} & 8400 \\
 & 3283 \\
3yx^2 = 3.20.7^2 \text{ ———————} & 2940 \\
 & 343 \\
x^3 = 7^3 \text{ ———————} & 343
\end{array}$$

Because there is no remainder, ∴ 27 is the root required.

From the above explanation the following Rule arises:—

1st. Point every third figure of the number whose root is to be extracted, beginning at the UNITS' place.

2nd. Find the greatest cube of the left-hand period, set the root in the quotient, and the power under the left-hand period, subtract it, and bring down the next period.

3rd. Square the quotient and multiply it by 300 for a trial divisor. See how often it is contained in the dividend, and place the root figure in the quotient.

4th. Multiply the trial divisor by the last root figure found, and multiply the figures in the quotient by 30, and this product by the last figure of the quotient squared, and place this product under the preceding one; then the cube of the last quotient figure added to the sum of the preceding products will give the *subtrahend*.

5th. Subtract the *subtrahend* from the dividend and bring down the next period, and proceed as at first.

Ex.—Required the cube root of 12167.

$$\begin{array}{rl}
 & 1\dot{2}16\dot{7}(23 \\
 & 8 \\
2^2 \times 300 = 1200) & \overline{4167} \text{ Dividend.} \\
 & 3600 = 1200 \times 3 \\
 & 540 = 2 \times 30 \times 3^2 \\
 & \underline{27 = 3^3} \\
 & 4167 \text{ Subtrahend.}
\end{array}$$

Here we first point as above directed, and find the nearest cube to the first point (12), which is 8, we put this 8 under the first point, and its root, (viz. 2,) in the quotient; we then subtract, and obtain 4, we then bring down the numbers in the next point, viz. 167; we then square the quotient (viz. 2), and multiply it by 300; we then try it into the dividend, viz. 4167, and find it goes 3; we put the 3 in the quotient. We then multiply the divisor by the 3, underneath which we place the quotient before found, multiplied by 30 and by the square of the last figure, viz., 3^2. Finally, we cube the last figure, viz., 3, and add all these products together; this sum we subtract from the dividend, and obtain 0 for remainder, which shows that the root is extracted, viz., 23 is the cube root of 12167.

EXAMPLES FOR PRACTICE.

(1) Find the cube roots of 985, 469, 930, and 835.

Ans. 9·949748, 8·148276, 9·761, 9·41663.

(2) Determine the cube roots of 5, 9, 11, 18, and 25.

Ans. 1·709976, 2·080084, 2·2398, 2·620741, and 2·924018.

(3) What are the cube roots of 12167, 300763, 205379, 511368, and 942673? *Ans.* 23, 67, 59, 82, and 97.

(4) Determine the cube roots of 5·088448, 22188·41, 00102743729, and ·000000366048.

Ans. 1·72, 28·1, ·1009, and ·0072.

(5) Required the approximate cube roots of 32·65, 1005, 96, 0079, 3·0092, 2315·68.

Ans. 3·19615, 10·01982, ·199163, and 1·43368.

(6) Find the approximate cube roots of $\frac{23}{729}$, $\frac{48}{155}$, $\frac{79}{243}$, $\frac{625}{10048}$ n decimals. *Ans.* ·292637, ·676555, ·687606, ·396256.

(7) There is a cube whose side is 10·5 inches; required the ide of another cube to be cut out of this that shall equal the $\frac{1}{10}$ part of the solid contents. *Ans.* 4·87366.

(8) Required the side of 5 equal cubes to be formed out of nother cube whose side is 8·5. *Ans.* 4·97083.

(9) A man has 5 equal cubes whose sides are 7·2 inches each, e wishes to form them into 3 equal ones. Required the sides of hese latter. *Ans.* 9·95287 inches each side.

(19) A person has 4 cubes; the side of the first is 3 inches, of he second 4 inches, of the third 5·5 inches, of the fourth 6 inches, f the fifth 8 inches. Required the side of a cube which shall contain all these. *Ans.* 9·24189 inches.

(11) Required the side of a cube whose solid contents are 256·82 inches. *Ans.* 6·35637 inches.

(12) A man abstracts $\frac{1}{5}$ of the solid contents of a cube whose ide is 3·5 inches. Required the sides of 2 equal cubes found out f the remainder. *Ans.* 2·57882 inches.

(13) Required the approximated side of the cube whose conents are 207·325 inches. *Ans.* 5·91886 inches.

(14) Required the cube root of 52·734375, 857·375, 561·515625, 498·677257, 92100·460096, 8309·286729.

Ans. 3·75, 9.5, 8·25, 7·93, 45·16, 20·09.

(15) Required the cube roots of $34\frac{21}{64}$, $\frac{1331}{6859}$, $\frac{125}{29791}$, $\frac{2197}{4913}$.

Ans. $\frac{4}{13}$, $\frac{11}{19}$, $\frac{5}{31}$, $\frac{13}{17}$.

(16) Find the cube roots of 439842·970125, 10·720765125, 34·248125. *Ans.* 76·05, 2·205, 3·25.

(17) A person wishes to send 500 oranges in a box, each range being 3 inches in diameter. Required the size of the box, upposing it to be a cube, and the oranges to be placed vertically ver each other. *Ans.* 23·8111 inches.

(18) A carpenter had to make a box $2\frac{1}{2}$ times larger than nother, whose sides were 34 inches long, 16 wide, and 9 deep. The employer stated that the length was to be 55 inches, and the readth 18; required the depth. *Ans.* 12·36363.

(19) A brewer has one large cooler, whose dimensions are 14 ft. long, 18 wide, and $1\frac{1}{2}$ deep; now he wishes to have another

vat made to fit in a niche which is 52 ft. long and 12 broad, which shall contain just treble the quantity of the old one. Required the depth of the new one. *Ans.* 5·71153+528 rem.

(20) A person has a cube of clay whose sides are 973 ft. long; he wishes to take out of the same, 5 cubes whose sides are 45 ft., 62 ft., 30 ft., 80 ft., and 20 ft. He requires to know the length of the side of the cube that can be formed out of the remaining clay. *Ans.* 972·699.

SURDS, OR IRRATIONAL QUANTITIES.

If the quantity whose root is to be extracted be a *prime*, it is evident that its root cannot be found in *finite* terms; an approximation however can always be effected.

In the expression $\sqrt{5}$ or $5^{\frac{1}{2}}$, which is called a *Quadratic Surd*, no number can be found, which when multiplied by itself will equal 5. The quantity is therefore said to be *incommensurable*.

In the expression $2^{\frac{3}{5}}$, the index $\frac{3}{5}$ means that the 2 is to be raised to the third power or cubed, and is then to have its fifth root extracted. And generally, in a fractional index, the numerator shows the power to which the quantity is to be raised, and the denominator shows the root to be extracted, after this operation has been performed.

EXAMPLES FOR PRACTICE.

(1) Determine the exact value of $\sqrt{3+2\sqrt{7}} - \sqrt{12-3\sqrt{7}}$. *Ans.* ·42364.

(2) What is the square of $2\sqrt{5}$, $5\sqrt{8}$, and the cube of $\sqrt[3]{7}\,.\,\sqrt{11}$? *Ans.* 20, 200, 255·3796.

(3) Find the exact value of $\sqrt{\sqrt[3]{72}-2} + \sqrt{\sqrt[3]{16}+3}$. *Ans.* 3·81814.

(4) Which is greater, $12\sqrt{7}$ or $7\sqrt{12}$; also $5\sqrt[3]{14}$ or $4\sqrt{9}$? *Ans.* $12\sqrt{7}$, $4\sqrt{9}$.

(5) Find the value of the expression $\sqrt[3]{22} \times \sqrt{\frac{7}{9}} \times 8\sqrt{3}$. *Ans.* 23·0837.

(6) Add $\sqrt{48}$ to $\sqrt{243}$, $\sqrt{180}$ to $\sqrt{500}$, $\sqrt{\frac{7}{32}}$ to $3\sqrt{4}$. *Ans.* $13\sqrt{3}$, or 18·7492448; $16\sqrt{5}$, or 35·77696; $\frac{25}{8}\sqrt{14}$, or 11·66765.

(7) Find the differences of $5\sqrt{98}$ and $3\sqrt{8}$, $20\sqrt[3]{81}$ and $6\sqrt[3]{375}$, $16\sqrt{\frac{5}{27}}$ and $8\sqrt{\frac{3}{5}}$. *Ans.* $29\sqrt{2}$, or 41·01209; $30\sqrt[3]{3}$, or 43·267488; $\frac{8}{45}\sqrt{15}$, or ·688533.

(8) Multiply $3\sqrt{42}$ by $4\sqrt{7}$, $2\sqrt{8}$ by $3\sqrt{5}$, and $6\sqrt[3]{9}$ by $4\sqrt[3]{3}$. *Ans.* 205·757136, 37·94733, 72.

(9) Divide $24\sqrt{32}$ by $3\sqrt{8}$, $15\sqrt{27}$ by $5\sqrt{3}$, and $29\sqrt{\frac{1}{2}}$ by $3\sqrt{\frac{1}{8}}$. *Ans.* 16, 9, $\frac{58}{3}$, $\frac{87}{4}$.

(10) What are the sum and difference of $2\sqrt{12}$ and $16\sqrt{7}$? *Ans.* Sum = 23·4484623, difference = 9·5920559.

(11)*Find the approximate values of $9 \times \sqrt[3]{\frac{3}{73}}$ and $\sqrt[3]{14} \times \sqrt{63} - 3)$. *Ans.* 3·10581; 11·89947.

The following is Dr. Hutton's method of extracting roots :—

Let P be the given power or number, n the index of the power, A the assumed power, r its root, R the required root of P. Then say,

As the sum of $n + 1$ times A and $n - 1$ times P
Is to the sum of $n + 1$ times P and $n - 1$ times A,
So is the assumed root r to the required R.

Or, as the said sum of $n + 1$ times A and $n - 1$ times P is to he difference between the given and assumed power, so is double he assumed root r to the difference between the true and assumed oots; which difference added to or subtracted, as the case reuires, gives the true root nearly.

That is, $(n + 1)\,A + (n - 1)\,P : (n + 1)\,P + (n - 1)\,A :: r : R$
Or, $(n + 1)\,A + (n - 1)\,P : P \backsim A :: 2r : R \backsim r$.

And the operation may be repeated as often as considered ecessary, to obtain the prescribed degree of accuracy, by using lways the last-found root for the assumed root, and its nth power or the assumed power A.

To extract the 5th root of 21053·8.

Here it appears that the 5th root is between 7·3 and 7·4, aking 7·3, its 5th power is 20730·71593. Hence we have P = 1035·8, $n = 5$, $r = 7·3$, and A = 20730·71593; then $n + 1 \bullet \frac{1}{2}$ A + $(n - 1)\,\frac{1}{2}\,P : P \backsim A :: r : R \backsim r$; that is,

$\times\, 20730·71593 + 2 \times 21053·8 \;:\; 305·084 :: 7·3 : ·0213605$

$$\begin{array}{r} 20730·71593 \\ 3 \\ \hline 62192·14779 \\ 42071·6 \\ \hline 104263·74779 \end{array} \qquad \begin{array}{r} 21053·8 \\ 2 \\ \hline 42071·6 \end{array} \qquad \begin{array}{r} 305·084 \\ 73 \\ \hline 915252 \\ 2135588 \\ \hline 2237·1132 \end{array}$$

$$\begin{array}{ll} ·0213605 & = R \backsim r \\ 7·3 & = r\text{, add} \\ \hline 7·321360 & = R \text{ true to the last figure.} \end{array}$$

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* In questions of this nature, logarithms will much abridge the labour. Barlow's Tables will materially assist in the solutions of these questions.

The following method is taken from the same work, page 71, for the Square Root:—

Let N denote the given number, and $\frac{n}{d}$ the fraction to which its square root is approximately equal; then if N be a near integer, or fractional value of the root, the fraction $\frac{n^2+N}{2n}$ will denote one still nearer.

Or for continuing the process the following is still more convenient:—

Let $\frac{n}{d}$ denote the last approximation; then the next is $\frac{2n^2-1}{2dn}$; or if intended for final conversion into the decimal scale, it is $\frac{n}{d}-\frac{1}{2dn}$.

Ex.—Extract the square root of 920.

Here $N=920$, and $n=30$ nearly. Hence by first formula $N\ 920=\frac{n^2+N}{2n}=\frac{1820}{60}=\frac{91}{3}$. Then from the second formula, putting $\frac{n}{d}=\frac{91}{3}$, we have $\frac{2n^2-1}{2dn}=\frac{2 \cdot 91^2-1}{2 \cdot 91 \cdot 3}=\frac{16561}{546}=$ 30·33150183, which differs from the truth only by 6 in the 10th place of figures, its true value being 30·33150177.—HUTTON'S *Tracts*, vol. i., pp. 457—459.

The most useful method of extracting roots is by a table of logarithms. This method generally affords accuracy to five or six numbers, which is sufficient for nearly all practical purposes.

Let us take the first question above:

Extract the cube root of 773.

$$\begin{array}{rl} \text{Here Log. } 773 & = 3\overline{)2{\cdot}8881795} \\ \text{No. } 9{\cdot}17754 & = \ \ {\cdot}9627265 \end{array}$$

No one acquainted with the use of logarithms would ever extract a root by any other method. Examples in this method will be found further on.

PART V.

ARITHMETIC OF PHILOSOPHICAL QUESTIONS.

On Sound.

Sound travels at the rate of 1142 feet per second. Therefore the number of seconds elapsed between the origin and arrival the sound be multiplied by 1142 feet, it will give the number feet between the places.

Ex.—(1) A cannon was fired, and after the flash was seen, 7 conds intervened before the sound was heard. Required the stance between the observer and gun.

Ans. 1 mile, 4 fur., 24 yds., 2 ft.

(2) A powder magazine blew up, the report was heard $4\frac{1}{2}$ conds after the light was visible. Required the distance.

Ans. 7 furlongs, 170 yards.

(3) The flash of a gun was seen, and 5 seconds after, the port was heard. Required the distance.

Ans. 1 mile, 0 fur., 143 yds., 1 ft.

(4) Supposing that in an engagement between two ships of ur the flash of the first gun fired was seen from the shore, and seconds after the report was heard, a boat's crew set off to the sistance of the nearest ship. Required, how long after the first n was fired they would be before reaching the ship, supposing ey pulled at the rate of 4 miles per hour.

Ans. 2 miles 6 fur. distance; 41 min. $23\frac{1}{4}$ sec.

(5) A meteor was seen, and a report heard $17\frac{3}{4}$ seconds after plosion. Required the distance from the observer.

Ans. 3 miles, 6 fur., 156 yds., $2\frac{1}{2}$ ft.

(6) Lightning from a cloud was observed $22\frac{1}{2}$ seconds before e thunder was heard. Required the distance of the cloud from e observer. *Ans.* $4\frac{305}{1056}$ miles.

(7) Five signal guns are fired, and the flash seen from each n and the report heard at the next station, after the following nes:—From A to B 17 seconds after firing; from B to C $19\frac{1}{2}$ conds; from C to D $22\frac{1}{4}$ seconds; from D to E $14\frac{1}{2}$ seconds; d from E to F $21\frac{1}{4}$ seconds. Required the distance of the stan of A and F. *Ans.* $20\frac{29}{66}$ miles.

On Falling Bodies.

To find the space fallen through.

Rule.—Multiply the square of the time by $16\frac{1}{12}$ feet; the oduct is the space.

Ex.—(1) A ball is observed to be 18 seconds in falling. Required the space passed. *Ans.* 5211 ft.

(2) A man dropped a ball from an eminence; it occupied $4\frac{3}{4}$ seconds before it touched the earth. How high was he? *Ans.* $362\frac{169}{192}$.

(3) A boy dropped a stone down a well; it occupied 6 seconds in reaching the bottom. How deep was the well? *Ans.* 579 ft.

To find the Velocity acquired by a Falling Body.

Rule.—1. Multiply the space in feet by $64\frac{1}{3}$ and extract the square root; the root is the velocity required.

(1) A ball has passed through 500 ft. Required its velocity. *Ans.* 179·35 ft.

(2) A ball was dropped from a steeple and reached the ground, distant 230 ft. Required its velocity on touching the ground. *Ans.* 121·64 ft.

Rule.—2. If the velocity of water be required, $\frac{1}{20}$ of the preceding root must be subtracted for the answer.

Ex.—If the height of a waterfall be 300 ft., what is the velocity of the water? *Ans.* 138·91.

To find the Velocity in a Given Time.

Rule.—Multiply the given time (turned into seconds) by $32\frac{1}{6}$, this gives the velocity per second.

Ex.—(1) Required the velocity in 12 seconds. *Ans.* 386.

(2) Required the velocity in $13\frac{1}{2}$ seconds. *Ans.* 434·25.

(3) Required the velocity in 9 seconds. *Ans.* $289\frac{1}{2}$.

(4) Required the velocity in 15 seconds. *Ans.* $482\frac{1}{2}$.

On Light, Heat, and Attraction.

The proportion of light, heat, or attraction of bodies, is as the square of the distance.

Thus, if a man is 3 feet from a fire, and another is 6 feet from it, the former receives 4 times as much heat as the latter: for $6^2 = 36$ and $3^2 = 9$, $\therefore \frac{36}{9} = 4$.

Also, if a book be held 2 feet from a candle, and another be held 3 feet, the former receives $2\frac{1}{4}$ times the light that the latter does: for $3^2 = 9$ and $2^2 = 4$, $\therefore \frac{9}{4} = 2\frac{1}{4}$.

Questions.

(1) One person is 7 feet from a fire, and another 4. What proportion of heat do they each receive? *Ans.* 3·0625 to 1.

(2) There are 2 fires 30 feet apart, the one, A, emitting double the heat of the other, B. Two persons, C and D, stand

etween them; C 16 feet from A, and D 9 feet from B. What roportion of heat will each receive from the 2 fires?

Ans. $2\frac{185}{286}$ and B $3\frac{34}{81}$.

(3) The earth's distance from the sun is 95000000 miles, its quare 9025000000000000, the distance of Venus from the sun 6879152 miles; the square of which is 47323051432295041. equired the ratio of the light the planet Venus receives to what he earth receives from the sun. *Ans.* ·1·907 to 2.

(4) Given the arth's distance as efore, and the fol- owing distances.

Mars 145000000 miles. Required the ratio of light and heat. *Ans.* $2\frac{1}{8}$ to 1.

Jupiter 494000000 miles. Ditto.

Ans. $\frac{1}{27}$ part of what the earth receives.

Saturn 907000000 miles. Ditto.

Ans. $\frac{1}{100}$ part of ditto.

EFFECTS.

RULE.—Place the reciprocals of all the terms one after another, ith the sign + between, as in Addition of Fractions; then reduce hem to a common denominator; add all the numerators together, nd divide this sum into the common denominator; the quotient the answer.

$$\frac{1}{14}+\frac{1}{15}+\frac{1}{17}=\frac{255}{3570}+\frac{210}{3570}+\frac{238}{3570}=\frac{703}{3570}.$$

$$\therefore 3570 \div 703 = 5\frac{55}{703} \text{ days.}$$

(1) A and B can do a piece of work in 14 days; B and C, in 5 days; and A and C, in 17 days. In what time will they do t working together? *Ans.* $5\frac{55}{703}$ days.

(2) A and B can dig a trench in 9 days; B and C, in 4 ays; and A and C, in 5 days. In what time will $\frac{5}{8}$ of the work e finished? *Ans.* $1\frac{71}{101}$ days.

(3) A can do a piece of work in 28 days, and B, in 33 days. n what time could they do it together? *Ans.* $15\frac{9}{61}$ days.

(4) C can do half as much work as B, and C can weave 48 ards of cloth in 67 days. In what time would B do 7 times the alf of C's work? *Ans.* $117\frac{1}{4}$ days.

(5) C produces a certain effect in $12\frac{1}{2}$ days, and B the same effect in $16\frac{1}{4}$ days. In what time will they finish $\frac{3}{4}$ of the effect, working together? *Ans.* $7\frac{3}{46}$ days.

(6) B can finish a piece of work in $8\frac{1}{2}$ days, C can do it in 3 days. How much of $\frac{1}{8}$ of the work could they do in 1 day?

Ans. $\frac{23}{51}$.

(7) A can produce a certain effect in 17 days, B can do the same in 13 days, and C can do the same in 19 days. In what time can they do it altogether? *Ans.* $5\frac{244}{791}$ days.

(8) A can build a boat in 16 days, B can build it in 19 days, and C in 28 days. In what time could they, all working together, build it? *Ans.* $6\frac{202}{321}$ days.

(9) B can build a wall in $7\frac{1}{2}$ days, and C can do it in $\frac{3}{4}$ of that time. How long will they both be in building it?

Ans. $3\frac{3}{14}$ days.

(10) A can make 57 pairs of shoes in $2\frac{1}{2}$ months, and B can make 47 pairs in $3\frac{1}{2}$ months. In what time would they both be (working together) making the 104 pairs? *Ans.* $2\frac{276}{317}$ months.

(11) A body (B) of men can pull down a village in $27\frac{3}{4}$ days? another body (b) can pull it down in 68 days. In what time can they both pull it down? *Ans.* $19\frac{271}{383}$ days.

(12) A can do a piece of work in 12 days; B, in 16 days; C, in 24 days; D, in 32 days; and E, in 40 days. In what time could they do the same piece of work supposing they all worked together? *Ans.* $4\frac{12}{117}$ days.

SIMPLE INTEREST BY DECIMALS.

Rule.—Reduce the shillings, pence, and farthings (if any), to the decimal of a £., by Rule, page 142; then multiply the principal by the interest of 1*l.* for one year, and by the time given; the product is the answer.

Case 1.

The method of finding the interest of 1*l.* for one year, at any rate per cent., is as follows:—

Ex. 1.—To find the interest on 1*l.* for one year, at 5 per cent.

	£. principal.		£. interest.		£. principal.		£. interest.
Here	100	:	5	:	1	:	·05

That is, if 100*l.* principal gains 5*l.* interest, what will 1*l.* principal gain? *Ans.* ·05 of a £.

Ex. 2.—Find the simple interest on 829*l.* 14*s.* 6*d.* for 1 year, at 4 per cent. Here

829*l.* 14*s.* 6*d.*=829·725

·04= { interest of 1*l.* for 1 year, at 4 per cent.

33·18900=33*l.* 3*s.* $9\frac{1}{2}$*d.*

Case 2.

Ex.—Find the simple interest on 215*l.* 16*s.* 8*d.* for 9 months, at $4\frac{1}{2}$ per cent. Here

215*l*. 16*s*. 8*d*.=215·8333
·045={interest on 1*l*. for 1 year, at 4½ per ct.
10791665
8633332
9·7124985
·75=decimal of 9 months.
485624925
679874895
7·284373875=7*l*. 5*s*. 8¼*d*.

Ex.—Find the simple interest on 429*l*. 16*s*. 7*d*. for 4½ years, at 6½ per cent.

Here 429*l*. 16*s*. 7*d*. = $429{\cdot}8291\dot{6}$
·065
214914580
257897496
27·93889540
4·5
13969447700
11175558160
125·725029300 = 125*l*. 14*s*. 6*d*.

This question might have been shortened by using only three decimals in each multiplicand.

Simple Interest.

Required the simple interest on the following sums:—

	£	*s*.	*d*.		£	*s*.	*d*.
(1)	4864	15	6	at 5 per cent.	243	4	9¼
(2)	3826	12	4	at 4 per cent.	153	11	3½
(3)	2197	7	8	at 5 per cent.	109	17	0¼
(4)	6832	15	9	at 6 per cent.	409	9	4
(5)	8329	19	10	at 10 per cent.	832	19	10¾
(6)	3824	13	9	at 10 per cent., for 3 years	1147	8	1½
(7)	6739	12	6	at 12½ per cent., for 8 years	6739	12	6
(8)	9654	5	6	at 4 per cent., for 8½ years	3282	9	0½
(9)	4832	9	7½	at 6 per cent., for 8 years	2319	11	9½
(10)	3065	18	6¾	at 7 per cent., for 5½ years	1180	7	7¾
(11)	6528	13	8½	at 6 per cent., for 6¾ years	2644	2	3¾
(12)	2547	16	9	at 5 per cent., for 1 year	127	7	10
(13)	2417	16	8	at 3 per cent., for 1 year	72	10	8¼

	£	s.	d.		£	s.	d.
(14)	9659	12	7	at 2½ per cent., for 1 year	241	9	9¾
(15)	1396	14	8½	at 6 per cent., for 1 year	83	16	0¾
(16)	4169	13	7	at 5 per cent., for 1 year	208	9	8
(17)	6734	18	9	at 7 per cent., for 1 year	471	8	10¾
(19)	2653	17	10¼	at 3½ per cent., for 1 year	92	17	8½

COMPOUND INTEREST BY DECIMALS.

Let p=principal.
A=amount.
a=amount of 1*l.* for 1 year.
r=interest of 1*l.* for 1 year.
t=time.

Then it is manifest, that if 1*l.* gain r (interest), that the amount for 1 year will be $1+r=a$.

By this rule we can compute certain amounts for various times.

As £1 : $1+r$ :: p : $p(1+r)$ 1st year's amount
Or £1 : a :: p : pa
1 : a :: pa : pa^2 2nd do. do.
1 : a :: pa^2 : pa^3 3rd do. do.
1 : a :: pa^3 : pa^4 4th do. do. and so on.

Therefore generally $pa^t = A$: t representing the time.

From which we obtain

$$p=\frac{A}{a^t}=\text{log.}\,A-\overline{\text{log.}\,a\times t}=\text{principal.}$$

$$a=\sqrt[t]{\frac{A}{p}}=\text{log.}\,A-\text{log.}\,p\div t.$$

$$t=\frac{\text{log.}\,A-\text{log.}\,p}{\text{log.}\,a}=\text{the time.}$$

If it be required to find the time in which any principal will double itself at any rate per cent. If we make $A=2\,p$ in last formula, we shall obtain

$$t=\frac{\text{log.}\,A-\text{log.}\,p}{\text{log.}\,a}=\frac{\text{log.}\;2\,p-\text{log.}\,p}{\text{log}\;a.}=\frac{\text{log.}\,2}{\text{log.}\,a}$$

Ex.—If the interest be at 4 per cent., then $a=1+{\cdot}04=1{\cdot}04$.

$$t=\frac{\text{log.}\;2}{\text{log.}\,1{\cdot}04}=\frac{{\cdot}3010300}{{\cdot}0170333}=17{\cdot}6732\text{ years.}$$

Table showing the amounts of £1, for any number of years.

yrs.	3 *per Cent.*	3½ *per Cent.*	4 *per Cent.*	4½ *per Cent.*	5 *per Cent.*
1	1·0300000	1·0350000	1·0400000	1·0450000	1·0500000
2	1·0609000	1·0712250	1·0816000	1·0920250	1·1025000
3	1·0927270	1·1087179	1·1248640	1·1411661	1·1576250
4	1·1255088	1·1475230	1·1698586	1·1925186	1·2155062
5	1·1592741	1·1876863	1·2166529	1·2461819	1·2762816
6	1·1940523	1·2292553	1·2653190	1·3022601	1·3400956
7	1·2298739	1·2722793	1·3159318	1·3608618	1·4071004
8	1·2667701	1·3168090	1·3685690	1·4221006	1·4774554
9	1·3047732	1·3628974	1·4233118	1·4860951	1·5513282
10	1·3439164	1·4105988	1·4802443	1·5529694	1·6288946
11	1·3842339	1·4599697	1·5394541	1·6228530	1·7103394
12	1·4257609	1·5110687	1·6010322	1·6958814	1·7958563
13	1·4685337	1·5639561	1·6650735	1.7721961	1·8856491
14	1·5125897	1·6186945	1·7316764	1·8519449	1·9799316
15	1·5579674	1·6753488	1·8009435	1·9352824	2·0789282
16	1·6047064	1·7339860	1·8729812	2·0223702	2·1828746
17	1·6528476	1·7946756	1·9479005	2·1133768	2·2920183
18	1·7024331	1·8574892	2·0258165	2·2084788	2·4066192
19	1·7535060	1·9225013	2·1068492	2·3078603	2·5269502
20	1·8061112	1·9897889	2·1911231	2·4117140	2·6532277
21	1·8602946	2·0594315	2·2787681	2·5202412	2·7859626
22	1·9161034	2·1315116	2·3699188	2·6336520	2·9252607
23	1·9735865	2·2061145	2·4647155	2·7521663	3·0715237
24	2·0327941	2·2833285	2·5633042	2·8760138	3·2250999
25	2·0937779	2·3632450	2·6658363	3·0054345	3·3863549
26	2·1565913	2·4459586	2·7724698	3·1406790	3·5556727
27	2·2212890	2·5315671	2.8833686	3·2820096	3·7334563
28	2·2879277	2·6201720	2·9987033	3·4297000	3·9201291
29	2·3565655	2·7118780	5·1186514	3·5840365	4·1161356
30	2·4272625	2·8067937	3·2433975	3·7453181	4·3219424
31	2·5000803	2·9050315	3·3731334	3·9138574	4·5380395
32	2·5750827	3·0067076	3·5080587	4·0899810	4·7649414
33	2·6523352	3·1119423	3·6483811	4·2740302	5·0031885
34	2·7319053	3·2208603	3·7943163	4·4663615	5·2533479
35	2·8138624	3·3335904	3·9460890	4·6673478	5·5160153
36	2·8982783	3·4502661	4·1039325	4·8773785	5·7918161
37	2·9852266	3·5710254	4·2680898	5·0968605	6·0814069
38	3·0747834	3·6960113	4·4388134	5·3262192	6·3854773
39	3·1670269	3·8253717	4·6163660	4·5658991	6·7047511
40	3·2620378	3·9592597	4·8010206	5·8163645	7·0399887

The method of using the preceding table.

Rule.—Multiply the amount of pounds in the table by the principal, the product will give the amount; from which subtract the principal, and we obtain the interest.

(1) Required the amount of 250*l.* for 15 years, at 3½ per cent.

Here in the column of years we find 15, opposite to which, and under 3½ per cent., we find 1·6753488, which we multiply by 250 (the principal), and obtain 418·8372=418*l.* 16*s.* 9*d.* (the amount). If we wish to find the compound interest, we subtract the principal from the amount. Thus, 418*l.* 16*s.* 9*d.* — 250= 168*l.* 16*s.* 9*d.*, the compound interest.

Ex. 1.—Find the compound interest on 460*l.* 16*s.* 6*d.* for 32 years, at 4½ per cent. *Ans.* 1423*l.* 18*s.* 9¾*d.*+

Ex. 2.—What is the amount of 700*l.* for 7 years, at 5 per cent.? *Ans.* 984*l.* 19*s.* 5*d.*+

Having given the amount, time, and rate per cent., to find the principal.

Rule.—Divide the tabular number found under the columns of the given rate and time, into the amount; the quotient is the principal.

Or in logarithms, log. A, by tabular number under rate and time=principal.

Ex.—What principal will amount to 324*l.* 8*s.* in 7 years, at 3½ per cent.?

$$\frac{324{\cdot}4}{1{\cdot}2722793} = 254{\cdot}9754l. = 254l.\ 19s.\ 7d.$$

$$\text{Or, } \frac{324{\cdot}4}{1.2722793} = \text{log. } 324{\cdot}4 - \text{log. } 1{\cdot}2722793 = 2{\cdot}5110808 - 0{\cdot}1045823 = 2{\cdot}4064985$$

Number answering to which=254·9754=254*l.* 19*s.* 6*d.*

A Table showing the value of an Annuity of 1l. for any number of years.

The following table is thus computed :—

The first year's amount is 1*l.*

And $1 \times 1{\cdot}03 + 1 = 2{\cdot}03$ =2nd year's amount.

$2{\cdot}03 \times 1{\cdot}03 + 1 = 3{\cdot}0909$ =3rd year's do.

$3{\cdot}0909 \times 1{\cdot}03 + 1 = 4{\cdot}183627$ =4th ditto, ditto, &c. &c.

yrs.	*3 per Cent.*	*3½ per Cent.*	*4 per Cent.*	*4½ per Cent.*	*5 per Cent.*
1	1·0000000	1·0000000	1·0000000	1·0000000	1·0000000
2	2·0300000	2·0350000	2·0400000	2·0450000	2·0500000
3	3·0909000	3·1062250	3·1216000	3·1370250	3·1525000
4	4·1836270	4·2149429	4·2464640	4·2781911	4·3101250
5	5·3091358	5·3624659	5·4163226	5·4707097	5·5256312
6	6·4684099	6·5501522	6·6329755	6·7168916	6·8019128
7	7·6624622	7·7794075	7·8982945	8·0191517	8·1420084
8	8·8923361	9·0516868	9·2142263	9·3800135	9·5491088
9	10·1591062	10·3684958	10·5827953	10·8021141	11·0265642
10	11·4638794	11·7313932	12·0061071	12·2882092	12·5778924
11	12·8077958	13·1419920	13·4863514	13·8411786	14·2067870
12	14·1920297	14·6019617	15·0258055	15·4640316	15·9171264
13	15·6177906	16·1130304	16·6268377	17·1599130	17·7129827
14	17·0863243	17·6769865	18·2919112	18·9321091	19·5986318
15	18·5989140	19·2956810	20·0235876	20·7840540	21·5785634
16	20·1568814	20·9710298	21·8245311	22·7193364	23·6574916
17	21·7615878	22·7050158	23·6975123	24·7417066	25·8403662
18	23·4144354	24·4996914	25·6454128	26·8550834	28·1323845
19	25·1168685	26·3571806	27·6712293	29·0635622	30·5320037
20	26·8703745	28·2796819	29·7780785	31·3714225	33·0659539
21	28·6764857	30·2694708	31·9692016	33·7831365	35·7192516
22	30·5367803	32·3289023	34·2479697	36·3033777	38·5052142
23	32·4528837	34·4604139	36·6178885	38·9370297	41·4304749
24	34·4264702	36·6665284	39·0826040	41·6891960	44·5019986
25	36·4592643	38·9498569	41·6459082	44·5652098	47·7270985
26	38·5530422	41·3131019	44·3117445	47·5706443	51·1134534
27	40·7096335	43·7590605	47·0842143	50·7113233	54·6691261
28	42·9309225	46·2906276	49·9675829	53·9933329	58·4025824
29	45·2188502	48·9107996	52·9662862	57·4230329	62·3227115
30	47.5754157	51·6226776	56·0849376	61·0070694	66·4388471
31	50·0026782	54·4294713	59·3283351	64·7523875	70·7607895
32	52·5027585	57·3345028	62·7014685	68·6662449	75·2988290
33	55·0778412	60·3412104	66·2095272	72·7562259	80·0637704
34	57·7301764	63·4531527	69·8579083	77·0302561	85·0669589
35	60·4620817	66·6740130	73·6522246	81·4966176	90·3203068
36	63·2759441	70·0076034	77·5983136	86·1639654	95·8363221
37	66·1742224	73·4578695	81·7022461	91·0413439	101·6281382
38	69·1594490	77·0288949	85·9703359	96·1382044	107·7095451
39	72·2342324	80·7249062	90·4091493	101·4644236	114·0950224
40	75·4012593	84·5502779	95·0255153	107·0303227	120·7997735

RULE.—Seek in the column of years for the given time, and in the continued line, and underneath the given per cent., is the tabular number to be used. Then multiply this tabular number by the annuity, and the product is the amount.

(1) What will an annuity of 60*l.* amount to in 28 years, at 4½ per cent. compound interest? *Ans.* 323*l.* 11*s.* 11¾*d.*

(2) Required the amount of 240*l.* forborne for 30 years, at 3 per cent. compound interest? *Ans.* 11418*l.* 1*s.* 11¼*d.*

(3) If an annuity of 80*l.* per annum be left unpaid for 25 years, what will be the value of it, interest being allowed at 4½ per cent.? *Ans.* 3565*l.* 4*s.* 4*d.*

AMOUNT OF ANNUITIES IN ARREAR.

A Table showing the present worth of £1 for any number of years.

The following Table is thus computed. If 1*l.* be divided by 1·03 we obtain for quotient 0·9708738, which is the present worth of 1*l.* for the first year. Again, 0·9708738 ÷ 1·03 = ·9425959; and if this be added to the present worth of 1*l.* for the first year, we obtain 1·9134697, or the present worth for two years; then ·9425959 ÷ 1·03 = 0·9151417, and if this be added to the present worth of 1*l.* for two years, or 1·9134697 + ·9151417, we obtain 2·7232481, or the present worth of 1*l.* for 3 years, &c.

Yrs.	3 *per Cent.*	3½ *per Cent.*	4 *per Cent.*	4½ *per Cent.*	5 *per Cent.*
1	0·9708738	0·9661836	0·9615385	0·9569378	0·9523810
2	1·9134697	1·8996943	1·8860947	1·8726677	1·8594105
3	2·8286114	2·8016370	2·7750910	2·7489643	2·7232481
4	3·7170984	3·6730792	3·6298952	3·5875256	3·5459506
5	4·5797072	4·5150524	4·4518223	4·3899766	4·3294768
6	5·4171915	5·3285530	5·2421368	5·1578723	5·0756922
7	6·2302830	6·1145440	6·0020546	5·8927008	5·7863735
8	7·0196922	6·8739556	6·7327448	6·5958859	6·4632129
9	7·7861089	7·6076866	7·4353315	7·2687903	7·1078218
10	8·5302028	8·3166054	8·1108956	7·9127180	7·7217351
11	9·2526241	9·0015511	8·7604765	8·5289167	8·3064144
12	9·9540040	9·6633344	9·3850735	9·1185806	8·8632518
13	10·6349553	10·3027385	9·9856476	9·6828522	9·3935732
14	11·2960731	10·9205203	10·5631227	10·2228251	9·8986412
15	11·9379350	11·5174109	11·1183872	10·7395455	10·3796583
16	12·5611019	12·0941168	11·6522954	11·2340148	10·8377698
17	13·1661183	12·6513206	12·1656616	11·7071912	11·2740665
18	13·7535129	13·1896817	12·6592967	12·1599916	11·6895872
19	14·3237989	13·7098374	13·1339391	12·5932934	12·0853210
20	14·8774747	14·2124033	13·5903260	13·0079363	12·4622105

Yrs.	3 *per Cent.*	3½ *per Cent.*	4 *per Cent.*	4½ *per ent.*	5 *per Cent.*
21	15·4150240	14·6979742	14·0291596	13·4047237	12·8211529
22	15·9369165	15·1671248	14·4511150	13·7844246	13·1630028
23	16·4436083	15·6204104	14·8568413	14·1477747	13·4885741
24	16·9355420	16·0583675	15·2469628	14·4954782	13·7986420
25	17·4131476	16·4815145	15·6220796	14·8282088	14·0939448
26	17·8768423	16·8903522	15·9827688	15·1466113	14·3751855
27	18·3270314	17·2853644	16·3295854	15·4513027	14·6430338
28	18·7641082	17·6670187	16·6630629	15·7428734	14·8981274
29	19·1884546	18·0357669	16·9837143	16·0218884	15·1410737
30	19·6004414	18·3920453	17·2920330	16·2888884	15·3724511
31	20·0004286	18·7362757	17·5884933	16·5443908	15·5928106
32	20·3887656	19·0688654	17·8735512	16·7888907	15·8026768
33	20·7657919	19·3902081	18·1476454	17·0228619	16·0025493
34	21·1318368	19·7006842	18·4111975	17·2467578	16·1929041
35	21·4872202	20·0006611	11·6646130	17·4610122	16·3741944
36	21·8322526	20·2904938	18·9082817	17·6660404	16·5468518
37	22·1672355	20·5705254	19·1425785	17·8622396	16·7112874
38	22·4924617	20·8480873	19·3678639	18·0499900	16·8678928
39	22·8082153	21·1024998	19·5844855	88·2296555	17·0170408
40	23·1147721	31·3550723	19·7927735	18·4015842	17·1590865

Let a = the annuity; t = time; r = rate per cent.; p = present worth.

Case 1.

When a, t, *and* r *are given to find* p.

Rule. $\left(a - \frac{a}{r^t}\right) \div (r - 1) = p$, or multiply the given annuity by the tabular present worth, and the product is p, or the present worth.

(1) What is the present worth of an annuity of 80*l.*, to continue 8 years, at 4 per cent. per annum?

By the Table 6·7327448 × 80=538·6195840=538*l.* 12*s.* 4¾*d.*

Case 2.

When p, t, r *are given to find* a.

Rule. $\frac{pr^t\,(r-1)}{r^t - 1} = a.$

(1) If I purchase an annuity for 273*l.* 18*s.*, to be continued 18 years, at 3½ per cent. per annum, what is the annuity?

By the Table 273·9 ÷ 13·1896817=20·8141=20*l.* 16*s.* 3½*d.*

CASE 3.

When a, p, r *are given to find* t.

RULE. $\frac{a}{p+a-pr} = r^t$: this divided by r till there is no remainder, will give t; the number of divisions being = to the number of years, or t.

(1) If the lease of a house let for 50*l.* per annum, be sold for 266*l.* 8*s.*, allowing 3½ per cent. per annum, what time had the lease to run?

By the Table 266*l.* 4*s.* ÷ 50 = 5·328, opposite to which number is 6 years.

The above are only given to show the mode of calculating a few cases of common occurrence. Any one wishing to study Annuities, should consult DE MORGAN'S *Treatise* on the subject, where every necessary information can be obtained.

THE SCALES OF NOTATION.

In our, or the decimal mode of writing numbers, it will be seen that every digit to the left hand of another is by an agreement ten times greater than if it occupied the next right hand place.

It is also well known that in the decimal scale, all numbers can be expressed by means of 9 digits and 0.

In the scale whose base is 5, we can express any number by means of 4 digits and 0; so of the scale whose base is 9, we can express any number by means of 8 digits and 0, &c.

To transfer any number in the decimal scale to any other scale.

RULE.—Divide the given number by the base of the intended scale to which it is to be transposed; and set the remainder as the unit figure of the intended scale. Divide also this quotient by the base of the intended scale, and set the remainder of this division as the ten's place of digits; divide this last quotient by the base of the intended scale, and set the remainder of this division as the hundred's place; proceed thus in dividing till the quotient is less than the base of the scale, or divisor. If there be no remainder, a 0 must be put for the digit corresponding to its place.

Ex.—Convert 42715 of the denary scale into a scale whose base shall be 7.

$$\begin{array}{r|r} 7 & 42715 \\ \hline 7 & 6102+1 \\ \hline 7 & 871+5 \\ \hline 7 & 124+3 \\ \hline 7 & 17+5 \\ \hline & 2+3 \end{array}$$

Hence 235351 is the number.

Where it will be perceived that our first division was

$$42715=6102\times 7+1.$$

And secondly, $42715=(871\times 7+5)7+1=871\cdot 7^2+5\cdot 7+1$

thirdly, $42715=124\times 7^3+3\times 7^2+5\cdot 7+1.$

$$42715=17\cdot 7^4+5\cdot 7^3+3\cdot 7^2+5\cdot 7+1.$$

$$42715=2\cdot 7^5+3\cdot 7^4+5\cdot 7^3+3\cdot 7^2+5\cdot 7+1. \quad \text{(A.)}$$

Where it will be perceived that the first number in 235351, viz., 2 is to be multiplied by 7^5, or 7 raised to the fifth power; that the 3 is to be multiplied by 7^4, or 7 raised to the fourth power; that the next number, viz. 5, is to be multiplied by 7^3; that the next, viz. 3, is to be multiplied by 7^2; that the next, viz. 5, is to be multiplied by 7. Also, it will be seen that every digit is 7 times as great as if its place had been next on the right. Therefore, 42715 is expressed in a scale whose base is 7, by the number 235351.

NOTE.—If the numbers in A. be multiplied out, we obtain

$$\left.\begin{array}{rr} 2\cdot 7^5= & 33614 \\ 3\cdot 7^4= & 7203 \\ 5\cdot 7^3= & 1715 \\ 3\cdot 7^2= & 147 \\ 5\cdot 7= & 35 \\ +1= & 1 \end{array}\right\} \text{the sum is 42715, our original number.}$$

(1) Let 4263 be transformed from a scale whose radix is 6, to a scale whose radix is 12.

$$\begin{array}{r|r} 12 & 4263 \\ \hline 12 & 213+3 \\ \hline 12 & 10+9 \\ \hline & 6 \end{array}$$

$\therefore$ The number required is 693.

EXPLANATION.—Here as the digits increase in a sixfold proportion, the division is performed in the following manner :—

We multiply the 4 of the 4263 by 6 (the base of the scale in which the number is), and add in the next number, (or 2), thus $\overline{4 \times 6} + 2 = 26$, we then divide this 26 by 12 (the base of the scale into which we wish to turn our number) and obtain 2 for quotient, and 2 for remainder, which latter we multiply by 6, and add in the 6 of the 4263, making a sum of 18, which we divide by 12, and obtain for quotient 1, and for remainder 6, which we multiply by 6, and add in the 3 of the 4263, and obtain for sum 39, which we divide by 12, and obtain for quotient 3 and for remainder 3; thus obtaining for our first quotient of 4263 by 12, the number 213, with remainder 3. We then divide the 213 by 12 in the same manner; viz., we multiply the 2 of the 213 by 6, and add in the next number (the 1), obtaining for sum 13, which we divide by 12, and obtain for quotient 1 and for remainder 1; we then multiply the latter by 6 and add in the 3, obtaining 9, into which 12 will not divide; we then multiply the 1 of the 10 by 6 and add in the 0, obtaining for sum 6, into which 12 is not divisible; our number therefore is 693.

Let us transfer the same number into a scale whose base shall be 9.

$$\begin{array}{r|l}
9 & 42715 \\ \cline{2-2}
9 & 4746+1 \\ \cline{2-2}
9 & 527+3 \\ \cline{2-2}
9 & 58+5 \\ \cline{2-2}
 & 6+4
\end{array}$$

Hence 64531 is the number. And from similar reasoning to the above, the number 42715 of the decimal scale = 64531 of the scale whose base is 9.

Also, it will be perceived $42715 = 6\,.\,9^4 + 4\,.\,9^3 + 5\,.\,9^2 + 3\,.\,9 + 1$
$= 39366 + 2916 + 405 + 27 + 1$

To reduce a number in any Scale into the Decimal Scale.

RULE.—Put the number to be reduced, and multiply the left-hand digit by the scale to which it is to be reduced, adding in the second digit; multiply this product by the base of the scale, adding in the third digit; multiply this product by the base of the scale, adding in the fourth digit, and so on, till we have added the last digit. The result is the answer.

Ex. 1.—Reduce 64531, expressed in the scale of 9, to its equivalent decimal scale.

$$\begin{array}{rrcl}
 & \cdot 64531 & & \\
 & 9 & & \\ \cline{2-2}\cline{4-4}
 & 58 & = & 6\times9+4 \\
 & 9 & & \\ \cline{2-2}\cline{4-4}
 & 527 & = & 58\times9+5 \\
 & 9 & & \\ \cline{2-2}\cline{4-4}
 & 4746 & = & 527\times9+3 \\
 & 9 & & \\ \cline{2-2}\cline{4-4}
\text{No. in decimal scale} & \underline{42715} & = & 4746\times9+1
\end{array}$$

Examples in Multiplication.

It is usual to express *ten* by *t*, and *eleven* by *e* in the duodenary scale.

Multiply 34 feet 8 inches by 5 feet 11 inches. These numbers when expressed in the duodenary scale, are 2 *t*, 8 by 5, *e*.

$$\begin{array}{r}
2\,t,\,8 \\
5,\,e \\ \hline
2794 \\
1254 \\ \hline
\underline{151{,}14}
\end{array}$$

In the duodenary notation the product is 151 ft. 1′ 4″ In the denary the product is

$$\begin{array}{l}
151\ \ 1\ \ 4 \\
12 \\ \hline
17 \\
12 \\ \hline
205 \text{ ft. } 1'\ 4'' \text{ in the denary.}
\end{array}$$

(2) There is a field containing 81564 ft. 1′ 6″ 8‴ square and 32 ft. 8 in. broad in the denary scale. Required its length in the duodenary scale. *Ans*. 1540, *t* 4.

$$\begin{array}{r|l}
12 & 81564 \\ \cline{2-2}
12 & 6797+0 \\ \cline{2-2}
12 & 566+5 \\ \cline{2-2}
12 & 47+2 \\ \cline{2-2}
 & 3+e
\end{array}
\qquad
\text{Also } 12 \,\Big|\, \begin{array}{l} 32 \\ \hline 2+8 \end{array}$$

∴ Contents in duodenary scale is 3*e*250,168 ft.

∴ Breadth „ 28,8

28,8)3 *e* 250,168(1540, *t* 4 length in duodenary scale.

$$\begin{array}{r}
288 \\
\hline
1265 \\
1174 \\
\hline
e10 \\
tt8 \\
\hline
2416 \\
2328 \\
\hline
tt8 \\
tt8 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
1540\ \ 10\ \ 6 \\
12 \\
\hline
17 \\
12 \\
\hline
208 \\
12 \\
\hline
2496\ \ 10\ \ 6
\end{array}$$

2496 10 6 ft. length in denary scale.

(1) Transpose 34692 of the septenary scale into another scale whose base shall be 12. *Ans.* 52 *e* 0.

(2) Transpose 5986 of the senary scale into another scale whose base shall be 11. *Ans.* 1 *t* 06.

(3) Transpose 48396 of the binary scale into another scale whose base shall be 12. *Ans.* 120.

(4) Also 48396 of the duodenary scale into a scale whose base shall be 2. *Ans.* 10111100100000010.

(5) Transpose 3256 from a scale whose radix is 7, to another whose base is 12. *Ans.* *e* 14.

(6) Transpose 20965 of the senary scale into another whose base shall be 9. *Ans.* 4045.

PART VI.

LOGARITHMS.

(1) Logarithms are series of numbers by which arithmetical operations are facilitated. Multiplication in common arithmetic being performed by addition; division by subtraction; involution by multiplication; and evolution by division. They are of most material use to astronomers, civil engineers, surveyors, &c. &c. The literal meaning of the word logarithms is *ratios of numbers*.

(2) Logarithms are the indices of a series of numbers increasing or decreasing in geometrical ratio, thus p^0, p^1, p^2, p^3, &c.; where if p represent 1, 2, or 10, or any other number as a *base*, the indices ^{0}, ^{1}, ^{2}, ^{3}, &c. will be the logarithms of certain other numbers.

If $p = 1$, then $p^0 = 1$, and in this case the logarithm of 1 is 0.
If $p = 2$, then $p^0 = 1$, „ „ 1 is 0.
If $p = 10$, then $p^0 = 1$, „ „ 1 is 0.

For any numeral or literal with 0 for index = 1.

Hence we perceive, that in every system of logarithms, that whatever number we take for the base of the system (the 1, 2, and 10 are the bases of the three systems here treated on), that the logarithm of 1 is 0.

(3) The system now generally used is that of Briggs, Professor of Geometry at Oxford, whose adaptation of Napier's or hyperbolic logarithms is nearly equal in value to the discovery of logarithms themselves. He substituted 10 for the base of his own system instead of 2·71828182459, &c. as Napier had done. The series of Briggs may be represented as follows:—

10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2, 10^3, 10^4, 10^5,

where $10^0 = 1$ as above stated. *Hence the tabular or common logarithm of any number is the index of a power of* 10, *which number being involved to the power denoted by the index produces the given number.*

Thus in $10^1 = n$ the log. is 1, and in 10^{-2} or $\frac{1}{10^2}$ the log. is -2; in the system where 10 is the base. So that we perceive in 10, the index of which is 1, that the log. of 10 is 1

$$\left.\begin{array}{llll} \text{In } 10^2 \text{ or } & 100 & \text{that the log. of} & 100 \text{ is } 2 \\ \text{In } 10^3 \text{ or } & 1000 & ,, \quad ,, & 1000 \text{ is } 3 \\ \text{In } 10^4 \text{ or } & 10000 & ,, \quad ,, & 10000 \text{ is } 4 \\ & \&c. & \&c. & \end{array}\right\} \text{where the base is } 10.$$

Here the 1, 2, 3 or 4 of 10^1, 10^2, 10^3 and 10^4 is called the *index* or *characteristic*, and it will be perceived that the index is always 1 less in number than the number of the places of digits in the given power; thus in 10 the log. is 1, which is 1 less than the digits in the 10, which are 2, viz. the 1 and 0; again in 10^2 or 100, the index is 2, being one less than the number of digits, viz. 1, 0, 0, which consist of 3 digits; also in 10^3 or 1000, the index is 3, being one less than the number of digits, which are 4, viz. 1, 0, 0, 0.

Now if we take the decreasing series, viz. $\frac{1}{10^1}$, $\frac{1}{10^2}$, $\frac{1}{10^3}$, which may be represented by 10^{-1}, 10^{-2}, 10^{-3}, we perceive that the indices are negative, it will thence be found that the index of a simple fraction is negative.

$$\text{Ex.—The log. of } \frac{1}{10} \text{ or } 10^{-1} = -1$$

$$,, \quad ,, \quad \frac{1}{10^2} \text{ or } 10^{-2} = -2$$

$$\text{The log. of } \frac{1}{10^3} \text{ or } 10^{-3} = -3$$

$$,, \quad ,, \quad \frac{1}{10^4} \text{ or } 10^{-4} = -4$$

$$\&c. \qquad \&c.$$

(4) It will also be readily perceived, that if the log. of 10 be 1, that the log. of any number less than 10 must consist of decimals, and we also perceive that if the log. of 100 be 2, that the log. of 85 or any other intermediate number between 10 and 100 will be between 1 and 2, that is, 1 with some decimals. The log. is in fact 1·9294180, so that the equation representing this will be $10^{1\cdot9294180} = 85$*.

Again. If the log. of 100 be 2, and the log. of 1000 be 3, the log. of any other intermediate number will be 2 with decimals. Ex. The log. of 893 is 2·9508515, so that the equation representing this will be $10^{2\cdot9508515} = 893$.

Hence $10^0 = 1$, therefore log. $1 = 0$ (Nos.)
$10^{\cdot3010300} = 2$, $\therefore$ log. $2 = \cdot3010300$
$10^{\cdot4771213} = 3$. $\therefore$ log. $3 = \cdot4771213$
$10^{\cdot6020600} = 4$, $\therefore$ log. $4 = \cdot6020600$
$10^{\cdot6989700} = 5$, $\therefore$ log. $5 = \cdot6989700$
$10^{\cdot7781513} = 6$, $\therefore$ log. $6 = \cdot7781513$
$10^{\cdot8450980} = 7$, $\therefore$ log. $7 = \cdot8450980$
$10^{\cdot9030900} = 8$, $\therefore$ log. $8 = \cdot9030900$
$10^{\cdot9542425} = 9$, $\therefore$ log. $9 = \cdot9842425$
$10^1 = 10$, $\therefore$ log. $10 = 1\cdot0000000$

(6) Hence generally if $10^x = n$, the log. of n is x.

The same mode of reasoning will apply to negative indices or logarithms.

Also if $p^x = q$ and $p^{x'} = q_,$ by the multiplication of powers,

$$p^x \times p^{x'} = q \times q_, \quad \text{Or } p^{x+x'} = qq_,$$

$\therefore x + x_, = \text{log. } q + \text{log. } q_,$, hence the rule for multiplication of logarithms.

(7) Again, if $p^x = q$ and $p^{x'} = q_,$ we obtain by dividing

$$\frac{p^x}{p^{x'}} = \frac{q}{q_,} \text{ or } p^{x-x'} = \frac{q}{q_,} \text{ or } x - x_, = \text{log.} \frac{q}{q_,} \text{ or}$$

or $x - x_, = \text{log. } q - \text{log. } q_,$ hence the rule for division of logs.

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* The 1 of the numbers 1·9294180 is called the *index* or *characteristic*, and the numbers ·9294180 are called the *mantissa*.

(8) Again, if $p^x = q$ and we raise both sides of the equation to the m^{th} power, we obtain $p^{mx} = q^m$ and $\therefore mx = m$ log. q, hence the rule for involution, viz. *that the log. of the* m*th* *power of any number is* m *times the log. of that number.*

(9) Again, if $p^x = q$ and we extract the $\frac{m}{n}$th root of both sides of the equation, we obtain

$$p^{\frac{m}{n}x} = q^{\frac{m}{n}} \text{ and } \therefore \tfrac{m}{n}x = \log.\ q^{\frac{m}{n}} \text{ or } \log.\ q^{\frac{m}{n}} = \frac{m}{n} \log.\ q$$

Hence the rule for evolution, viz. *that the log. of any root of any number is found by multiplying the log. of the number by the numerator of the root, and dividing the product by the denominator*. Our space will not admit of further investigation of the theory of logs., but the following formula is presented to the reader's notice, by which he will be enabled to calculate the log. of any number above 1000.

(10) To find the log. of any number below 1000, let him consult PROFESSOR YOUNG'S *Elementary Essay on the Construction of Logarithms**.

To find the Briggean or common log. of any number above 1000. Ex.—Of 3967. Formula.

$$\log.\ n = \tfrac{1}{2}\ \{\log.\ (n+1) + \log.\ (n-1)\} + \text{M}\left(\frac{1}{2n^2-2}\right)$$

Here n represents the number whose log. is to be found, and M the modulus of the system, viz. ·4342944819, &c.

$$\therefore \text{lg. } 3967 = \tfrac{1}{2}\left\{\begin{array}{l} \text{lg.}(n+1) = \text{lg. } 3968 = 3{\cdot}5985717 \\ \text{lg.}(n-1) = \text{lg. } 3966 = \underline{3{\cdot}5983527} \\ \hphantom{\text{lg.}(n-1) = \text{lg. } 3966 = {}} 7{\cdot}1969244 \end{array}\right\} = 3{\cdot}5984622$$

$$\text{And M}\left(\frac{1}{2n^2-2}\right) = {\cdot}43429 \text{ \&c.}\left(\frac{1}{2 \times 3967^2 - 2}\right)$$

$$= {\cdot}43429 \text{ \&c.}\left(\frac{1}{3107070178 - 2 = 310707016}\right) = {\cdot}0000000001$$

$$\therefore \log.\ 3967 = 3{\cdot}5984622 + {\cdot}0000000001 = 3{\cdot}5984622 \text{ \&c.}$$

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* The best tables of logarithms are those by Taylor, Gardiner, Hutton, Babbage, Caillet. The smaller works on this subject are Lalande, Hassler, Renaud, and those published in the *Library of Useful Knowledge*.

(11) The following prime numbers with their logs. to 100 are extracted from BABBAGE's *Logarithms*.

Nos.	Logarithms.	Nos.	Logarithms.
2	·3010300	47	1·6720979
3	·4771213	53	1·7242759
7	·8450980	59	1·7708520
11	1·0413927	61	1·7853298
13	1·1139434	67	1·8260748
17	1·2304489	71	1·8512583
19	1·2787536	73	1·8633229
23	1·3617278	79	1·8976271
29	1·4623980	83	1·9190781
31	1·5682017	89	1·9493900
41	1·6127839	97	1·9167717
43	1·6334685		

The indices of all the numbers above 10 have been added for the convenience of the learner.

(12) The logs. of the first 10 natural numbers and the primes to 100 being given, we shall now proceed to shew how the logs. of any other numbers may be found.

Ex. 1.—To find the log. of 20.

Here $20 = 10 \times 2 = \text{log. } 10 + \text{log. } 2 = 1{\cdot}0000000 + {\cdot}3010300 = 1{\cdot}3010300$

Ex. 2.—To find the log. of 96.

Here $96 = 4 \times 4 \times 6 = 4^2 \times 6 = 2 \text{ log. } 4 + \text{log. } 6 = 2 \times {\cdot}6020600 + {\cdot}7781513 = 1{\cdot}9822713$

Ex. 3.—To find the log. of 801.

Here $801 = 89 \times 9 = \text{log. } 89 + \text{log. } 9 = 1{\cdot}949300 + {\cdot}9542425 = 2{\cdot}9046325$

Ex. 4.—Find the logs. of the following numbers: 165, 511, 196, 852.

NOTE.—The log. of a fraction can be found by finding the log. of the numerator and subtracting the log. of the denominator therefrom, or by reducing the fraction to a decimal and finding its logarithm.

(13) Method of using logarithmic tables.

If the number sought for be below 1000 the logarithm is found opposite to it, but if it be above 1000, look for the three first numbers on the left hand of the top of the page: the remaining part of the number is found as directed in the next paragraph.

Ex.—Suppose we are required to find the log. of 265897 in Hutton's or any other tables. We first look on the top of the page for 265; on the top of the column in this page is 26500, we look down the column till we find 2658, and then look on the top row of numbers and find 9. Underneath the 9, and opposite 2568 we find 0·4247020, which is the log. of 26589, but we want the log. of 265897; to find the part to be added for the 7 we look in the right hand column in the *proportional parts*, and find opposite 7 the number 114, which we add to the log. already found, thus,

Log. of 26589 =	·4247020	We then prefix 5 for the index, agreeably to Art. 3, and obtain 5·4247020, which is the log. of 265897.
In the proportional parts opposite 7 we find	114	
	5·4247020	

(14) It is proper to observe, that the log. of any number, as 32549, being 4·5125379; that the log. of its $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, or $\frac{1}{10000}$th part, will be as below:—

	Nos.	Logs.
	32549	4·5125379
If divided by 10..	3254·9	3·5125379
„ 10..	325·49	2·5125379
„ 10..	32·549	1·5125379
„ 10..	3·2549	0·5125379
„ 10..	·32549	—1·5125379
„ 10..	·032549	—2.5125379
„ 10..	·0032549	—3.5125379
„ 10..	·00032549	—4·5125379

We thence perceive that in every division of the number 32549 by 10, that the index of the log. decreases by unity till it arrives at 0, when the index of the next number, ·32549, is—1.

By Art. 3, the index of $\frac{1}{10} = -1$, and that of $\frac{1}{10^2} = -2$, &c., &c.

(15) We infer from this, a ready mode of finding the index of any decimal, which to a beginner is generally the most difficult process he has to effect in logarithms.

RULE.—If the *value* of the first *significant* number be the *first* decimal, the index will be—1; if the *second* decimal, the ndex will be—2; if the *third* decimal, the index will be—3, &c., nd so on for any decimal place of the first *significant* figure.

Ex.—The index of ·32549 is—1, because the first significant number, viz. 3, is the *first* decimal.

The index of ·032549 is —2, because the first significant number, viz. the 3, is the second decimal.

„ ·0032549 is —3, because the first significant number, viz. the 3, is the third decimal; and so of any other decimal.

Ex. 1.—It is required to find the log. of the number ·00729.

Here we look for 729 on the top of the page, disregarding the 2 ciphers prefixed; and we find for 729 the log. ·8627275, we prefix — 3 for index, because the 7 of the 729 is the third decimal, .·. log. of ·00729= —3·8627275.

(2) To find the log. of ·000009578.

Here we look for the number 9578, disregarding the 5 ciphers which precede it, and we find in the logs. ·9812748, we then prefix — 6, because 9 is the sixth decimal.

(16) To find the Number answering to a Logarithm.

Ex.—Let the log. be 3·9817251.

Here we first look in the logs. for 981 (disregarding the index), and we find the nearest log. in the tables to be ·9817235, and the number answering thereto, to be 95879. We then subtract ·9817235 from ·9817251, and obtain for difference, 18, opposite which in the proportional parts is 4, which we affix to 958798, and obtain 958794; and in order to know where to put the decimal point, observe this rule:—That the number of integers is one more than the number of the index (when the index is positive); and when the index is negative, that the first digit of the number found will be the first, second, third, &c. decimal, according as the index is — 1, — 2, — 3, &c. In this instance the integers will be 4, because the index is + 3.

Ex. 1.—Find the number answering to the following Logarithms:—

Logs.		Nos.
3·03233706	*Ans.*	1077·391
5·8680969	*Ans.*	738069.
4·7466588	*Ans.*	53803·2
2·8057067	*Ans.*	639·303
4·8310809	*Ans.*	67776·8
—3·8440439	*Ans.*	·00698303
—2·9681741	*Ans.*	·0929339

Multiplication of Logarithms.

Rule.—Find the logarithms of the numbers to be multiplied then add them together, and find the number answering to the resulting logarithm.

Ex.—Required the product of 7296 and 325·49.

Here log. 7296 .. 3·8630848
Also log. 325·49 .. 2·5125377
2374774 .. 6·3756225
Multiply ·00

Here the log. found in the table is ·8630848, and we prefix 3, because there are 4 numbers in 7296; *i.e.* we prefix one less than the number of digits. The same method was adopted in finding the log. of 325·49, only the learner must remember that the index is one less than the number of integers, when there are any integers, and that the negative index is the same in number as the decimal of the first significant figure; in other words, will be — 1 if the first significant figure be the first decimal; will be — 2 if the first significant figure be the second decimal; will be — 3 if the first significant figure be — 3; Art. 15.

Multiply ·00329 by 62·4
Here the log. of ·00329 is .. —3·5171959
„ of 62·4 is .. 1·7951846
Number answering thereto ·205396 .. —1·3123805

(1) Multiply 3068 by 2096 *Ans.* 6430528
(2) „ 37591 by ·0028 *Ans.* 105·2548
(3) „ ·963957 by ·00004835 *Ans.* ·00004660732
(4) „ 2469·7 by ·037 *Ans.* 19·3789
(5) „ ·003729 by ·052 *Ans.* ·673093
(6) „ ·257 by ·1986 and this product by ·00068 × 3·215 *Ans.* ·000111584
(7) „ 25·63 × 7.829 × 4153· × ·00000062 *Ans.* ·516664
(8) „ 205·7 × ·324 × ·625 × ·4098 *Ans.* 17·0699
(9) „ ·000372 × 4083 × ·06953 *Ans.* ·105605

DIVISION OF LOGARITHMS.

RULE.—Division of Logarithms is performed by subtraction.

Divide
(1) 5·6027 by 3·25 *Ans.* 1·723907
Log. 5·6027 = 0·7483974
Log. 3·25 = 0·5118834
Number 1·723907 = ·2365140
(2) 26·87 by ·00035 *Ans.* 12167·45
(3) 4·732 by ·0098 *Ans.* 482·857
(4) 64·5 by ·009325 *Ans.* 6916·89
(5) ·00072 by 52·98 *Ans.* ·00001359
(6) ·09687 by 29·63 *Ans.* ·0032693
(7) ·00634 × 528·7 by 2·006 *Ans.* 1·66875
(8) ·2473 × 62·8 by 3·709 × 524·7 × ·002 *Ans.* 3·99011

INVOLUTION.

Find the logarithm of the given number, and multiply it by the index of the power to which it is to be raised.

Ex.—(1) Find the 5th power of 6·24 *Ans.* 9460·69 &c.

$$\begin{array}{r} \text{Log } 6{\cdot}24 = {\cdot}7951846 \\ 5 \\ \hline \text{Number } 9460{\cdot}69 = 3{\cdot}9759230 \end{array}$$

(2) Find the 3rd power of 7·96 *Ans.* 504·358 &c.
(3) „ 7th „ 2·03 *Ans.* 142·06 &c.
(4) „ 4th „ ·0027 *Ans.* ·0000000000000000000000104603 &c.
(5) Find the value of $(3{\cdot}072 \times 8{\cdot}25)^7$ *Ans.* 6716194000
(6) „ $(6429{\cdot} \times {\cdot}073)^8$ *Ans.* 23538 &c. to 22 figs.
(7) „ $(2{\cdot}695 \times 7{\cdot}24 \times {\cdot}0003)^5$ *Ans.* 0000000000006872163
(8) „ $\left(\frac{{\cdot}0096 \times 8{\cdot}257}{{\cdot}096 \times 2{\cdot}07}\right)^3$ *Ans.* 06346817
(9) „ $\left(\frac{26\ 98 \times 3{\cdot}5 \times 7{\cdot}24}{36{\cdot}4 \times 9{\cdot}2}\right)^8$ *Ans.* 301·7132
(10) „ $\left(\frac{2{\cdot}49 \times 4{\cdot}35 \times 700{\cdot}9 \times 2{\cdot}4}{3{\cdot}208 \times 6095{\cdot} \times {\cdot}03}\right)^3$ *Ans.* ·02996949
(11) „ $({\cdot}025)^{18}$
Ans. ·0000000000000000000000000000000145519 &c.
(12) „ $({\cdot}002)^{15}$
Ans. ·000000000000000000000000000032768

EVOLUTION.

RULE.—Find the logarithms of the power whose root is to be extracted; then divide by the index of the root.

Ex.—See question 2 below.

Here log. of

$${\cdot}040926^{\frac{1}{5}} = -\overset{5)}{2}{\cdot}6123599 = (-\overset{5)}{2} - 3 + {\cdot}6123599 + 3) =$$
$$-1{\cdot}7224719 \text{ answering to the number } {\cdot}527863.$$

The learner will perceive that in the division a new process has been introduced; we allude to the division of −2 by 5. It will be perceived that in order that we may divide the −2 by 5, that we increase the +2 by −3, making −5, which is divisible by 5, giving −1 for quotient; and that whilst we added −3 to the negative index, we also added +3 to the following mantissa, making it 3·7224719. It will thence be perceived that we have neither increased nor diminished our numbers by the process.

(1) Extract the 5th root of 29·47 *Ans.* 1·967324
(2) And $\cdot 040926^{\frac{1}{5}}$ *Ans.* ·527803
(3) Show that $328{\cdot}29^{\frac{1}{3}}$ *Ans.* 6·89846+

(4) And $2{\cdot}053^{\frac{1}{9}}$ *Ans.* $1{\cdot}08322+$

(5) Also that $\cdot0003257^{10}$ *Ans.* $\cdot4480038+$

(6) And $(\frac{232}{719})^{\frac{2}{3}}$ *Ans.* $\cdot4704419$

(7) And $(\frac{403}{751})^{\frac{6}{7}}$ *Ans.* $\cdot586522$

(8) Also $(3{\cdot}1416\times{\cdot}0356\times721{\cdot}4)^{\frac{7}{11}}$ *Ans.* $70{\cdot}75651$

(9) Find the value of $\left\{\left\{\frac{3^{\frac{1}{3}}}{7^{\frac{1}{2}}}\right\}^{\frac{2}{13}}\times{\cdot}062\times\left\{\frac{57^3}{71^2}\right\}^{\frac{1}{5}}\right\}^{\frac{5}{6}}={\cdot}166236$

EXAMPLES FOR PRACTICE.

(1) $\dfrac{32{\cdot}407}{12{\cdot}32}=2{\cdot}63143$ (2) $\dfrac{\cdot8246}{4-(1{\cdot}248)^3}=4{\cdot}01023$

(3) $\sqrt[5]{982{\cdot}73}=3{\cdot}96722$ (4) $\dfrac{\cdot0286\times325{\cdot}8}{205{\cdot}6\times7{\cdot}832}={\cdot}00581224$

(5) $3\sqrt[7]{9}=4{\cdot}10621$ (6) $\sqrt[9]{\dfrac{204{\cdot}2}{3{\cdot}097}}=1{\cdot}59366$

(7) $(\frac{2}{3})^{\cdot05}={\cdot}979936$ (8) $(\frac{5}{9})^{\cdot175}={\cdot}902354$

(9) $(\frac{82}{5})^{\cdot345}=2{\cdot}62494$ (10) $\sqrt[7]{\cdot08729}\times\sqrt[8]{3{\cdot}046}={\cdot}55309$

(11) $\sqrt[3]{\cdot58\sqrt{\frac{5}{3}}}={\cdot}771216$ (12) $\sqrt[5]{\left(\dfrac{209{\cdot}5\times7{\cdot}2}{\cdot5}\right.}=3{\cdot}679647$

(13) $\sqrt[101]{8}=1{\cdot}0208$ (14) $\sqrt[253]{2{\cdot}06}=1{\cdot}0012406$

(15) $\sqrt{\sqrt{\cdot09}\times\sqrt[3]{5}\times(\cdot007)^3\times\sqrt[3]{7}}={\cdot}19088$

To find a fourth proportional to three given numbers.

RULE.—Add together the logarithms of the second and third erms, subtract the logarithms of the first term from the sum of he logarithms of the second and third terms; then find the atural number answering to the resulting logarithm; and it ill be the fourth proportional required.

Ex.—If x represent the fourth proportional, let it be required find x from the terms 6, 17, and 5. Thus, $6 : 17 :: 5 : x$, r $6x=17\times5$, or $x=\frac{17\times5}{6}$, $\therefore x=\log.17+\log.5-\log.6$.

$$\begin{array}{rr} \log.\ 17 = & 1{\cdot}2304489 \\ \log.\ 5 = & \underline{\cdot6989700} \\ & 1{\cdot}9294189 \\ -\log.\ 6 = & \underline{\cdot7781513} \\ \text{Nat. No. } 14{\cdot}1666 = & 1{\cdot}1512676 \end{array}$$

$\therefore x=14{\cdot}1666.$

Solutions of questions in Simple Interest, Compound Interest, Annuities, &c., by Logarithms.

1. SIMPLE INTEREST.

Find the simple interest on 4908*l.* for 20 years, at 7½ per cent.

$$\begin{aligned} \text{Here log. of } \pounds 4908 &= 3{\cdot}6909046 \\ \text{log. } {\cdot}075 &= 2{\cdot}8750613 \\ \text{log. } 20 &= \underline{1{\cdot}3010300} \\ \text{Nat. No.} = \pounds 7362 &= 3{\cdot}8669959 \text{ Hence 7362 } \textit{Ans.} \end{aligned}$$

COMPOUND INTEREST.

Find the compound interest on 250*l.* 10*s.* for 13½ years, at 8¼ per cent.

Here the amount of 1*l.* for 1 year, is 1·0825.

$\therefore 1{\cdot}0825^{20} \times 250{\cdot}5 =$ compound interest.

In logarithms, 20 log. 1·0825 + log. 250·5

$$\begin{aligned} \text{Log. } 1{\cdot}0825 &= 0{\cdot}0344279 \\ & \qquad\quad \underline{20} \\ & \quad {\cdot}6885580 \\ \text{Log. } 250{\cdot}5 &= \underline{2{\cdot}3988077} \\ \text{Nat No. } 1222{\cdot}828 &= 3{\cdot}0873657 \end{aligned}$$

∴ 1222*l.* 16*s.* 6¾*d.* is the amount.

Consequently, 1222*l.* 16*s.* 6¾*d.*—250*l.* 10*s.*=972*l.* 6*s.* 6¾*d.* the compound interest of 250*l.* 10*s.* for 20 years.

To find the magnitudes of the Planets with respect to each other.

RULE.—Cube the diameters of the planets, and divide the greater cube by the less, the quotient is the answer.

Ex.—Required how much the magnitude of the Earth exceeds that of Mercury. *Ans.* 16·8.

			Miles.	
Reqd. the same ratios of the other planets and the Earth.	Venus	diameter	7498	*Ans.* 1·19827
	Mars	do.	4218	*Ans.* 6·73
	Jupiter	do.	89069	*Ans.* 1398·9 larger than the earth.
	Saturn	do.	78730	*Ans.* 966·11 do.
	Herschel	do.	35112	*Ans.* 85·6984 do.

Many useful problems can be obtained by varying the above data.

Questions of this description are almost immediately solved by means of logarithms.

PART VII.

MENSURATION OF SUPERFICIES AND SOLIDS.

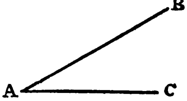
GEOMETRICAL DEFINITIONS.

GEOMETRY literally signifies measuring the earth. It is now generally applied to the mensuration and properties of lines, angles, surfaces, and solids.

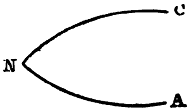
A *point* is without parts or magnitude, and is therefore *imaginary*.

A *line* has length, but no breadth or thickness. A line lying in the same direction, and in the same plane, is a *right* line; Archimedes defines this "the shortest distance between two points."

A *curve* line is a line either equally distant from a centre, as the circumference of a circle, a section of a cone, as an ellipse, or it is formed from the hyperbola, parabola; or from unfolding a string wound round a solid; or by the motion of a point in the felly of a wheel running on a plane, or &c. &c.

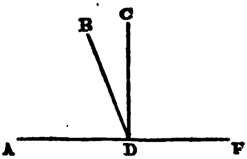


An *angle* is the inclination or opening of two straight lines, it is either called a rectilineal angle, as B A C; or if formed of two curve lines, as C N A, it is called a curvilinear angle.



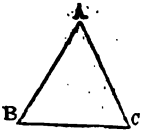
Angles assume various names, according to the inclination of the lines forming them.

They are called *right* when one line is perpendicular to another, as A D C, or E D C; *acute*, when they form an angle *less* than a right angle, as A D B; *obtuse*, when *greater* than a right angle, as B D E. To express any angle, the letter at the angular point must be the centre one of the three letters.

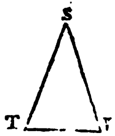


Triangles assume different names, according to the nature of their angles. Thus, if a triangle contains a right angle, it is called a *right-angled* triangle; if it contains an *obtuse* angle, it is called an *obtuse*-angled triangle; if it contains only acute angles, it is called an *acute-angled* triangle. Other names are given with reference to *the sides* of the triangle.

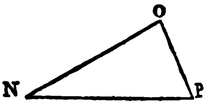
If all the sides are *equal,* it is called an *equilateral* triangle, as A B C.



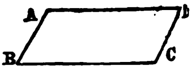
If 2 sides are *equal,* it is called an *isosceles,* S T V.



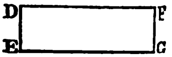
If all the sides are *unequal,* it is called a *scalene triangle,* as N O P.



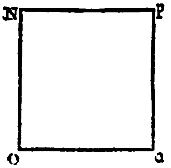
A *parallelogram* is a figure having its opposite sides equal and parallel, as A B C D.



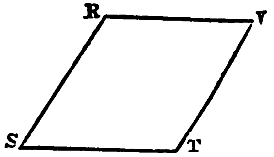
If the angles be right angles, the figure is called a *rectangle,* as D E F G.



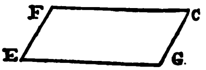
If the angles of the rectangle be right angles, and all its sides equal, it is called a *square,* as N O P Q.



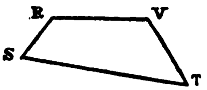
A *rhombus* has all its sides equal, but only its *opposite* angles equal, as R S T V.



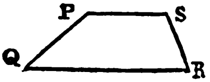
A *rhomboid* has its opposite sides and opposite angles equal, as F E G C.



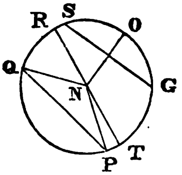
A *trapezium* is a figure having 4 unequal sides, as R S T V.



A *trapezoid* is a figure having 2 of its opposite sides parallel, as P Q R S.



A *circle* is a plane figure bounded by a continuous curved ine, called the *periphery* or *circumference*, which is equidistant from a point within, s O P Q.



This point, N, is called its *centre*.

A line passing through its centre is alled its *diameter*, as T R.

And N T and N R are called *radii*.

An *arc* is part of the circumference (as G O S), which is cut off by a line (as G S) called its *chord*.

The *space* contained by the arc and its chord is called a *segment;* if the chord pass through the centre, it is a *semicircle*, as T O R; if above the centre (as G S), the segment is less than a emicircle; if below (as P Q), it is greater than a semicircle.

A *sector* is part of a circle formed by two radii and an arc, as Q N P: if the radii are at right angles, the figure is called a *quadrant*, as R N O.

MENSURATION OF SUPERFICIES.

To find the area of any parallelogram.

RULE.—The length multiplied by the breadth or altitude will be the area required.

(1) A garden is square, each side measuring 45 yds.; required ts superficial content. *Ans.* 2025.

(2) A rectangular board is 21½ ft. long and 9 inches wide; required its superficial content. *Ans.* 16 ft. 1½ in.

(3) The roof of a house is a parallelogram, whose sides are 35 and 19 ft.; required half its contents. *Ans.* 332·5.

(4) A rhombus has its sides 14 ft. long, and its perpendicular height 9 ft. 6 in.; required its superficial content. *Ans.* 131 ft.

(5) A grass plot is 189 ft. long and 144 ft. wide. It contains 5 square beds, the sides of which are 10 ft. each. Required the quantity of square yards of grass. *Ans.* 3024 yds.

To find the area of a triangle.

RULE.—Half the base multiplied by the whole perpendicular, or the whole base multiplied by half the perpendicular, will give the area required.

Ex.—(1) Given the base 140 ft., and perpendicular 17½; required its contents. *Ans.* 1225.

(2) base 73·9 ft., perp. 14¼. *Ans.* 535·775.
(3) „ 49·25 ft., „ 17·9. *Ans.* 41·862.5
(4) „ 302·5 ft., „ 20·15. *Ans.* 3047·6875.
(5) „ 10 ft., „ 7·5. *Ans.* 112·5.

The area of any triangle may also be found when two sides and their included angle are given, by multiplying either side by half the other, and that product by the natural sine of the angle. The product is the area.

Ex.—Required the area of a △ which has two sides, equal to 20 and 50 yds,, and whose included angle is 27° 35′.

$$\begin{array}{rcr} \text{Here nat. sine of } 27^\circ\, 35' & = & \cdot 4630382 \\ \frac{1}{2} \bullet 20 \bullet 50 & = & 500 \\ \hline \text{yds.} & & \underline{231\cdot5191000} \end{array}$$

To find the area of a sector of a circle, the angle and radius being given.

RULE.—1. Multiply the square of the radius by the number of seconds in the angle, and that product by 206265 × 2, or 412530.

2. The length of the arc multiplied by the radius, and this product divided by 2 will give the area.

3. Or say, as 360° : length of arc in degrees :: area of the circle : area of the sector.

Ex.—(1) When the radius is 14, and the chord of the whole arc 16, what is the area? *Ans.* 117·95 inches.

(2) Required the area of the sector whose arc is 150°, and radius 32 inches. *Ans.* 1340·416 inches.

The following ratios were calculated many years ago by the Author by means of continued fractions:—

(1) 1:3
(2) 7:22 .. This is generally called Archimedes' Ratio.
(3) 106:333
(4) 113:355. .. This ratio was discovered by Metius, a Dutch mathematician; true to the 6th fig.
(5) 33102:103993
(6) 33215:104348
(7) 66317:208348
(8) 99532:312689
(9) 265381:833719
(10) 364913:1146408
(11) 1360120:4272943
(12) 1725033:5419351
(13) 25510582:80143857 .. True to 14 places: this number gives but a difference of $\frac{1}{33}$ part of a barleycorn in calculating the circumference of the earth's orbit; reckoning its diameter in round Nos. 190000000 miles.
(14) 52746197:165707065
(15) 78256779:245850922
(16) 131002976:411557987
(17) 340262731:1068966896

(18) 811528438:2549491779.. **True to 17 places of decimals.**
(19) 1963319607:6167950454
(20) 4738167652:14885392687
(21) 16177822563:50824128515
(22) 20915990215:65709521202.. True to 19 places of decimals.
(23) 58009802993:182243170919
(24) 78925793208:247952692121
(25) 215861389409:678148555161
(26) 510648572026:1604249802443
(27) 2258495677513:7095147764933
(28) 9544641282078:29984840862175
(29) 20347778241669:67064829489283
(30) 29892419523747:97049670351458
(31) 50240197765416:164114499840741
(32) 80132617289263:261164170192199
(33) 531035901500994:1731099520993935
(34) 5715626903366203:1819340549079508

The use of the preceding Table is to find the ratio of the liameter to the circumference when the diameter is 1.

Ex.—The second form gives $\frac{22}{7} = 3{\cdot}1428+$; the third form ives $\frac{333}{106} = 3{\cdot}1415+$

Van Ceulen determined the ratio of the circumference to the liameter to 26 places of decimals; Mr. Rutherford, of Woolwich ollege, has lately calculated it to 208 places of decimals. The ollowing are the Nos.:

3·14159 26535 89793 23846 26433 83279 50288 41971
69399 37510 58209 74944 59230 78164 06286 20899
86280 34825 34211 70679 82148 08651 32823 06647
09384 46095 50582 23172 53594 08128 48473 78139
20386 33830 21574 73996 00825 93125 91294 01832
80651 744, &c.

To find the circumference of a circle.

Multiply the diameter by 3·14159265, or, which is generally ufficient, by 3·1416; or say, as 113 : 355 :: diameter : to the ircumference.

Ex.—(1) The diameter of a circle is 42·75 yards; required s circumference. *Ans.* 102·8874.

(2) The diameter of a circle is 420 yards; required its cirumference. *Ans.* 1319·472.

To find the length of an arc of a circle.

Rule.—1. Subtract the chord of the whole arc from 8 times e chord of half the arc; the remainder, divided by 3, will give e length of the arc *nearly*.

2. The number of degrees in the *arc* multiplied by the radius, and this product by ·01745 will give the length of the arc.

Ex.—(1) The chord of the whole arc is 80 inches and the chord of half the arc is 50 inches; required the length of the arc. *Ans.* $106\frac{2}{3}$ inches.

(2) Required the length of the arc when the chord of half the arc is 40 and the versed sine 30 inches. *Ans.* 89·03 inches.

(3) The number of degrees in an arc is 50°, and the radius 5 inches; required the length of the arc. *Ans.* 4·3625 inches.

Given the area of a circle to find its diameter.

Rule.—Divide the area by ·7854, and extract the square root of the quotient, which will give the diameter.

Ex.—(1) The area of a circle is 754.7694 feet, what is its diameter? *Ans.* 31.

(2) The area of a circle is 2·0589115, required its diameter. *Ans.* 2·15.

To find the area of a circle.

Rule.—Square the diameter, and multiply this square by ·7854. The product is the area.

Ex.—(1) The diameter of a circle is 12; required its area. *Ans.* 213·0976.

(2) The diameter of a circular field is 2140 yards; required its area. *Ans.* 3616817·84.

(3) A circular grass plot is $214\frac{1}{5}$ yards in diameter; required its area. *Ans.* 36035·440056.

(4) There is a building which has a circular roof of $19\frac{1}{2}$ feet in diameter; required the number of square feet in it, and the expense of painting at $4\frac{1}{4}$*d.* per foot. *Ans.* 298·64835 sqr. ft.; 5*l.* 5*s.* $4\frac{1}{4}$*d.*

(5) In a circular field the diameter is 729 feet; required the area. *Ans.* 417393·7614.

To find the arc of a circular sector, the radius and angle being given.

The number of seconds in the angle multiplied by the radius, and this product divided by 206265, will give, very nearly, the number of units in the arc.

Ex.—There is a circular sector which contains an angle of 45° 15′, and the radius of which is 4 inches; required the length of the arc. *Ans.* 3·15+

To find the area of an internal circle which touches the three sides of a triangle.

Rule.—The half sum of the sides divided into the area of he triangle will give the radius, then double the radius, squared, multiplied by ·7854, will give the area of the circle.

Ex.—The three sides of a △ are 20, 15, and 25 feet; required he area of the inscribed circle. *Ans.* 78·54 feet.

When the three sides of a △ are given—

Rule.—Subtract each side from half the sum of the sides, hen multiply the three differences by the half sum of the sides, nd extract the square root of the product for the area.

Ex.—(1) The three sides of a △ are 400, 560 and 740, equired its area. *Ans.* 210242·1.

(2) The three sides of a △ are 140, 350, 470; required the rea. *Ans.* 14565·7.

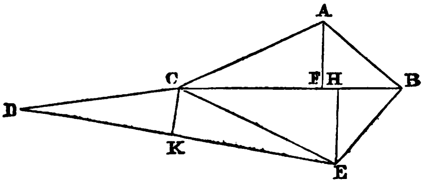
(3) The three sides of a △ are 40, 70 and 90; required the rea. *Ans.* 1341·6.

Irregular Figures.

Divide the given figure into trapeziums and triangles; then ind the area of every trapezium; the sum of all the areas will be he area sought.

Ex. (1.)—A figure consists of two trapeziums, having the iagonal of the top trapezium 590, and the upper perpendicular '8, and the under perpendicular 84; the bottom trapezium has its iagonal 596, its upper perpendicular 49, and its lower one 86; equired its contents? *Ans.* 88020.

(2) There is an irregular figure which is divided into triangles s follows:



A B=15 ft.; B C=240 ft.; H E=32 ft.; C K=19 ft., and D C = 342 ft.; required its area? *Ans.* 987$\frac{2}{3}$ yds.

To find the area of a trapezium or trapezoid:

Rule.—Divide the trapezium into two △s. Then multiply half the base by the sum of the two perpendiculars, or the whole base by half the sum of the two perpendiculars.

Rule for a Trapezoid.—The half sum of the two parallel ides multiplied by the perpendicular height will be the area sought.

Ex. (1.)—The diagonal of a trapezium is 344 feet, and the perpendicular on the one side is 75 feet, and that on the other side is 84 feet; required the superficial contents. *Ans.* 27348.

(2) The diagonal of a trapezium is 236 yards, and the two perpendiculars are 62 and 79 yards; required its contents.
Ans. 16638.

(3) The two perpendiculars of a trapezium are 84 and 62·5, and the diagonal 1095·7; required its contents. *Ans.* 80260·025.

(4) Given in a trapezium the diagonal = 763·5, the two perpendiculars 175·5 and 480·5; required the superficial contents.
Ans. 243048.

(5) There is a field in the shape of a trapezium, whose diagonal is 630 feet, and the upper perpendicular is 39·5 feet, the lower one is 83 feet; required the contents of the field, also half the area of the upper field, when 100 square feet have been taken therefrom.

Ans. Total content 38587·5 ft.; area of up. △ = 12342·5.

For a trapezoid.

(1) The two parallel sides of a trapezoid are 368 and 593, and the perpendicular is 72·5; required its contents.
Ans. 34836·25.

(2) The two parallel sides of a trapezoid are 509·6 and 719·4, and the perpendicular is 62·03; required its contents.
Ans. 38117·435.

REGULAR POLYGONS.

RULE.—The perimeter of the polygon, multiplied by the perpendicular from its centre to any side, and this product divided by 2, will give the area.

Or, if the side of the polygon be squared and multiplied by the tabular area in the following table, the product will give the area.

Number of sides.	Names	Areas, or multipliers.	Radius of circumscribed circle.
3	Trigon or Triangle	0·4330127	0·5773503
4	Tetragon or Square	1·0000000	0·7071068
5	Pentagon	1·7204774	0·8506508
6	Hexagon	2·5980762	1·0000000
7	Heptagon	3·6339124	1·1523824
8	Octagon	4·8284771	1·3065628
9	Nonagon	6·1818242	1·4619022
10	Decagon	7·6942088	1·6180340
11	Undecagon	9·3656399	1·7747324
12	Dodecagon	11·1961524	1·9318517

Ex (1.) Required the area of a heptagon whose side is 12.
Ans. 523·2833856.

(2) Required the area of a dodecagon whose side is 30.
Ans. 10076·53716.

(3) Find the area of a pentagon whose side is 15.
Ans. 301·083545.

(4) Find the area of an octagon whose side is 7.
Ans. 236·5953779.

To find the superficies of a right cone.

RULE.—Multiply half the circumference by the whole slant height, or half the slant height by the circumference of the base.

Ex. (1.) A right cone is 30 inches in diameter, and height 52; required its superficies. *Ans.* 2550·35088 inches.

(2) There is a sugar-loaf, whose diameter at base is 8 inches, and height 16; required its superficial contents?
Ans. 207·219936 inches.

To find the superficies of a right cylinder.

RULE.—The circumference of the base multiplied by the altitude, with double the area of the base added, will give the superficial contents. For the solid contents, multiply the altitude by the area of either end.

Ex. (1.) Required the superficial and solid contents of a right cylinder, whose diameter is 20 inches, and length or axis 42.
Ans. 3267·264 superficies.

(2) There is a garden roller, whose length is 48 inches, and diameter 10; required its superficies. *Ans.* 1507·92.

To find the area of an ellipsis.

RULE.—The product of the transverse and conjugate diameters and the number ·7854 will give the area.

Ex. (1).—The two axes of an ellipse are 80 and 90; required its area. *Ans.* 5654·88.

(2) Find the area of an ellipse whose two axes are 40 and 15. *Ans.* 471·24.

(3) If the two axes of an ellipse be 56 and 82, what is its area? *Ans.* 3606·5568.

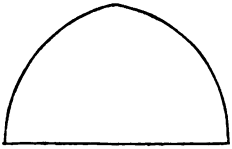
To find the area of an elliptical segment.

RULE.—Find the area of a circular segment, which has the same altitude and vertical axis. Then state : the vertical axis :

the axis which is parallel to the base of the segment :: the circular segment's area : area of elliptical segment.

Ex.—Find the area of the elliptic segment, whose altitude is 30, and its vertical axis 90, and its parallel axis 40.
Ans.

To find the area of a parabola.



RULE.—Two-thirds of the base multiplied by the perpendicular height will give the area.

Ex. (1)—The base of a parabola is 14, and height 4. Required its area. *Ans.* 37·33.

(2) If the base of a parabola be 20, and the height 6, what is the area? *Ans.* 80.

(3) Given the base 56, and the height 20. Required the area. *Ans.* 376·66'.

To find the area of a cycloid.

RULE.—Multiply the area of the generating circle by 3, the product is the area.

Ex. (1).—The diameter of the generating circle is 8. Required the area of the cycloid. *Ans.* 150·7958.

(2) The area of the generating circle of a cycloid is 113·0976. Required the area. *Ans.* 339.2928.

(3) The area of a cycloid is 530145; required the diameter of the generating circle. *Ans.* 150.

To find the solid content of a cone or pyramid.

RULE.—Multiply $\frac{1}{3}$ of the height of the pyramid or cone by the area of the base.

Ex. (1)—There is a pyramid whose height is 25 feet, and its diameter at base is 10 feet; required its solidity. *Ans.* 1636·25.

(2) A cone is 4 feet in diameter at its base, and 7 feet high. Required its solidity. *Ans.* 29·3216.

(3) A cone has its diameter 20 feet, and has also a square base, and its height 14 feet. What is its solidity?
Ans. 1498,74.

To find the solid content of a prism.

Rule.—Multiply the length of the prism by the area of its base.

Ex. (1)—There is a prism whose sides are 6 inches, and length 12 inches. Required its solidity. *Ans.* 187·0614 inches.

(2) Suppose that the area of the end of a prism is 124·32 inches, and its length 12 feet. Required its solidity. *Ans.* 491·92 solid feet.

To find the area or solidity of any of the platonic bodies.

Rule.—Square the lineal edge, and multiply this by the tabular number. The product is the superficial content. For the solidity, cube the lineal edge, and multiply it by the tabular number for solidity.

Ex. (1).—In a hexahedron the length of a ridge of a side is 7. Required its area. *Ans.* 294.

(2) An icosahedron has the length of the ridge of its side 14. Required its area and solidity.
Ans. Area, 1697·409; solidity, 5984·664.

No. of Sides.	Name.	Superficial Multiplier.	Solid Multiplier.
4	Tetrahedron	1·73205	0·11785
6	Hexahedron	6·00000	1·00000
8	Octahedron	3·46410	0·47140
12	Dodecahedron	20·64572	7·66311
20	Icosahedron	8·66025	2·18169

To find the superficies of a sphere.

Rule.—Multiply 3·14159265, or 3·1416 by 4 times the square of the radius.

Ex. (1).—A sphere is 40 inches in diameter. Required the superficial contents. *Ans.* 5026·56.

(2) The radius of a sphere is 17 inches. Required the solid contents. *Ans.* 907·9224 inches.

To find the solid content of a sphere.

Rule 1.—Cube the radius, and multiply it by 3·14159265 $\times \frac{4}{3}$, or 4·1887902.

Rule 2.—Square the diameter, and multiply it by the circumference, and take $\frac{1}{6}$ of the product for the solidity.

Rule 3.—The diameter cubed and multiplied by ·5236 will give the solidity.

Rule 4.—The circumference cubed and multiplied by ·01688 will give the solidity.

Ex. (1).—What is the solid content of a globe, supposing its circumference to be 20 inches? *Ans.* 4612·4 inches.

(2) If the diameter of a cannon-ball be 6½ inches, what is its solidity? *Ans.* 143·79365 inches.

To find the solidity of a spheroid.

A spheroid is either *oblong* or *oblate*. *Oblong* when it revolves round the lesser axis, and *oblate* when it revolves round the greater axis.

Rule.—Square the revolving axis, and multiply this by the fixed axis, and this product by ·5236.

Ex.—The two axes of a spheroid are 10 and 12. Required its solidity as an oblong and oblate spheroid.

$10^2 \times 12 \times ·5236 = 628·32 =$ solidity of oblate spheroid.
$12^2 \times 10 \times ·5236 = 753·984 =$ „ oblong „

To find the solidity of a parabolic conoid.

Rule.—Multiply the area of the base by half the height for the solidity.

Ex. (1).—If the height of a parobolic conoid be 14, and the area of its base 204·53, what is the solidity? *Ans.* 1431·71.

TIMBER MEASURING.

Square Timber.

To find the solid contents of square or four-sided Timber.

Rule.—The mean breadth multiplied by the mean thickness, and this product by the length, gives the solid content.

Ex. 1.—A piece of timber is 18 feet long, the breadths of the greater end are 2 feet 4 inches, and 1 foot 8 inches, and those of the less ends 12 inches and 10 inches; required the solid contents. *Ans.* 49 feet.

(2) There is a piece of timber whose breadths of the less ends are 1 foot 8 inches, and 1 foot 2 inches, and those of the greater end 26 inches and 12 inches, it is 40 feet long; required the solid contents. *Ans.* 82½ feet.

Round Timber.

Rule.—The square of the quarter girt, multiplied by the length, the product is the content.

The mean girt is the girt taken equidistant from the ends; sometimes it is found by adding the circumferences of the ends and dividing by 2, the quotient is the mean girt.

Should the tree not taper regularly, several circumferences must be taken at equal distances; then if the sum of them be divided by the number, the mean girt is obtained.

Ex. (1)—There is a piece of timber 35 feet 6 inches long, mean quarter girth is 33 inches; required its content.

Ans. $267\frac{31}{32}$ feet.

(2) A piece of timber is 26 feet long, its girth at the larger end is 15 feet, and at the smaller 3 feet; what is its content?

Ans. $131\frac{5}{6}$ feet.

(3) Required the contents of a tree whose mean girth is 6 feet 4 inches, and length 36 feet 9 inches. *Ans.* $93\frac{73}{192}$ feet.

ARTIFICERS' WORK.

MODE OF COMPUTATION.

(1) Glaziers and masons compute their work by the square foot.

(2) Painters, plasterers, paviors, slaters, tilers, &c., by the square yard of 9 feet.

(3) Bricklayers and plasterers by the square yard of 9 feet; sometimes by the square rod or pole, which contains $30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet.

(4) The standard thickness of brickwork is $1\frac{1}{2}$ brick thick.

(5) Calculations are frequently abridged by using decimals instead of duodecimals.

1. OF GLAZIERS' WORK.

Observations.—Round windows are measured as if square.

Ex. (1) A house has 2 tiers of windows, and 2 windows in each tier, the breadth of each being 4 feet 2 inches; the height of the first tier is 7 feet 6 inches, and that of the second 5 feet 8 inches. Required the expense glazing, at $15\frac{1}{2}$*d.* per foot?

Ans. 8*l.* 0*s.* $0\frac{1}{4}$*d.*

(2) A window 45 feet long and 12 feet 8 inches high, is to be glazed; what will be the expense, at $12\frac{3}{4}$*d.* per foot?

Ans. 30*l.* 5*s.* $7\frac{1}{2}$*d.*

2. OF MASONS' WORK.

Observations.—All walls, columns, steps, and blocks of stone, are measured by the cubic foot of 7·28 inches. All slabs, pavements, chimney-pieces, coping, &c., by the square foot.

The workmanship is measured by the square foot.

The materials are measured by the cubic foot.

Ex. 1.—There is a court-yard 32 feet 8 inches long, and 17 feet 2 inches broad; what will be the expense of paving, at 3*s*. 4*d*. per yard. *Ans*. 92*l*. 10*s*. 10*d*.

(2) There is a foot-path 3 feet 6 inches broad, and 79 feet 8 inches long; what will be the expense of paving it, at 3*s*. 5½*d*. per foot? *Ans*. 49*l*. 18*s*. 3½*d*.

(3) What would the paving of a court come to, at 2*s*. 10½*d*. per square foot, whose length is 42 feet 6 inches, and breadth 27 feet 4 inches. *Ans*. 135*l*. 4*s*. 10¼*d*.

3. OF PAINTERS' WORK.

Observations.—In measuring mouldings, the line or tape employed is thrust into them.

Windows are usually charged at a certain price each.

Double measure is allowed for carved mouldings and things of similar description.

Ex. 1.—What will be the expense of painting a room whose sides are pannelled, and whose length round is 135 feet 6 inches, height 14 feet 6 inches, two doors, each 7 feet high and 4 feet 10 inches broad, which are to be charged for as double space, deducting a fire-place 4 feet 8 inches high by 4 feet 2 inches broad, and three windows, each being 7 feet 6 inches high and 4 feet 2 inches broad, the cost per yard being 8½*d*. *Ans*. 7*l*. 8*s*. 10½*d*.

4. OF PLASTERERS' WORK.

Observations.—The work assumes different names, according o the *ground* on which it is done.

If on walls, it is called *rendering*.

If on laths, it is called *ceiling*.

Deductions are allowed for doors, windows, and chimneys.

A barrel of cement (3 cwt.) is 5 bushels.

36 bushels of cement } are required to cover a rod of brick-
36 „ sand } work.

4½ square yards require 1 bundle of laths and 500 nails.

Plasterers' work is frequently measured by the 100 square feet.

Ex. 1.—A ceiling is to be plastered, its length 32 feet 8 inches, and breadth 20 feet 6 inches; what was paid for it, at 9*d*. per yard? *Ans*. 2*l*. 15*s*. 9¾*d*.

Ex. 2.—A room is to be plastered; it is 27 feet 9 inches long, 11 feet 6 inches high, and 17 feet 2 inches broad; there are 2 doors, each 7 feet high, 4 feet 2 inches broad, 4 windows, each 7 feet 6 inches high, and 3 feet 8 inches broad. Required the expense of rendering, which is 6*d.* per yard, and plastering, at 2½*d.* per yard. *Ans.* 5*l.* 2*s.* 0½*d.*

5. Of Bricklayers' Work.

Observations.—Chimneys are generally allowed as if solid.

A brick measures 8½ inches long, 4 inches broad, and 2½ thick.

Four courses, with the mortar, 1 foot in height.

A thousand bricks usually weigh 2⅔ tons.

A standard rod of brickwork will require 27 bushels of lime and about 3 loads of sand.

4500 bricks are required for one standard rod of brickwork.

If the superficial content of a wall be multiplied by the number of half bricks in the thickness, and divided by 3, the quotient will give the standard thickness.

Ex.—(1) In a wall, whose circumference is 247 feet 8 inches, and height 12 feet 3 inches, how many yards of standard brickwork are therein, supposing it to be 3½ bricks thick?

Ans. 1562¼ square yards.

(2) What is the content in standard brickwork of a wall which is 172 feet long, and 27 feet 10 inches high, and 3 bricks thick?

Ans. 9575 feet.

The following Table is extracted from Davies' edition of HUTTON's *Mathematics*. "Taking 4500 for the bricks employed, including waste, in a standard rod 272 feet face, and a brick and a half thick, the following Table will serve to determine the number of bricks in any proposed case.

Area of the face of the wall in feet.	No. of bricks required for 1, 2, 3, 4, &c. feet, at the respective thicknesses.				
	1 brick.	1½ brick.	2 bricks.	2½ bricks.	3 bricks.
1	11·02947	16·54412	22·05883	27·57353	33·08824
2	22·05883	33·08824	44·11765	55·14707	66·17648
3	33·08824	49·63236	66·17648	82·72060	99·26472
4	44·11765	66·17648	88·23531	110·29413	132·35296
5	55·14706	82·72060	110·29414	137·86766	165·44120
6	66·17648	99·26472	132·35296	165·44121	198·52944
7	77·20589	115·80884	154·41180	193·01473	231·61768
8	88·23531	132·35296	176·47062	220·58827	264·70592
9	99·26472	148·89708	198·52945	248·16180	297·79416

"The left-hand column exhibiting the area of the face of a wall in feet, the number of bricks required for 1 brick thick, 1½ bricks thick, &c., are shown in the corresponding horizontal column under the appropriate heading. For greater numbers, being 10 times, 100 times, 1000 times, &c., the number of square feet specified in any part of the left-hand column, take 10 times, 100 times, 1000 times, &c., the given numbers under the proper head."

5 square feet at 2 bricks will require 110·294 bricks.
50 " " 1102·94 do.
500 " " 11029·4 do.

6. CARPENTERS' WORK.

Observations.—Mouldings are measured by *running* measure; *i. e.*, by length. Joists are measured by running measure, an allowance of ⅔ of the thickness being made for the part inserted in the wall.

Partitions are measured by their superficies.

In grain centering double measure is allowed.

Staircases are measured by passing a line close over them, and then multiplying this length by the stair's length.

Doors are calculated by their superficies; panelled doors are allowed for as double measure.

Battens are from 4½ to 7 inches, deals average 9 inches, and planks 11 to 12 inches in width.

Ex. (1).—What is the content of a floor 56 feet 10 inches long, and 16 feet 8 inches broad? *Ans.* 947 feet 2⅔ inches.

(2) Required the number of squares in a floor 272 feet 4 inches long, and 24 feet 8 inches broad. *Ans.* 67·17365 + squares.

7. SLATERS' AND TILERS' WORK.

Observations.—The contents of the generality of roofs are found by multiplying the girt over from eaves to eaves, by the length of the ridge. But when the roof is of a *true-pitch, i. e.*, when the vertex meets in a right angle, 1½ times the breadth of the building is the girt.

EXAMPLES.—What is the content of a tiled roof 53 feet long, whose whole girt is 28 feet 4 inches? What would it amount to at 27*s.* 4*d.* per square? *Ans.* 20*l.* 10*s.* 5*d.*

RULE FOR THE MOTION OF BODIES.

Rule.—As the difference of the speed of the two bodies, is to the *distance gone* by one, so is the motion of the quicker body to the distance required.

Ex. (1.)—One body moves 3 miles per hour, another 5; the first has already travelled 9 hours when the second starts to overtake it. Required the distance the latter must travel before it overtakes the former.

$$\begin{array}{r}5\\3\\\hline 2\end{array} : 27 :: 5 : \tfrac{135}{2} = 67\tfrac{1}{2} \text{ miles. } \textit{Ans.}$$

(2) Suppose a sound was heard at the moon, distant from the earth 240000 miles, and suppose that it proceeded towards the earth for 4½ days; at this time another sound started off from the earth towards the moon, travelling at double the usual rate. How long will it be before they meet, after the last started?

Ans. 2 days, 18 hrs., 44 min., 37 $\frac{233}{571}$ sec.

(3) Two rays of light, travelling at the rate of 200000 miles per second, move, the one, A, from the planet Saturn, distant 900000000 of miles from the sun; the other, B, from the moon; the two planets being in opposition. Required how long it will be before they will meet; the moon being 240000 miles from the earth, and the earth 95000000 miles from the sun.

Ans. 47 min. 16$\frac{3}{200}$ sec.

(4) Three couriers, A, B, and C, start from London to a certain place, distant 1700 miles. A goes 7 miles an hour; B. 10, until he overtakes A; he then proceeds at A's pace; C follows, going 12 miles an hour. A starts 20 hours before B, and B 16 hours before C. Required when they will all be together, and how far from London at the hour of meeting?

Ans. 6⅔ days after A started; 1120 miles from London.

(5) Two bodies move. One weighs 209·75 lbs.; and the other 165·7 lbs. The greater body is impelled with a force 6 times greater than the lesser. What is the proportion of their velocities? *Ans.* 1 and 7·5709.

(6) One body contains 32·95 times the matter of the other, and is impelled with a force 13½ times that of the other. Required the ratio of their velocities. *Ans.* 2·4407 to 1.

(7) Two friends start at the same time, one from Liverpool, and the other from London, distant 202 miles. A travels at the rate of 3½ miles per hour; and B at the rate of 5½ miles per hour. After travelling 7 hours, B discovered he had left some-

thing in London, and consequently returned, still travelling at the same rate. He then, after quitting London, travels at the rate of 8 miles per hour, hoping to retrieve his lost time. What distance will they meet from Liverpool?

Ans. $95\frac{12}{23}$ miles from Liverpool.

(8) A and B start from Dover, distant 70 miles from London, to pass through London on their road to St. Alban's, distant 25 miles from London. A starts $3\frac{1}{2}$ hours before B, and travels at the rate of $5\frac{1}{2}$ miles per hour; B goes at the rate of $6\frac{3}{4}$ miles. When will he overtake A, and how long will A have been on his journey?

Ans. In $15\frac{2}{5}$ hrs., and A will have been on his journey $18\frac{9}{10}$ hrs.

PART VIII.

CHRONOLOGY.

Explanations of the various terms used in the Calendar.

Calendar is an almanac. The word is derived from *Calendæ*, the first days of the Roman months, or from *Calendarium*, a sort of pocket-book kept by rich men at Rome, into which were entered their accounts and daily transactions. The Calendar at present used in ecclesiastical affairs, consists of the months and days of the months, on which are marked the fasts and festivals, changes of the moon, and planets' places, &c.

Julius Cæsar, as has already been remarked, made the year to consist of 365 days 6 hours; the 6 hours being disregarded till the expiration of every fourth year, when a day was added after the VI. Cal., making a double VI. Cal. Hence the name Bissextile, or twice sixth.

Sosigines, a celebrated astronomer of Alexandria, was brought to Rome by Cæsar and effected these alterations. Flavius, a scribe, arranged the order of the Roman festivals and composed a new calendar. His mode of computing the days by calends, nones, and ides after the old manner was published and received the sanction of the dictator. This constituted the Julian Calendar, and it existed for more than 16 centuries, viz. to 1582, in the time of Pope Gregory, who, having discovered that the vernal equinox, which was on the 21st of March at the Council of Nice in 325, then fell on the 18th, caused 10 days to be dropped between 4th and 15th of October. He likewise, in order to make the solar and civil years agree more correctly, ordered that every even century *only* should be leap year, and not any *uneven* umber of centuries, divisible by 4, so that in 7000 years, here will scarcely be a difference of 1 day. This alteration was ntroduced into our own country in 1752, when the days between eptember 2 and 14 were ordered to be dropped. So that the nonth of September was to contain only 19 days.

It was customary amongst the Romans to reckon backwards n marking the days of the month. Thus, as the ides were on he 13th of the months, except in March, May, and October, he days before the *ides* were reckoned 1, 2, 3, &c. before the des; and the days following the *ides* were reckoned 19, 18, 17, 6, &c. before the kalends, as may be seen in any Roman alendar.

The *cycle of the moon*, or *Metonic cycle*, is a period consisting f 19 years, at the end of which time the new and full moons ommence precisely on the same day of the month as at the ommencement of the cycle. For in 19 years 1½ hour there are 35 lunations.

The *golden number* is the *number* of this period, *i. e.*, the umber of years that have elapsed in the lunar cycle.

The *epact* is the *moon's age* at the commencement of the ear.

The *Roman indiction*, instituted by Constantine A. D. 312, as a period consisting of 15 years; at which time the Romans ollected the taxes imposed on conquered countries. It is useless our present Calendar. To find the Roman indiction, add 3 to e year of Christ, and ÷ by 15, the remainder is the Roman ndiction.

The *cycle of the sun*, or as it ought to be called, the cycle of e Sunday Letter, is a period of 28 years, after which the same ays of the month occur on the same days of the week as in the receding cycle. The days of the week are connected with the ays of the year by means of the first seven letters of the alphabet; so that

stands for January 1
" 2
" 3
" 4
" 5
" 6
" 7
" 8
" 9
&c., &c.

And as these 7 letters are regularly repeated in order through the days of the year, it is evident that Sunday must always have the same letter throughout the year.

Ex.—In the year 1842, if January 1 be represented by A, B will represent Jan. 2, C Jan. 3, and so on; and if Jan. 1 fell on Saturday (as it did in 1842) A will represent this day, and B Jan. 2, Sunday; hence the Sunday etter was B for the year 1842.

The *Julian Period* is a period of 7980 years; at the expiration of which time, the three cycles, viz., the *lunar cycle*, the *solar cycle*, and the *Roman indiction* will commence together. It is found by multiplying together the cycles of years belonging to each; viz., lunar cycle 19, solar cycle 28, and Roman indiction 15, which gives for product 7980 years. The Julian Period may always be found by adding 4713 to the year of Christ. Thus $4713 + 1843 = 6555$, is the year of the Julian Period. It is useless in our Calendar.

There was also a *cycle* called the *Victorian cycle*, which had its origin from Victorius about the year 457. He multiplied the number of years in the cycle of the moon by the number in the cycle of the sun (viz. 19×28) which gave a period of 532 years. This cycle was more correct in its results than the 84-year cycle, which was received into the Western Churches, and by which the British Churches computed Easter. It was continued after the separation of Britain from the Roman Empire.

The *Dominical letter* is the letter which denotes the Lord's day, or Sunday.

The *number of direction* is a number which must be added to the 21st of March, the sum giving the day of the month in March or April on which Easter falls. Thus, in the year 1842, the number of direction was 6, which, when added to March 21st, gives for Easter day March 27th.

Many fasts and festivals apparently depend upon the sun's entrance into the signs of the Zodiac.

	Sun enters
Annunciation of the Virgin Mary, 25th March	Aries, 21st March.
St. Mark's day, 25th April	Taurus, 19th April.
Corpus Christi	Gemini, 20th May.
John the Baptist, 24th June	Cancer, 21st June.
St. James, 25th July	Leo, 22nd July.
St. Bartholomew, 24th August	Virgo, 20th August.
St. Michael, 29th September	Libra, 20th September.
St. Simon and St. Jude, 28th Oct.	Scorpio, 23d October.
St. Andrew, 30th November	Sagittarius, 22nd Nov.
St. Thomas, 21st Dec., Jesus, 25th Dec., or Christmas Day	Capricornus, 21st Dec.
St. Paul, 25th January	Aquarius, 20th January.
St. Matthew, 24th February	Pisces, 19th February.

Having found Easter Sunday by the Rule, page 284, the following Table will give the time of the moveable feasts which depend on it:—

NAME OF FEAST.	Time before in Weeks.		Time after in Weeks.	REMARKS.
Septuagesima Sunday...	9	...	...	The Sundays which fall after the Epiphany to Septuagesima Sunday are called Sundays after Epiphany.
Sexagesima Sunday.....	8	Easter		
Shrove Sunday, or Quinquagesima do.	7			
Shrove Tuesday and Ash Wednesday are the Tues. and Wed. following Quinquagesima..................	...	...	...	The Sundays which fall between Ash Wed. and Easter Sunday are called Sundays in Lent.
Quadragesima Sunday..	6			
Palm Sunday	1	..		
Good Friday, two days				
Low Sunday...............	...	...	1	The Sundays which fall between Easter Sunday and Whit-Sunday are called Sundays after Easter
Rogation Sunday........	...	...	5	
Ascension Day, or Holy Thursday, the Thursday following Rogation Sunday		...		
Whit Sunday	...	...	7	
Trinity Sunday	...	...	8	The Sundays following Trinity Sunday to Advent are called Sundays after Trinity.

To find the Golden Number.

If 1 be added to the given year of Christ, and the sum be divided by 19, the remainder will be the Golden Number; if nothing remains, 19 is the Golden Number.

Note.—1 is added because the Æra of Christ commenced in the second year of the cycle.

Ex.—Find the golden number for 1842.

1842
1
19)1843(97
171
133
133
19 G. N.

To find the Epact.

If 1 be subtracted from the Golden Number, and the remainder be multiplied by 11, the product will be the Epact; if it should exceed 30 divide the product by 30, and the remainder will be the Epact.

Ex.—Find the epact for 1842.

The G. N. already found is 19, then,

19
1
18
11
30)198(6
180
18 Epact.

The epact of any year being known, that of the next year may also be known by adding 11 to the given year's epact. But should the sum exceed 30, subtract 30 therefrom, the remainder is the epact. If the preceding year's epact be 29 (being the last year of the cycle), then having added 11 to the preceding year's epact, we subtract the 29 to make the epact for the next or first year of the cycle, because the intercalary month has but 29 days.

To find the Dominical or Sunday Letter.

Rule.—To the year of Christ add its fourth part, increased by 6, divide the sum by 7, and the remainder taken from 7 leaves the number of the letter; reckoning A 1, B 2, C 3, D 4, E 5, F 6, and G 7. In leap years the rule will give the letter answering to the months following February to the end of the year. At the commencement of a leap year two letters are used in reverse order, thus G F would be used in 1844 to the end of February.

Required the Dominical letter for 1842.

$$\begin{array}{r} 1842 \\ 460 \\ \hline 2302 \\ 6 \\ \hline 7\overline{)2308} \\ \hline 329 + 5 \end{array}$$

Then 7 — 5 = 2 or B.

Required the Dominical letter for 1829.

$$\begin{array}{r} 1829 \\ 457 \\ 6 \\ \hline 7\overline{)2292} \\ \hline 327 + 3 \end{array}$$

Then 7 – 3 = 4 = D.

To find on what Day of the Week in any Month the 1*st*, 2*nd*, 3*rd*, 4*th*, &*c. day will fall.*

The following doggrel should be committed to memory, remembering that the words in the first line represent the *first* six months of the year, and those in the last line the *last* six.

Thus, *At* represents January, *Dover* represents February, and so on.

At Dover Dwell George Brown Esquire
Good Christopher Finch And David Friar.

Rule.—Find the Sunday letter for the given year, then the first day of the month will either be before or after the Sunday letter in the alphabetical order of the letters.

An example will illustrate it.

Let it be required to find on what day of the week the 19th of October, 1842, falls. In this year the Sunday letter is B, and the word answering to the month of October is *And*, therefore as B is the Sunday, and the initial letter of *And* is A, it follows that the first of the month fell on a Saturday, because it is the letter before B, the Sunday letter, therefore the 19th day must fall on a Wednesday.

Again. Let it be required to find on what day of the week the 4th of December will fall. Here, as before, the Sunday letter is B and the initial letter of *Friar* (the word answering to January) is F; and as F, in the order of the alphabet, is four letters from B, it follows that the *first* of the month will fall on a Thursday, and consequently that the fourth day will be on a Sunday.

7	1	2	3	4	5	6	No. of days of week.
A Sat.	B Sunday	C Monday	D Tues.	E Wed.	F Thurs.	G Friday	

This is for the year 1842. Any other year may be found in a similar manner by putting 1 over the Sunday letter.

Note.—Easter Sunday is the first Sunday after the *first* Full Moon, which happens upon, or next after, the 21st of March; and if the Full Moon falls on a Sunday, *Easter-day* is the Sunday after.

To find the Paschal Full Moon and Easter-day *for the Year.*

From the No. 50 subtract the sum of the epact for the given year, and the No. 6 (if the sum exceed 30 subtract 30 therefrom), the remainder is the Paschal full moon, or Easter limit.

Again. Find the number of the Dominical letter for the given year, and add 4 thereto, subtract the sum from the Easter limit (found as above), and then the remainder from the next higher number divisible by 7 without remainder. If this last remainder be added to the limit before found, the sum will give the number of days from March 1st to Easter-day, both days being included.

Ex.—Find the Paschal Full Moon, and thence Easter-day for 1842.

$$\begin{array}{rl} & 50 \\ \text{Epact} + 6 = 18 + 6 = & \underline{24} \\ & 26 \ \text{Easter limit.} \end{array}$$

$$\begin{array}{lr} \text{No. of Dominical letter} \ldots\ldots\ldots & 2 \\ \text{No. to be added} \ldots\ldots\ldots\ldots\ldots & \underline{4} \\ \text{Sum} & \underline{6} \\ \text{Then } 26 - 6 = & 20 \end{array}$$

Also 21, (next higher No. to 20) divisible by 7 without remainder; hence, $21 - 20 = 1$: the remainder to be added to the Easter limit 26; $\therefore 26 + 1 = 27$; consequently Easter Sunday was on 27th March.

To find the Moon's Age.

Rule.—To the epact add the day of the month and the No. of the month from March, to the given month.

Ex.—Required the Moon's age on November 9th, 1843.

Here the epact for 1843 is

$$\begin{array}{lr} \text{Epact} \ldots\ldots\ldots\ldots\ldots\ldots = & 0 \\ \text{Day of month} \ldots\ldots\ldots\ldots = & 9 \\ \text{No. of month after March} = & \underline{8} \\ & 17 \end{array}$$

This rule is tolerably correct; generally answering to within a day of the truth.

EXAMPLES.

(1) On what day does Easter Sunday fall in the year 1873? *Ans.* April 13th.

(2) Required the epact, golden number, Sunday letter, and Easter-day of the year 1887.
Ans. Epact 6, G. N. 7, D. letter B, Easter Sunday April 10th.

(3) What is the epact for the year 1867? *Ans.* 25.

(4) On what day is Whitsunday in the year 1865? *Ans.* June 4th.

(5) Find the epact for 1859. *Ans.* 26.

(6) Find the Dominical letter for 1854. *Ans.* A.

(7) State the epact, golden number, and Sunday letter for 1880. *Ans.* Epact 18, G. N. 19, Sunday letter D C.

(8) On what day of the week will November 9th fall in the years 1843 and 1846? *Ans.* In 1843 on Thursday, in 1846 on Tuesday.

(9) What are the epact, Dominical letter, and golden number or 1859? *Ans.* Epact 26, dom. letter B, G. N. 17.

(10) What is the epact for 1869? *Ans.* 17.

(11) What is the number of direction for 1843? *Ans.* 26.

(12) Required the day of the week on which November 15th will fall in the year 1843. *Ans.* Wednesday.

(13) Find the epact and Dominical letter for 1885.
Ans. Epact 14, G. N. 5, Dom. letter D.

(14) On what day of the month and in what month will Easter Sunday fall in the year 1853? *Ans.* March 27th.

(15) Are the years 1800 and 2000 leap-years?
Ans. Second is leap-year, first is not.

(16) What year is leap-year coming after the year 2196? *Ans.* 2204.

PART IX.

HYDRODYNAMICS is divided into Hydrostatics and Hydraulics.

HYDROSTATICS

s a science which treats on the *pressure* of fluids. The word terally means, "stopping or balancing of water."

The motion or pressure of fluids is in every direction.

I. The pressure of water or of any other fluid on a horizontal ase is found by multiplying the specific gravity of the fluid by he cubic feet in the column of water.

Ex.—There is a cylindrical tub whose diameter is 6 feet, and he height of the top of a tube placed vertically therein is 7 feet above the surface. Required the pressure on the bottom of the tub.

Here $6^2 \times \cdot 7854 = 28\cdot 2744$ area of base.

And $28\cdot 2744 \times 7 = 197\cdot 9208$ cubic feet of water.

Specific gravity of water 1000 ozs., ∴ $197\cdot 9208 \times 1000 = 197920\cdot 8$ ozs. *Ans.* 5·52234875 tons.

(2) A cylindrical tub having its diameter 7 feet, and the height of the top of a tube inserted vertically therein is 8 feet; required the pressure on the bottom? *Ans.* 307876·8 ozs.

II. To find the pressure on *flood-gates.*

Rule.—1. Multiply 31·25 by the square of the depth, and this product by the breadth (all in feet) for the pressure in lbs.

2. Multiply ·2727 by the square of the depth, and the product by the breadth for the pressure in cwts.

3. If the gate be wider at top than at bottom,

III. Multiply 31·25 by $\frac{1}{3}$ of the difference between the top and bottom, and this product added to the product of the breadth at bottom and the square of the depth will give the pressure in lbs.

Ex.—(1) A flood-gate is 17 feet broad, and 13 feet below the water. What pressure does it sustain in lbs.? *Ans.* 89381·25 lbs.

(2) If a flood-gate be 19 feet broad at top, and 15 feet below the water, what is the pressure it sustains? *Ans.* 133593·75 lbs.

(3) The top of a flood-gate is 20 feet, and the depth of water below 10 feet, required its pressure in cwts. *Ans.* 545·4 cwts.

(4) Required the pressure in lbs. that a flood-gate sustains, the top of which is 28 feet broad and the bottom 23 feet broad, the depth of the water being 14 feet. *Ans.* 4518·416 lbs.

IV. To find the pressure of a *Bramah press.*

Rule.—Divide the area of the *cylinder*, or large piston, by the area of the pump piston, and multiply the quotient by the pressure of the pump piston.

Ex.—(1) If the diameter of the cylinder of a Bramah press be 10 inches, and that of the diameter of the pump piston be 2 inches, and the pressure applied to the pump piston be 60 lbs.; what is the power of the press? *Ans.* 500 lbs.

(2) If the area of the cylinder be 300 square inches, and the area of the pump piston be 14 square inches, and the pressure of the pump piston be 48 lbs., what power is the press? *Ans.* 1028·544 lbs.

(3) If the area of the cylinder be 1200 square inches, and the area of the pump piston be 26 square inches, and the pressure of the pump piston be 90 lbs., with what pressure will the press act? *Ans.* 4153·77 lbs.

V. To find the quantity of water discharged through a slit or notch in the side of any vessel open at top, or the quantity flowing over a weir.

RULE.—Multiply 5·4 by the area of the stream running through the notch, &c., and this by the square root of the depth; $\frac{2}{3}$ of this product is the number of cubic feet flowing through per second.

Ex.—(1) A notch is 5 inches broad, the depth of water $4\frac{1}{2}$ inches. Required the quantity which will run out in 37 seconds. *Ans.* 6357·5361 cubic feet.

(2) Given the breadth of a weir 32 yards, the depth of the water flowing over it 13 inches; what quantity will pass over in $1\frac{3}{4}$ hours? *Ans.* 403.242 ft.

To find the cubic contents of cisterns in imperial gallons.

Let C = cistern's content in feet; c = content in inches.
D = any dimension in feet; L = length; B = breadth; d = depth.
δ = diameter, if cylindrical.
n = number of gallons.

$$\left.\begin{array}{ll}(1)\ \text{Then} & C \times 6{\cdot}232 \\ (2) & c \times {\cdot}003607 \\ (3) & \dfrac{c}{277{\cdot}274}\end{array}\right\} = \text{content in imperial gallons.}$$

$$(4)\ \frac{n}{(L \times B) \times 6{\cdot}232} = d \qquad (5)\ \frac{n}{(L \times d) \times 6{\cdot}232} = B.$$

$$(6)\ \frac{n}{(B \times d) \times 6{\cdot}232} = L.$$

(7) For cylindrical vessels.

$\delta^2 \times L \times 4{\cdot}895$ = content, D being in feet.
$\delta^2 \times L \times {\cdot}034$ = do. if δ be in inches.
$\delta^2 \times L \times {\cdot}002832$ = do. if δ and 4 be in inches.

Ex.—(1) A cylinder contains 4284 cubic feet; required its content in imperial gallons. *Ans.* 26697·888 imp. galls.

(2) The contents of a cylinder are 72540 inches; required its content in imperial gallons. *Ans.* 261·65178 imp. gall.

(3) Required the length of a cylinder which shall contain 7820 imperial gallons, the breadth being 20 feet and depth 14 feet. *Ans.* 4·48.

HYDRAULICS

Is the science which treats on the *motion* of fluids. The word is of Greek origin, and from two words, signifying *water* and *a pipe*, on account of certain musical instruments used by the Greeks.

I. To find the quantity of water that flows through a pipe, river, or canal.

Rule.—Multiply the area by the velocity at the part where the area is taken.

Observation.—The greatest velocity of a running stream is in the middle of the stream.

II. Formula for determining the velocity of a stream at the bottom.

The square root of the number of seconds passed over by a floating body doubled, and the velocity of the top subtracted, with 1 added, will give the velocity of the stream at the bottom.

Ex.—Suppose the velocity at the top of a stream to be 25; required its velocity at the bottom.

$$25-(2\times\sqrt{25})+1=25-10+1=16$$

The *mean* velocity is $25-\sqrt{25}+\cdot5=25-5+\cdot5=20\cdot5$.

III. To find the velocity of water runnning through canals or pipes, or from the side of a vessel.

$$\left.\begin{array}{l}\text{Let } H \text{ be the height of the head}\\ \quad D \text{ the diameter of the pipe}\\ \quad L \text{ the length of the pipe}\\ \quad V \text{ the velocity}\end{array}\right\}\text{then}\left\{\begin{array}{l}\sqrt{\left(\dfrac{57\times H\times D}{57\times E\times D}\right)}\cdot23\tfrac{1}{3}\\ =V\text{, the velocity that the}\\ \text{fluid issues from the pipe.}\end{array}\right.$$

Ex.—Suppose a cistern is 8 feet high, and that it has a tube 14 inches long, opening 20 inches from the bottom, which tube is $3\frac{1}{2}$ inches in diameter; required the velocity that the water will issue from the orifice.

Here the height of the head is $8\times12=96$ inches.

And $H=92-20=72=$ height of the head of the fluid above the orifice.

$$\text{Then}\sqrt{\left(\frac{\cancel{57}\times\overset{36}{\cancel{72}}\times\cancel{3\cdot5}}{\cancel{57}\times\underset{7}{\cancel{14}}\times\cancel{3\cdot5}}\right)}\cdot23\tfrac{1}{3}=\sqrt{\tfrac{36}{7}}\times23\cdot33=2\cdot267''7\times$$

$23\cdot33=52\cdot9070741$ inches per second.

IV. To find the quantity issuing from the orifice per second.

Rule.—Multiply the preceding result by the area of the orifice for the quantity. Or multiply 5·4 by the square root of the depth in feet, and this product by the area of the outlet.

Ex.—If the orifice be circular, then $\frac{3{\cdot}5 \times 3{\cdot}1416}{2} = \frac{1}{2}$ circumference $= 5{\cdot}4978$ inches for area, hence $53{\cdot}2929 \times 5{\cdot}4978 = 292{\cdot}9947$* cubic inches per second.

V. M. Prony gives the following formula:—

Let $A =$ area of orifice in square feet.
$H =$ the height of the water.
$t =$ time.
$g =$ force of gravity at end of 1 second.
$Q =$ quantity of water discharged in cubic feet.
Then $Q = 0{\cdot}61938 \cdot At\sqrt{2g} \cdot H.$ (1)
Or $Q = 4{\cdot}818 \cdot At\sqrt{H}$ for any orifice. (2)
$Q = 3{\cdot}7842 \cdot d^2 \cdot t\sqrt{H}$ where d represents the diameter if the orifice be circular.

From (2) of these equations the following formula may be obtained:—

$$A = \frac{Q}{4{\cdot}818 \cdot t \cdot \sqrt{H}};\ t = \frac{Q}{4{\cdot}818 \cdot A \cdot \sqrt{H}};\ H = \frac{Q}{(4{\cdot}818 \cdot A \cdot t)^2}$$

To find the resistance of any plane surface, whilst moving through a fluid.

The resistance any plane surface encounters in passing through a fluid with any given velocity, is equal to the weight of a column whose height is the space a body would have to fall through in free space to acquire that velocity, and whose base is the surface of the plane.

Ex.—If a plane 20 inches square move through water at the rate of 16 feet per second, then

$\frac{16^2}{64} = 4$ feet, the space a body would require to fall through, to acquire a velocity of 16 feet per second; and 4 feet $= 48$ inches. Therefore $20 \times 48 = 960$ cubic inches=the column's solidity $\therefore$

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* Logarithms facilitate all calculations of this kind.

$\therefore \cancel{1728} : \cancel{960} :: 1000$ (the weight of a cubic foot of water) :

$$\begin{array}{rrr} \cancel{144} & \cancel{80} & 5 \\ 9 & 5 & 9\overline{)5000} \end{array}$$

555·111 oz., or 34·694 lbs. amount of resistance the plane meets with at the above velocity.

The following Table will give the cubic contents of a pipe one inch in diameter: it is extracted from HYDRAULICS, *Library of Useful Knowledge*:—

Feet in length.	Quantity in Cubic Inches.	Weight in Avoirdupoise Ounces.	Gallons, Wine Measure.
1	9·42	5·46	0·0407
2	18·85	10·92	0·0816
3	28·27	16·38	0·1224
4	37·70	21·85	0·1632
5	47·12	27·31	0·2040
6	56·55	32·77	·2423
7	65·97	38·33	·2448
8	75·40	43·69	·3264
9	84·82	49·16	·3671
10	94·25	54·62	·4080
20	188·49	109·24	·8160
30	282·74	163·86	1·2240
40	376·99	218·47	1·6300
50	471·24	273·09	2·0400
60	565·49	327·71	2·4480
70	659·73	382·33	2·8560
80	753·98	436·95	3·2640
90	848·23	291·57	3·6700
100	942·48	546·19	4·6800
200	1884·96	1092·38	8·1600

This Table will determine the capacity of a pipe of any other size.

RULE 1.—Multiply the tabular number opposite the pipe's length by the square of the diameter of the pipe; the product will give the inches, avoirdupoise ounces and wine gallons contained therein.

RULE 2.—The square of the diameter of any pipe in inches ill give the number of lbs. avoirdupoise contained in 3 feet of ngth. This divided by 10 (because 10 lbs. avoirdupoise = 1 nperial gallon) will give the number of imperial gallons.

Ex.—Required the number of gallons in 3 feet of pipe, whose iameter is 7 inches.

Here $7^2 \div 10 = 4{\cdot}9$ ∴ it contains $4\frac{9}{10}$ gallons.

Examples on the preceding formula.

(1) A stream moves at the top at the rate of 32 inches in a econd. Required its least velocity, or velocity at bottom. Also s mean velocity.

Ans. 21·6863 velocity at bottom; 31·84315 do. mean.

(2) There is a reservoir whose depth is 4 feet; it has a tube ½ inch bore, which lets the water escape at the distance of 12 nches from the bottom. It is required to find the velocity of the ater, and the quantity discharged in a second.

Ans. 60·3556 inches velocity; 106·6533205 cubic inches, quantity discharged per second.

(3) If the velocity of the top of a stream be 18 inches in a econd, required its velocity at the bottom?

Ans. 10·5148 velocity at bottom.

(4) There is a floodgate 16 feet broad, and 14 below the ater. Required the pressure it sustains in lbs. *Ans.* 98000 lbs.

(5) A Bramah press has the diameter of its large cylinder 6 inches, and the area of its tube is 9 inches. The pump is orked with a power of 64 lbs. With what pressure will the ump act? *Ans.* 1493·7984 lbs.

OF THE DIFFERENT KINDS OF ENGINES.

1. *Newcomen's Engine.*—The principle of this engine is as ollows:—The engine consists, like most other engines, of a hollow cylinder, with a solid piston. The piston is worked by a orking beam, to which it is attached. The steam enters the ylinder underneath the piston, and by its elastic force the piston s forced upwards to the top of the cylinder, when cold water is njected, which condenses the steam, and consequently forms a acuum beneath the piston-rod; this rod is consequently forced own by atmospheric pressure. There are many disadvantages ttending this engine, amongst which are, a waste of steam and uel. This is also called the *atmospheric engine.*

2. *Watt's Engine.*—Watt improved on the preceding by ondensing the steam in a separate vessel, and thus maintained he cylinder of a uniform temperature, thereby saving steam and

fuel. His second improvement consisted in admitting steam both above and below the piston. This latter engine is called *the double acting low-pressure engine*. His third improvement consisted in shutting off the steam when a portion of the stroke had been passed through by the piston. In this state the engine is called *the expansive engine*.

3. The *high-pressure engine* differs from the last kind, principally from the admission of steam of a high temperature into the cylinder, alternately above and below the piston; which steam, unlike that in the last, is not condensed, but suffered to escape. The steam in this engine acts only by its own elastic force, whilst in Watt's Engine the piston is forced down by atmospheric *pressure*.

Useful remarks on the properties of Steam and Steam Engines,

2000 cubic feet can be heated to 70° or 80° Fahrenheit, by 1 cubic foot of boiler.

200 cubic feet can be warmed by 1 cubic foot of surface of steam-pipe.

Watt discovered that steam in passing again to a state of water, emits 945° of latent heat.

Dr. Black discovered that 810° heat Fahrenheit become latent in steam.

The *atmospheric* pressure is generally taken at 15 lbs. per square inch.

The *working*, or effective pressure, is taken at ⅔ of 15, or 10 lbs. per square inch.

3½ or 4 gallons cold water are required by engines per minute for every horse power.

3 cwt. of Newcastle coals, are said to be equivalent to .. .. ..
4 cwt. of Glasgow coals.
7 cwt. of culm.
9 cwt. of wood.

In *low-pressure engines*,
16 lbs. of coals,
56 do. wood,
35 do. culm,
are consumed for 1 horse power per hour.

The maximum of 1 horse power is usually estimated at 1000 lbs. raised 13 feet high in one minute.

One bushel of coal will raise from 25 to 32 millions of lbs. to the height of 1 foot.

One lb. of coals will evaporate 10 lbs. of water at 212°.

One cubic inch of water will produce 1728 cubic inches, or a cubic foot, of steam.

I. To estimate the power of a steam engine.

RULE.—The product of the effective pressure on a square inch multiplied by the area of the piston in square inches, and this product by the length of stroke in feet, and this product by the number of strokes per minute divided by 44000, will give the number of horse power of the engine.

Ex.—Required the power of a low-pressure engine, whose cylinder is 34 inches, length of stroke 7 feet, making 18 double strokes per minute.

To find the area of the piston, let d = diameter of the piston, and C the circumference, then

$$\frac{d \cdot C}{4} = \text{area of piston.}$$

Hence we obtain $\frac{34 \times (34 \times 3\cdot1416)}{4} = 907\cdot9224$ area of piston in square inches.

Again, if 12 lbs. is the effective pressure, 7 ft. the length of stroke, 18 the number of double strokes in a minute,

$$\text{Then } \frac{907\cdot9224 \times \overset{6}{\cancel{12}} \times 7 \times 18 \times \cancel{2}}{\begin{array}{c}\cancel{44000}\\ \cancel{22000}\\ 11000\end{array}} = \frac{686389\cdot3344}{11000}$$

62·3990304 horse power.

II. To find the power of a *high-pressure engine.*

Let F = the force of steam in the boiler.
A = the area of the piston.
V = its velocity per minute.

$$\text{Then } \frac{\{(0\cdot9 \times F) - 6\} \times A \times V}{44000} = \text{horse power.}$$

$$\text{Or } \{(0\cdot9 \times F) - 6\} \times A \times V \times \cdot0000227^{*} = \text{horse power.}$$

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* The reciprocal of $44000 = \frac{1}{44000} = \cdot0000227$.

The following Table will give "the elasticity of steam," at different temperatures, Fahrenheit. It is extracted from various authors, but principally from M. Arago.

Elasticity of steam, the atmospheric pressure being reckoned as unity.	Degree of Fahrenheit.	Elasticity of steam, the atmospheric pressure being reckoned as unity.	Degree of Fahrenheit.
1	212 °	13	380·6
1½	234	14	386·9
2	250·5	15	392·8
2½	263·8	16	398·4
3	275·2	17	403·8
3½	285·	18	408·9
4	293·7	19	413·7
4½	300·3	20	418·4
5	307·5	21	422·9
5½	314·2	22	427·2
6	320·3	23	431·4
6½	326·2	24	435·5
7	331·7	25	439·3
7½	336·8	30	457·1
8	341·7	35	472·7
9	350·7	40	486·6
10	358·7	45	499·1
11	366·8	50	510·6
12	374·		

The first column has been calculated by the following formula, Let t = temperature of the steam, F the force of steam in inches of mercury.

Then $\left(\frac{t + 100}{177}\right)^6 = F$, and this divided by 30, will give the number of atmospheres of pressure.

This formula will serve up to 24 atmospheres; after which number the following must be employed.

Let t = temperature by Centigrade thermometer, 100° being regarded as unity, and t the excess above 100°, then the elasticity will be equal to $(1 + t \times {\cdot}7153)^5$.

By Rule 1. Ex.—The cylinder of a high-pressure steam-engine has a piston 6 inches diameter, with a 15-inch stroke, making 34 double strokes in a minute, steam being admitted whose elastic force is equal to 8 atmospheres on the interior of the cylinder; required the horse power.

NOTE.—⅙th of the effective pressure is allowed for friction.

Therefore at 15 lbs. pressure we get 15 × 8 = 120, deducting ⅙th we obtain 100 lbs. effective pressure per inch.

$$\text{Again, } \frac{6\,(6 \times 3\cdot1416)}{4} = 28\cdot2744 \text{ area of piston.}$$

$$\text{And } \frac{\overset{14\cdot1372}{\cancel{28\cdot2744}} \times \cancel{100} \times \overset{\cdot25}{\cancel{1\cdot25}} \times \overset{17}{\cancel{34}} \times \cancel{2}}{\begin{array}{c}\cancel{44000}\\ \cancel{220}\\ \cancel{44}\\ \cancel{22}\\ 11\end{array}} = \frac{60\cdot0831}{11} = 5 \text{ h. power.}$$

By Rule 2. The pressure of steam on the boiler of an engine is 34 lbs. per square inch, the diameter of the cylinder is 20 inches, length of stroke 4 feet, and the engine makes 40 double strokes per minute.

$$\text{Area of piston} = \frac{12\,(12 \times 3\cdot1416)}{4} = 113\cdot0976 \text{ inches.}$$

Velocity of piston = 4 × 40 × 2 = 320 feet per minute.

$$\therefore \{(0\cdot9 \times 34) - 6\} \times 113\cdot0976 \times 320 \times \cdot0000227 = 20\cdot21$$
horse power.

III. To calculate the velocity of the piston when the engine is at its maximum.

$120 \times \sqrt{L} = V$, where L = length of stroke and V the velocity.

Ex.—Suppose an engine has a 3-feet stroke, then,

$120 \times \sqrt{3} = 207\cdot84$ feet, velocity of piston per minute.

Therefore, as the engine has a 3-feet stroke per minute, we have

$$\frac{207\cdot84}{6} = 34\cdot64, \text{ about } 34\tfrac{1}{2} \text{ times nearly.}$$

(2.) If the engine has a 5-feet stroke, then we have

$120 \times \sqrt{5} = 268\cdot32$ velocity of piston per minute, and

$$\frac{268\cdot32}{10} = 26\cdot832 = 26\tfrac{3}{4} \text{ nearly.}$$

IV. To find the weight of the fly-wheel of an engine.

RULE.—Divide the product of the horse power of the engine and the number 2000 by the square of the velocity that the wheel's circumference moves in feet per second.

Ex. (1.)—If the diameter of a fly-wheel of a 32-horse power engine be 22 feet, and it makes 21 revolutions per minute, then, $22 \times 3{\cdot}1416 = 67{\cdot}1152$=circumference in feet, and $67{\cdot}1152 \times 21 = 1409{\cdot}34192$ feet, which the circumference moves through in one minute, then

$$\frac{1409{\cdot}34192}{6\quad 0} = 23{\cdot}48 \text{ feet per second.}$$

$$\text{Hence } \frac{32 \times 2000}{(23{\cdot}48)^2} = 116{\cdot}08 \text{ cwts.} = 5 \text{ tons } 16 \text{ cwts., weight of the fly.}$$

V. To find the pressure per square inch on a *safety valve.*

CASE 1.

When the weight is placed on the top of the *valve.*

RULE.—Multiply the area of the valve by the pressure per square inch, the product is the whole weight; from which formula any of the other quantities can be obtained. Thus, let A= area of valve; W=whole weight; P=the pressure, then

$$A \times P = W \,(1); \text{ or } P = \frac{W}{A}\,(2); \text{ or } A = \frac{W}{P}\,(3).$$

Ex. Suppose a weight of 60 lbs. to be put upon a valve, the area of which is 14 inches, then by formula 1,

$$\frac{60}{14} = 4\tfrac{3}{7} \text{ lbs. pressure per square inch.}$$

Any of the other quantities can be found by substitution.

CASE 2.

When the weight acts by a lever.

RULE.—The product of the weight, multiplied by the area of the valve in inches, and by the difference between the length of the lever and the distance of the valve from the fulcrum.

RULE.—The product of the weight, and the area of the valve in inches, will give the whole pressure of the valve. Then if the whole pressure be multiplied by the distance of the valve from the fulcrum, and this product be divided by the difference between the length of the lever and the distance of the valve from the fulcrum, the quotient will give the weight to be hung at the end of the lever to give the pressure required.

Ex. If the length of the lever be 25 inches, and the diameter of the valve be $3\frac{1}{2}$ inches (its area will = 9·62115 inches), the distance between the fulcrum and the valve 4 inches, then in order to give 80 lbs. pressure per square inch on the valve, $80 \times (9{\cdot}6)^2 = 769{\cdot}60$ lbs. the whole pressure on the valve, $\frac{769{\cdot}6 \times 4}{25 - {\cdot}4} = \frac{3078{\cdot}4}{21} = 146{\cdot}6$ lbs. will be the weight hung at the end of the lever to give the pressure required.

VI. The sum of sensible and latent heat of steam at any temperature is a *constant* quantity. This constant is 1180°.

Ex. Suppose the temperature of steam to be 320°; where the elastic force of it is equal to 5 atmospheres or 75 lbs. per square inch, we wish to discover the latent heat. Let S = sensible heat, and L the latent heat, then

$$320 - 32 = 188 = S$$
$$\therefore 1180 - 188 = 992 = L.$$

VII. To determine the quantity of water requisite to condense any quantity of steam at any temperature.

Let T = heat of the steam; t = temperature of warm water: τ = temperature of cold water. Then to find the quantity of cold water necessary to condense any quantity of steam.

$$\frac{T - t}{t - \tau} \times \text{cubic feet of steam} = \text{the quantity required.}$$

SPECIFIC GRAVITY.

Table of specific gravity of various solids and fluids.

EARTHS AND STONES.	
Agate	2·590
Alum	1·714
Amber	1·100
Amethyst, Oriental	3·391
Basalt	2·421 to 3·000
Beryl, Oriental	3·549
Borax	1·714
Brick	2·000
Camphor	0·985
Caoutchouc	0·933
Cornelian	2·613
Chalcedony	2·609 to 2·650
Chalk	2·252 to 2·657
Coals	1·020 to 1·300
Copal	1·045
Diamond	3·521
Emerald	2·600 to 2·770
Garnet, precious	4·000 to 4·230
Glass, crown	2·520
—— green	2·642
—— flint	2·760 to 3·000
—— plate	2·942
Granite	2·613 to 2·955
Gunpowder (solid)	1·745
———— (loose)	·836
Honey	1·450
Slate	2·672
Marble	2·742
Limestone	3·179

FLUIDS.	
Water	1·000
Acid, Muriatic	1·200

Acid, Nitric ..	.. 1·271	Ebony ..	.. 1331
—— Sulphuric	.. 1·860	Lignum Vitæ ..	.. 1333
Alcohol of commerce ..	0·835		
Ammonia aqulaus	.. 0·875	METALS.	
Oil of Turpentine	.. 870	Cast Iron ..	.. 7207
Olive Oil ..	.. 915	Bar Iron ..	.. 7788
		Hard Steel ..	.. 7816
WOODS.		Soft Steel ..	.. 7833
Cork ..	.. 246	Cast Brass ..	.. 8395
Elm	.. 556	—— Copper ..	.. 8788
Mahogany, Hondurus .	560	—— Silver ..	.. 10474
———— Spanish ..	852	—— Lead ..	.. 11352
Cedar ..	.. 596	Mercury ..	.. 13568
Walnut ..	.. 671	Gold (pure) ..	.. 19258
Fir	.. 694	Gold Coin ..	.. 17647
Beech ..	.. 696	Platinum (pure)	.. 19500
Teak	.. 745	———— wire	.. 21041
Oak, English	.. 970	Tin, Cornish	.. 7291
—— American	.. 872		

The unit for solids is water, which is taken at 1000 ozs. the cubic foot. As water is taken as above, and gold at 19258, it follows that gold is above 19 times heavier than water.

I. *To find the weight or magnitude of bodies.*

1. To find the magnitude of a body from its weight.

RULE.—Let $W =$ wt. of body in ozs., and S its specific gravity in the table; also C its cubic feet. Then $\frac{W}{S} = C$.

Ex. (1).—Required the cubic feet in ½ a ton of lead.

Here $10 \times 112 \times 16 = 17920$ oz. in half a ton, and $\frac{17920}{11352} = 1{\cdot}4$ cubic feet.

(2) What is the number of cubic feet in 30 cwt. of lignum vitæ? *Ans.* 40·33 cubic feet.

(3) What is the number of cubic ft. in 14 cwt. of caoutchouc?

Here $14 \times 112 \times 16 = 25088$ oz. in 14 cwt., and $\frac{25088}{933} = 26{\cdot}8$ cubic feet.

II. *To find the weight of a body from its magnitude.*

RULE.—Assume the same letters as before. Then $C \times S = W$.

Ex. (1).—What is the wt. of 10 cubic feet of granite of 2955 sp. gravity?

Here $10 \times 2955 = 29550$ oz.

(2) Required the wt. of 17 packages of mercury, each $2\frac{1}{2}$ ft. square. *Ans.* 21200 oz.

(3) Required the wt. of a log of ebony, which is 30 feet long and $1\frac{1}{2}$ feet square. *Ans.* $89842\frac{1}{2}$ oz.

III. *To find the weight of two ingredients compounded.*

Take the difference between the 3 specific gravities, viz. of of each ingredient and the compound, and also of the ingredients themselves. Then

If $W =$ weight of heavy body
w = its spec. grav.
X = wt. of lighter body
x = its spec. grav.
G = wt. of compound
g = its spec. grav.

Then

$$\begin{cases} \dfrac{(g-x)\times w}{(w-x)\times g} \times G = W \\[2ex] \dfrac{(w-g)\times x}{(w-x)\times g} \times G = X \end{cases}$$

Ex.—A composition weighs 260 lbs.; it consists of gold and silver, and its specific grav. is 15240. Required the wt. of each metal.

Here $w = 19326$; $x = 10744$; $g = 15240$; $X = 160$

$$\frac{4496 \times 19326 \times \overset{\cancel{4}2}{\cancel{1}6\cancel{0}}}{\underset{4291}{\cancel{8582}} \times \underset{381}{\cancel{15240}}} = \frac{173779392}{1634871} = 106{\cdot}29 \text{ gold.}$$

$\therefore$ 160 − 106·29 = 53·71 silver.

ON MECHANICAL CENTRES.

I. The *centre of gravity* is a point in any body from which if the body be suspended, the body will remain at rest in any situation in which it may be turned.

I. CENTRES OF GRAVITY.

Want of space prevents our pointing out the methods of mechanically finding the centre of gravity of plane or solid figures. This knowledge, if required, can be obtained from the generality of works on Civil Engineering or Mechanics.

The centre of gravity of a line, circle, prism, parallelogram, cylinder, circumference of a circle, sphere, regular polygon is the geometrical centre of these planes or solids.

The centre of gravity of the following figures is,

(1) In a *triangle* $\frac{2}{3}$ distance of a line drawn from any ∠ bisecting opposite side reckoning from the angle.

(2) In a *trapezium*. Divide the trapezium into 4 Δs by diagonals; then find the centres of gravity of each triangle, and join these points by diagonals; the point of intersection of these diagonals is the centre of gravity.

(3) In a section of a circle. Let $r =$ radius of circle; $c =$ chord of arc; $l =$ length of arc; then

$$\frac{r \times c}{l} = \text{distance of centre of gravity from centre of circle.}$$

(4) In a *parabolic space*, $\frac{3}{5}$ of distance of the axis from vertex.

(5) In a *paraboloid*, $\frac{2}{3}$ of distance of the axis from vertex.

(6) In a *cone* or *pyramid*, $\frac{3}{4}$ of distance of the axis from vertex.

Ex. (1) If a line bisecting the base of a Δ be 16 inches long, where is the centre of gravity?

Here $\frac{2}{3}$ of $16 = \frac{32}{3} = 10{\cdot}666$ distance from vertex.

(2) Required the centre of gravity of a cone whose axis is 22 inches. *Ans.* 16·5 from vertex.

(3) In a paraboloid the axis is 19 inches from the vertex. Required the centre of gravity. *Ans.* 12·66 inches.

II. CENTRE OF OSCILLATION.

The *centre of oscillation* in any vibrating body is a point in the axis of vibration, such, that if all *the matter of* the body were collected, and had the same force acting on it, would, if attached to the same axis, perform all its vibrations in the same time.

NOTE.—The centre of oscillation is situated in the straight line passing through the centre of gravity. The centre of oscillation always coincides with the centre of percussion.

The length of the pendulum vibrating seconds at the equator is 39·027 inches; at London is 39·1393 inches: at the poles, is 32·197 inches.

To find the time of one vibration of any pendulum.

I. RULE.—As the square root of the length of a pendulum vibrating seconds in the latitude of London is to the square root of the length of the given pendulum, so is 1 to the time of one vibration.

Ex.—Required the time of one vibration of a pendulum whose length is 16 inches, in the latitude of London.

$: \sqrt{31{\cdot}1393} : \sqrt{16} :: 1 : {\cdot}71681$, time of one vibration.

(1) What is the length of a pendulum vibrating $\frac{1}{4}$ seconds at London?

$(60)^2 : (15)^2 :: 39{\cdot}1393$

$16 : 1 :: 39{\cdot}1393 = 2{\cdot}4462$ inches in length.

(2) The lengths of pendulums have the same proportion to each other as the squares of the number of vibrations made in a given time.

(3) To find *the length* of a pendulum, making any *given number* of vibrations in a minute.

$\frac{140850}{n^2} = l$, when $n =$ number of vibrations, and $l =$ the length.

(4) The length of a pendulum in inches is = length of a pendulum at London $\times \left(\begin{matrix}\text{time of one}\\ \text{vibration}\end{matrix}\right)$ or it is $\frac{\text{Length}}{(\text{No. of vibrations})^2}$.

II. The following formula will express the *time of one vibration* in seconds, of any pendulum.

$\frac{1}{n}$ or $\sqrt{\frac{l}{39{\cdot}1393}}$ where $n =$ number of vibrations in a second, and $l =$ length of pendulum.

Ex. (1) If the number of vibrations be $\cdot 5$, then $\frac{1}{\cdot 5} = 2$ seconds, time of one vibration.

(2) If it be required to find the length of the pendulum,

Then $39{\cdot}1393 \times 2^2 = 156{\cdot}5572$ inches.

III. To find the number of vibrations of any pendulum in a second.

$\sqrt{\frac{39{\cdot}1393}{l}} =$ number of vibrations, or $\frac{1}{\text{time of one vibration.}}$

Ex.—Suppose a pendulum is $19{\cdot}5196$ inches long; required the number of vibrations in a second.

$$\sqrt{\frac{39{\cdot}1393}{12{\cdot}314}} = 3{\cdot}178 \text{ vibrations in a second.}$$

III. CENTRE OF PERCUSSION.

If a sword, stick, &c., strike any resisting object, as another sword or stick, &c., there is a point in the blade of the former, in which, at the time of contact, no shock will be received by the hand. The name of this point is the *centre of percussion*.

The same rules will give the distance of either the centre of oscillation or percussion from the axis of motion.

To find the distance of the centre of oscillation or percussion from the axis of motion.

The following are taken from DAVIES' HUTTON'S *Mathematics*, a work which the author of this cordially recommends to his reader's attention.

Expressions for the distances of the centres of oscillation for the several figures, suspended by their vertices and vibrating flatwise, viz.:—

1. Right line, or very thin cylinder, centre of oscillation $= \frac{2}{3}$ its length.
2. Isosceles $\triangle, = \frac{3}{4}$ of its altitude.
3. Circle $= \frac{5}{4}$ radius.
4. Common parabola $= \frac{5}{7}$ of altitude.
5. Any parabola $= \dfrac{2\,m+1}{3\,m+1} \times$ its altitude.

Bodies vibrating laterally, or in their own plane.

6. In a circle $= \frac{3}{4}$ of diameter.
7. In a rectangle suspended by one $\angle = \frac{2}{3}$ of diagonal.
8. Parabola suspended by its vertex $= \frac{5}{7}$ axis $+ \frac{1}{3}$ parameter.
9. Parabola suspended by middle of its base $= \frac{4}{7}$ axis $+ \frac{1}{2}$ parameter.
10. In a sector of a circle $= \dfrac{3 \text{ arc} \times \text{radius}}{4 \text{ chord}}$
11. In a cone $= \frac{4}{3}$ axis $+ \dfrac{(\text{rad. of base})^2}{5 \text{ axis}}$
12. In a sphere $=$ rad. $+ d + \dfrac{2 \text{ rad.}^2}{5\,(d + \text{rad.})}$ where d is the thread by which it is suspended.

Ex. (1).—What is the centre of percussion of a pole 48 inches long?

$\frac{2}{3} \times 48 = 32$ inches from the axis of motion.

(2) Required the centre of percussion in a sphere whose diameter is 20 inches, and which is suspended by a string 32 inches long.

Here $10 + 32 + \dfrac{2(10)^2}{5(32+10)} = 42 + \dfrac{200}{210} = 42{\cdot}952$ inches.

IV. CENTRE OF GYRATION.

The distance of the centre of gyration, from the axis of motion, is a mean proportional between the distances of the centres of gravity and percussion from that axis.

The distance R, the centre of gyration, from C, the centre or axis of motion, in some of the most useful cases, is as below.

In a cylinder or straight line, whose axis of motion is in one end $CR =$ length × ·5775.

In a circular wheel of uniform thickness, $CR =$ rad. $\sqrt{\frac{1}{2}}$.

In the periphery of a circle revolving about the diameter, $CR =$ rad. $\sqrt{\frac{1}{2}}$.

In the plane of a circle revolving about the diameter, $CR = \frac{1}{2}$ rad.

In the surface of a sphere revolving about the diameter, $CR =$ rad. $\sqrt{\frac{2}{3}}$.

In a solid sphere revolving about the diameter, $CR =$ rad. $\sqrt{\frac{2}{5}}$.

In a plain ring, formed of circles whose radii are R, r, revolving about its centre, $CR = \sqrt{\dfrac{R^2 + r^2}{2}}$

In a cone revolving about its vertex, $CR = \frac{1}{2} \sqrt{\frac{12}{5}a^2 + \frac{3}{5}r^2}$.

In a cone revolving about its axis, $CR = \sqrt{\dfrac{R^3 + r^3}{3\,(R + r)}}$

The above are taken from the before quoted work.

Ex. (1) In a solid globe, revolving about its diameter, which is 3 feet, the distance of the centre of gyration is = 18 × ·6324 = 11·3832 inches.

(2) There is a rod 82·3 inches long. Required the distance of the centre of gyration from the centre of motion.

Ans. 47·32825 inches.

It was intended to have introduced a short treatise on the Mechanical Powers; but the work has already exceeded the limits originally proposed by the author.

MISCELLANIES.

I. *Table of powers of the 9 digits.*

Digits	1	2	3	4	5	6	7	8	9
Powers	1	1	1	1	1	1	1	1	1
	2	4	8	16	32	64	128	256	512
	3	9	27	81	243	729	2187	6561	19683
	4	16	64	256	1024	4096	16384	65536	262144
	5	25	125	625	3125	15625	78125	390625	1953125
	6	36	216	1296	7776	46656	279936	1679616	10077696
	7	49	343	2401	16807	117649	823543	5764801	40353607
	8	64	512	4096	32768	262144	2097152	16777216	134217728
	9	81	729	6561	59049	531441	4782969	43046721	387420489

II. *To find the distance of an object at sea.*

RULE.—Multiply the square root of the height of the eye in feet, by the number 1·2247, and it will give the product in miles.

Ex. A man stood on a rock 45 feet high, and saw a sailing boat just making its appearance in the horizon. Required his distance from the boat.

$$\sqrt{45} \times 1{\cdot}2247 = 6{\cdot}7082039 \times 1{\cdot}2247 = 8{\cdot}2152876 \text{ miles.}$$

(2) A tower is 35 feet high; a man standing on a ladder, whose eye is 14 feet from the top of the tower, sees the hull of a ship; required his distance from it. *Ans.* 8·5729 miles.

III. *To find the magnifying power of an astronomical telescope or Galilean telescope.*

RULE.—The focal length of the *object* glass divided by that of the eye-glass, will give the magnifying power.

Ex. An astronomical telescope has an object glass whose focal length is 3 feet, and whose eye-glass has a focal length of $\frac{1}{10}$ of an inch. Required the power of the telescope. *Ans.* 360.

IV. *Useful properties of numbers.*

(Extracted from various Works.)

1. Every square number ends in one of these 5 figures, 1, 4, 5, 6, 9; or with an even number of ciphers preceded by one of those numbers.

2. The progression of the cubes 1, 8, 27, 64, 125, &c. of the natural numbers 1, 2, 3, 4, 5, &c., possesses this remarkable property; that if any number of its terms whatever from the beginning, be added together, their sum will always be a *square*.

Ex. $1+8=9$; $1+8+27+64+125=225$.

3. Every square number is divisible by 3, or becomes so when diminished by unity. Thus $4-1$; $25-1$; and by 4 when diminished by 1.

Ex. $121-1=120$; $169-1=168$.

4. Every square number is likewise divisible by 5, or becomes so when increased or diminished by 1.

Ex. $36-1=35$; $49+1=50$.

(5) Every number is either a square or divisible into 2, 3, 4, or more squares.

Ex. $30=25+4+1$; $81=64+16+1$; $120=81+25+9+4+1$.

V. *Formula for reducing the different scales of heat to each other.*

Let C=degrees of centigrade; F do. of Fahrenheit; R do. of Reaumur.

$$\text{Then } C^\circ=\frac{F^\circ-32^\circ}{6\times0{\cdot}3};\ F^\circ=(C^\circ\times6\times0{\cdot}3)+32^\circ;$$

$$R^\circ=\frac{4(F-32^\circ)}{9};\ F^\circ=\tfrac{9}{4}R^\circ+32^\circ.$$

Ex. (1) Required the degree of Fahrenheit when the centigrade stands at 65°.

Here $F^\circ=(65^\circ\times6\times0{\cdot}3)+32^\circ=390\times{\cdot}3=117^\circ+32=149^\circ$.

(2) If Fahrenheit's thermometer stands at 87°, at what degree will Reaumur's stand?

$$R^\circ=\frac{4(87^\circ-32^\circ)}{9}=\frac{220^\circ}{9}=24\tfrac{4}{9}^\circ$$

VI. *To find the height of a mountain by the barometer.*

The mercury falls about $\frac{1}{10}$ of an inch for every 32 feet ascended; but a more exact formula is as follows :—

Let d be difference between 32° and mean temperature.
" D " barometric hts. in 10ths of inch.
" δ " mercurial temperature.
" m The mean barometric height.
" E correct elevation.

$$\text{Then } E = \frac{30\,(D\,87 \pm \cdot 21\,d)}{m} \pm 2\cdot 833\,\delta$$

The formula may be given in words.

1. Multiply the difference between 32° and the mean temperature of the air by ·21, and add the product to 87, if the mean temperature be above 32°, but subtract it if below.

2. Multiply the sum or difference found above, by thirty times the difference between the barometric heights, in tenths of inches, and divide the product by the mean of the barometric heights, the quotient will give the approximated elevation.

3. Multiply the difference between the mercurial temperatures by 2·833 feet, and add this product to the approximated elevation, if that of the upper barometer be the greatest, otherwise subtract it; the result will be the corrected elevation in feet.

KEITH'S *Trigonometry*, 7th edit. p. 97.

PART X.

MISCELLANEOUS QUESTIONS.

(1) Divide 769 moidores amongst 75 people; required the amount each will receive in pounds sterling.
Ans. 13*l.* 16*s.* 10$\frac{6}{75}$*d.*

(2) Required the value of 9845 reams of paper at 14*s.* 6*d.* per ream. *Ans.* 7137*l.* 12*s.* 6*d.*

(3) It is said 70000 larks are sold in London in a single season; supposing each fetches 1$\frac{1}{2}$*d.*, what is the total amount?
Ans. 437*l.* 10*s.*

(4) If the circumference of the earth be 25020 miles, how many [illegible] barleycorns would reach half round it?
Ans. 4755801600 half barleycorns.

(5) If 57950 tons of butter be consumed in London and the county of Middlesex annually, what is the expense at 9$\frac{1}{2}$*d.* per lb.?
Ans. 5138233*l.* 6*s.* 8*d.*

(6) There were from 10th October, 1827, to 5th April, 1828, 4960293 barrels of strong beer brewed in London; required the value of it if 1 gallon was sold for 8*d.*? *Ans.* 165343*l.* 2*s.*

(7) What number is that from which if the one-tenth part be taken, and the remainder be divided by 75, the quotient shall equal 152? *Ans.* 10260.

(8) Blackfriars' Bridge cost 152840*l.* in building; how many passengers must cross it daily at 1*d.* each, for 20 years, reckoning 365 days in a year, to repay the shareholders? *Ans.* 5024$\frac{64}{73}$.

(9) Westminster Bridge cost 382500*l.* in building. Supposing 4500 passengers were to cross it daily at $\frac{1}{2}$*d.* each, how many years would elapse before its full value would be received? *Ans.* 113 years + 1230375 remainder.

(10) In 1829 the window tax produced 1151079*l.* 17*s.* 5$\frac{1}{2}$*d.*; what did each house pay supposing there were 259682 houses rated? *Ans.* 4*l.* 8*s.* 7$\frac{3}{4}$*d.* + 84008.

(11) In 1829 the legacy duty received in England was 1066091*l.* 10*s.* 2*d.*; how many persons' estates paid, supposing each to have paid 86*l.* 9*s.* 4*d.*? *Ans.* 12332 + 4149 remainder.

(12) In 1829 cotton manufactures were exported to the value of 1646849*l.* 8*s.* 4*d.* and 12248 vessels cleared outwards, what did each on an average take in value in this article? *Ans.* 134*l.* 8*s.* 5*d.* + 10132.

(13) In 1829 the land tax was 1188428*l.* 9*s.* 9*d.*, now supposing there are 76981 estates, required the average each paid. *Ans.* 15*l.* 9*s.* + 1440.

(14) In 1829 the unfunded debt was 7410898836*l.* in Great Britain, if the population of London and its suburbs was 1698247, what would each receive? *Ans.* 436*l.* + 654144 remainder.

(15) In 1829 rough rice paid duty 20995*l.* 5*s.* 3*d.*, if that from British colonies paid 1*s.* per quarter, and that from foreign countries paid 1*l.*, and if 13 qrs. of British were imported to 1 of foreign, how many quarters were imported? *Ans.* 12724$\frac{13}{33}$.

(16) In 1829 the quantity of gypsum imported paid a duty of 1*s.* 3*d.* per ton from British possessions, and from all other places 1*l.* 11*s.* 8*d.*, the net produce received was 7*l.* 6*s.* 4*d.*, how many tons were imported from abroad supposing the British imported was 80 tons? *Ans.* 1$\frac{44}{95}$ tons.

(17) In 1829 the timber duties amounted to 1556038*l.* 2*s.* 11*d.* the articles consisted of balks, ufers, battens, paling, deals, teakwood, &c., if there were 59 different kinds what did each sort pay, supposing that the battens and deals paid double of the other articles? *Ans.* 25508*l.* 16*s.* 5$\frac{1}{4}$*d.* + $\frac{11}{61}$.

(18) Supposing there are 9758 streets, alleys, and courts in London and Southwark, and the population to be 1800000, what will be the average in each street? *Ans.* 184 + 4528 remainder.

(19) "The sun never sets on our possessions;" how many miles will it have passed over on the equator at half-past 5 in the evening, supposing the earth to be 25020 miles in circumference and reckoning from noon? *Ans.* 5733$\frac{3}{4}$ miles.

(20) How many shillings would reach from London to Edinburgh, each being $\frac{3}{4}$ of an inch in diameter, and the distance being 400 miles? *Ans.* 33792000 shillings.

(21) The population of the United States is about 16000000, and the male slaves are probably 850000; what is the ratio of the slave population to the free? *Ans.* 1 to 18$\frac{14}{17}$.

(22) Suppose a person can count 170*l.* per minute, and to be employed 8 hours 30 minutes daily, and six days per week; how long would he be in counting the national debt, now 753000000*l.*?
Ans. 1447 weeks, 3 days $\frac{35}{289}$.

(23) A balloon is said sometimes to have moved 1 mile in a minute; how long would it be in going round the earth in latitude 45°, where a degree measures 42·43 miles?
Ans. 10 days, 14 hours, 34 minutes $\frac{2}{15}$.

(24) A government contractor has to furnish 3251 horses, at an average price of 25*l.* 12*s.*; now he buys 2749 @ 22*l.* 18*s.*, 350 @ 27*l.* 12*s.*, and the remainder @ 40*l.*; whether does he gain or lose, and how much? *Ans.* He gains 4532$\frac{1}{2}$*l.*

(25) How many bricks 9 inches long, 4$\frac{1}{2}$ broad, and 3 inches deep, can be made from 4 cubic yards of earth? *Ans.* 1536.

(26) If I pay 14*l.* 12*s.* for picking cotton, what weight can I have picked, supposing I pay 1*s.* 8$\frac{1}{2}$*d.* for 5 lb.?
Ans. 854$\frac{26}{31}$ lbs.

(27) Suppose I have 450 flag-stones, each 48 inches broad, and wish to pave a court 120 yards long and 4 wide, what length must the flag-stones be? *Ans.* 3$\frac{1}{5}$ feet.

(28) A body in motion, whose weight is 48lbs., is impelled at the rate of 150 feet per second; required the velocity of another, whose weight is 23$\frac{1}{2}$ lbs., moved by the same force.
Ans. 306·3 feet.

(29) A wall 78 yards long, 9 feet high, and 9 inches broad, is to be built; required the number of bricks, supposing 1000 will fill 11$\frac{1}{4}$ square yards. *Ans.* 140400

(30) A cubic foot of stone was 13 inches broad and 5 thick; required its length. *Ans.* 26$\frac{38}{65}$ inches.

(31) A cubic yard of stone was 44 inches long and 13 inches thick; required its breadth. *Ans.* 81$\frac{81}{143}$ inches.

(32) A joist is 20 feet long, 9 inches broad, and 3 thick; required the depth of another, 5 inches broad, and 27 feet long, which shall be three times the size. *Ans.* 12 inches deep.

(33) What is the difference between eleven hundred pounds one shilling, and eleven hundred shillings and one pound?
Ans. 1044*l.* 1*s.*

(34) The wind is stated to travel at the rate of 100 miles per hour, in a hurricane that tears up trees; required what distance it will go in 7$\frac{1}{2}$ seconds at this rate. *Ans.* 366$\frac{2}{3}$ yards.

(35) If the weight of the atmosphere be 11522211494201773089 lbs. avoir., and if the earth's superficies be 79991479257628409 8 square inches, what is the pressure an inverted vase sustains, whose superficial contents are 27 square inches? *Ans.* 388$\frac{4}{5}$ lbs.

(36) It is well known there are twelve signs of the zodiac, and each sign contains 30°; how many degrees of right ascension will the sun have when he rises 14° 12″ in Cancer?
Ans. 104° 12″

(37) As climates depend on the length of the longest day at any place, and as they are found by subtracting 12 from the length of the longest day (except for the polar circles), and turning the remainder into half-hours, the number of which shows the climate; from this explanation, find the climates of places whose longest days are 16 hours 12 minutes, 13 hours 10 minutes, 17 hours 18 minutes. *Ans.* 9th climate, 3rd climate, 11th climate.

(38) What is the difference between 5 times eleven hundred and thirteen, and eleven times five hundred and thirteen.
Ans. 78

(39) Divide sixty-nine millions and five by three hundred and seventy-two and a half. *Ans.* 185234$\frac{136}{149}$.

(40) What is the difference between the 19th part of 7239 and the 15th part of 5700? *Ans.* 1

(41) The national debt is about eight hundred millions. Now supposing that three millions and a half were paid off annually by means of the sinking fund, how many years would it be before it was wholly paid off. *Ans.* 228$\frac{4}{7}$ years.

(42) A lottery prize came up 36,000*l.*, there were deductions amounting to 1294*l.* 12*s.* The remainder was divided amongst 5 families, consisting of 7 persons each; how much would each family and person receive? *Ans.* 991*l.* 11*s.* 7$\frac{3}{4}$*d.* each person.
4957*l.* 18*s.* 3$\frac{1}{4}$*d.* each family.

(43) A cubic foot of water weighs 1000 ozs.; how many lesser cubes, each containing 3$\frac{1}{2}$ ozs., could be formed out of a cubic foot of water? *Ans.* 285$\frac{5}{7}$ cubes.

(44) Supposing the circumference of the earth in latitude 35° to be 14894 miles, at what rate per hour are the inhabitants in that latitude carried? *Ans.* 620·58 miles.

(45) The planet Mars goes round the sun in about 1 year

and 322 days; how many years of the planet Mercury, which are about 88 days in length, would be contained in a year of the planet Mars? *Ans*. $7\frac{71}{88}$.

(46) If there are 850 millions of inhabitants on the earth, and if $\frac{1}{34}$ part dies annually; how many people will die in 22 years? *Ans*. 550000000.

(47) The sum of two numbers is 927030, and the lesser is 29681; required their quotient and difference.

Ans. Quotient $30\frac{6919}{29681}$, difference 867668.

(48) A. and B. start from London, A. goes due west, at the rate of $6\frac{1}{4}$ miles per hour, and B. due north at the rate of 9 miles per hour. Now supposing A. was not to start till 6 hours after B., required at the end of the 10th day, how far they will be apart; and suppose on the 10th day each increases his day's journey by $1\frac{3}{4}$ miles, and that they determine to meet; required how long, after starting towards each other, they will be before meeting; and the distance each man will have travelled from his first setting out; the time of a day's travelling being 10 hours. *Ans*. 6·889 days; A.'s distance = 1030·118 miles; B. = 632·005, and distance on 10th day = 1074·78.

(49) A. B. and C. are three vessels, A. sails 6 knots per hour, B. 9, and C. 11. A. goes south 5 days, B. north 7 days, and C. due west 8 days, all casting anchor at the end of their respective times; required the distance of A., B., and C. from each other. *Ans*. B. and C. 2957·48 m.; A. and B. 2232 m.; C. and A. 2149·17 m.

(50) There are two bodies put in motion; the one A. has moved 2960 feet before the other B. starts. B.'s rate is 700 per second and A.'s 300; required the number of feet the ball B. must traverse before it overtakes A. *Ans*. 5180.

(51) A hare is 230 yards before a dog when he starts after her, the dog runs at the rate of 12, and the hare at the rate of 9 miles per hour; required the time the dog will be before he overtakes the hare. *Ans*. $2\frac{27}{44}$ minutes.

(52) A slaver was seen 16 miles off, a steam frigate gave chase, the slaver sailed at the rate of 10 knots an hour, and the steamer at the rate of $14\frac{1}{2}$; required the distance run before capture, and time engaged in overtaking the slaver.

Ans. $51\frac{5}{9}$ miles, $3\frac{5}{9}$ hours.

(53) Two war steamers exactly of the same force and rate of sailing, observe each other 21 miles off. They bear up, but when 12 miles apart, the steamer A. meets with an accident in her machinery, and therefore can only sail at the rate of 3 miles an hour. They still proceed on the same tack, and when 2 miles

distant, A. by B.'s firing is still further disabled, and compelled to drop anchor; required how long B. will be from the time of starting till she comes up with A., and the distance B. sails. They sailed 10 miles an hour at starting.

Ans. $85\frac{2}{13}$ minutes, time of B.'s sailing, $14\frac{5}{26}$ distance sailed by B.

(54) A starts from London 4 hours before B., and travels at the rate of 6½ miles per hour, and B. starts after him at the rate of 7 miles per hour; required how long B. will be, from the time of A.'s starting, before he overtakes A. *Ans.* $50\frac{2}{7}$ hours.

(55) A. starts from Edinburgh, distant from Dover 472 miles, at 10 o'clock on Friday morning, travelling at the rate of 8¼ miles per hour; and B. starts from Dover on Sunday morning at 9 o'clock, travelling at the rate of 6¾ miles per hour; required on what day and hour they will meet, and how far B. will be from Edinburgh at the time of meeting.

Ans. $434\frac{7}{80}$ miles from Edinburgh.
Sunday at 37 minutes past 2 P.M.

(56) A privateer sees a merchantman 16 miles off and gives chase; the privateer sails 8½ knots an hour, and the merchantman 7¾; required how long the privateer will be before it overtakes the merchantman, and distance run.

Ans. 181⅓ miles distance; 21⅓ hours.

(57) A ship of war fires a gun, the flash is seen, and the report reaches the observer 12¼ seconds after; required the distance between the ship and the observer.

Ans. 2 miles, 5 fur., 43 yards, ½ feet.

(58) A stone is observed to be 5 seconds falling to the ground from an eminence; required the height from which it fell.

Ans. $402\frac{1}{12}$ foot.

(59) A man dropped a stone down a well, and found 8 seconds elapsed before it struck the water at the bottom; required the depth of the well. *Ans.* 1029⅓.

(60) Divide 7299*l.* 19*s.* 6¼*d.* by 64½.

Ans. 113*l.* 3*s.* 6½*d.* + 104.

(61) Gave 44*l.* 16*s.* for 56 yards of cloth; what cost 5 English ells? *Ans.* 5*l.*

(62) If 400 men can dig a trench 500 feet long, 30 wide, and 50 deep in 20 days of 6 hours long, how many men will it take to dig another 890 feet long, 160 wide, and 50 deep in 50 days of 8 hours long? *Ans.* 1139⅕.

(63) Supposing a ball to be projected on the equator eastward round the globe, and to rebound at the distance of every 600 feet, how many rebounds would it take before returning to the

point of starting; and what time would it occupy, supposing each rebound to be completed in 30 seconds.

Ans. 220176 reboundings, and 10 w. 6 d. 10 h. 48m.

(64) What is the discount on 500*l.* for 1 year and 8 months, at 5 per cent.? *Ans.* 38*l.* 9*s.* $2\frac{3}{4}$*d.* + $\frac{1}{13}$.

(65) Find the least common multiple of 2, 4, 8, 12, 26, 56, and 182. *Ans.* 2184.

(66) Find the simple interest on 29684*l.* 12*s.* $9\frac{1}{4}$*d.* for $4\frac{1}{2}$ years at 5 per cent. *Ans.* 6679*l.* 0*s.* $10\frac{1}{4}$*d.*

(67) Add $\frac{1}{3}$, $\frac{2}{9}$, $\frac{3}{16}$, $\frac{7}{24}$, $\frac{5}{48}$, and $\frac{7}{96}$ together. *Ans.* $1\frac{61}{288}$.

(68) Divide $\frac{2}{3}$ of 96 by $\dfrac{\dfrac{2\frac{1}{2}}{3}}{\dfrac{\dfrac{4\frac{1}{2}}{2}}{\frac{3}{4}}}$ *Ans.* $129\frac{3}{5}$.

(69) Multiply $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{9}$ of $\dfrac{28\frac{1}{2}}{6}$ by $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$. *Ans.* $\frac{19}{48}$.

(70) Multiply ·000726 by 59·68. *Ans.* ·04332768.

(71) Extract the square root of ·0000986 to 4 places.

Ans. ·0099 + 59.

(72) Extract the cube root of 964·472.

Ans. 9·8 + 23280 remainder.

(73) If 10 bushels of wheat at 7*s.* per bushel, and 20 bushels of rye at 6*s.* per bushel, and 84 bushels of rye at 10*s.* per bushel be mixed together, what can I sell the mixture at per bushel?

Ans. 9*s.* $0\frac{1}{4}$*d.* + $\frac{13}{19}$.

(74) In 4969 guineas, 730 shillings, 593 crowns, and 675 half crowns, how many sixpences, twopences, and $10\frac{1}{2}$*d.*, and of each an equal number. *Ans.* $71177\frac{7}{37}$.

(75) A. B. C. and D. set out to go round the globe, travelling on the equator; A. goes 15 miles per day, B. 26, C. 30, and D. 55; A. and B. go round from east to west, and C. and D. from west to east; required how far apart A. and C. will be (reckoning from face to face) on the 90th day after starting; and how far B. and D. reckoning as before, on the 60th day; also how far A. and C., reckoning as before, on the 49th day after starting, and likewise when B. and D. would meet, supposing the circumference of the earth to be 25020 miles.

Ans. A. and C. 20970 miles apart; B. and D. 20160 ditto; A. and C. 22815 ditto at 49th day after starting; $308\frac{8}{9}$ days before B. and D. would meet.

(76) Find the least common multiple of 2, 9, 16, 27, 48, and 81. *Ans.* 1296.

(77) Divide ·025 by ·00098725 to two places of decimals. *Ans.* 25·32 +

(78) Extract the square root of ·000079684 to five places of decimals. *Ans.* ·00892 +

(79) Extract the cube root of ·00042982951 to three places of decimals. *Ans.* ·075 +

(80) Gave $9\frac{3}{4}$*d.* per lb. for plums; at what rate must I retail 3 cwt., so that I shall lose $16\frac{1}{4}$ per cent.? *Ans.* 11*l.* 2*s.* $9\frac{1}{4}$*d.* + $\frac{1}{5}$.

(81) A. and B. trade together for 12 months; A. put in 5000*l.* and at the end of 3 months 500*l.* more, but took out that sum and 700*l.* besides, at the end of 10 months; B. put in 4000*l.* and at the end of 6 months took out 2000*l.*, and 4 months after 1000*l.* more; they gained 1000*l.*, what was each man's share?
Ans. A. 647*l.* 4*s.* $10\frac{181}{961}$*d.*, B. 353*l.* 15*s.* $11\frac{529}{961}$*d.*

(82) How many piastres of 3*s.* $4\frac{1}{2}$*d.* each, can be exchanged for 50*l.* sterling, and 50 Spanish dollars @ 4*s.* 9*d.* each? *Ans.* $366\frac{2}{3}$.

(83) Reduce $\frac{5}{9}$, $\frac{6}{7}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{12}{18}$, $\frac{5}{36}$ to a common denominator.
Ans. $\frac{700}{1260}$, $\frac{1080}{1260}$, $\frac{756}{1260}$, $\frac{882}{1260}$, $\frac{840}{1260}$, $\frac{175}{1260}$.

(84) Add $2\frac{1}{2}$ to $\frac{5}{7}$ of 6. *Ans.* $6\frac{11}{14}$.

(85) From $4\frac{3}{4}$ take $\frac{2}{9}$ of $11\frac{1}{2}$. *Ans.* $2\frac{7}{36}$.

(86) Multiply $5\frac{3}{4}$ by $(1\frac{1}{7} + 7)$. *Ans.* $46\frac{23}{28}$.

(87) Divide $54\frac{3}{7}$ by $\frac{2}{5}$ of $(6\frac{1}{2} — 3\frac{1}{8})$. *Ans.* $40\frac{20}{63}$.

(88) Reduce 9*s.* $2\frac{1}{4}$*d.* + $\frac{1}{9}$ to the fraction of a half-crown. *Ans.* $\frac{397}{108}$.

(89) Reduce 19*s.* $10\frac{1}{2}$*d.* to the decimal of 1*l.* *Ans.* 99375.

(90) Add ·00723 + 729 + . 0·92168 + 31·709 together. *Ans.* 761·73791.

(91) From ·0009 take ·000009328. *Ans.* ·000890672.

(92) Multiply 29·09 by 38·607. *Ans.* 1123·07763.

(93) Divide ·00072 by 384. *Ans.* ·001875.

(94) Reduce 5 grs. 10 drs. avoirdupoise to the decimal of a quarter of a lb. *Ans.* 1·40625.

(95) If I give 195*l.* 12*s.* for 185 yards of cloth, what is that per yard? *Ans.* 1*l.* 1*s.* $1\frac{3}{4}$*d.*+$\frac{1}{185}$.

(96) 19 boys have 3 guineas and a half to divide amongst them; what will each receive? *Ans.*. 3*s* $10\frac{1}{4}$*d.*+$\frac{13}{19}$.

(97) 275 sailors took a prize of 12957*l.* 12*s.* value; what is the share of each sailor? *Ans.* 47*l.* 2*s.* $4\frac{1}{4}$*d.*+$\frac{221}{275}$.

(98) If 79 oxen cost 1298*l.* 16*s.*, what is the value of half an ox? *Ans.* 8*l.* 4*s.* $4\frac{3}{4}$*d.*+$\frac{70}{79}$.

(99) 3 cwt. of cheese was bought for 7*l.* 9*s.* 8*d.*; what was the value of $\frac{3}{4}$ of a lb.? *Ans.* $3\frac{3}{4}$*d.* $\frac{128}{336}$.

(100) If a pipe of wine was sold for 107*l*. 8*s*., what would three dozen cost, supposing one dozen to contain $2\frac{3}{4}$ gallons?

Ans. 7*l*. 0*s*. $7\frac{1}{2}$*d*.+$\frac{6}{7}$.

(101) A compositor sets up an Old Testament, containing 2728000 letters, what would he receive if $6\frac{1}{2}$*d*. is paid for 1000 letters? also in the New Testament, there are 838,380 letters: how much would he receive, at the same rate, for setting up these?

Ans. 73*l*. 17*s*. 8*d*. for Old Test.; and 22*l*. 13*s*. 11*d*.+386 for New Testament.

(102) The word *and* occurs 46227 times, and the word *Jehovah* 6855 times; what ratio exists between these numbers?

Ans. $6\frac{1699}{2285}$.

(103) There were in London 1270 hackney coaches, each of which, on an average, earned 15*s*. per day, what was the annual expense of these vehicles to the public? *Ans*. 347662*l*. 10*s*.

(104) In the Old Testament there are 39 books, and 929 chapters; what is the ratio of the books to the chapters? In the New Testament there are 27 books, and 260 chapters; required as before. *Ans*. $23\frac{32}{39}$ Old Test.; $9\frac{17}{27}$ New Test.

(105) What may I spend per week and month if I have 493*l*. 12*s*. per annum?

Ans. 37*l*. 19*s*. 4*d*. per month; 9*l*. 9*s*. 10*d*. per week.

(106) If 760 men can dig a trench 490 yards long, 36 wide, and 30 deep, in 54 days of 10 hours long, how many men will it require to dig another 580 yards long, 45 wide, and 42 deep, in 56 days of 12 hours long? *Ans*. $1265\frac{5}{98}$.

(107) Find the simple interest on 69843*l*. 19*s*. $10\frac{1}{2}$*d*. for 16 years, at $5\frac{1}{4}$ per cent. *Ans*. 58668*l*. 19*s*. 1*d*.+$\frac{14}{25}$.

(108) Find the discount on 73249*l*. 16*s*. 8*d*. for 6 years, at $5\frac{1}{3}$ per cent. *Ans*. 17757*l*. 10*s*. $8\frac{1}{4}$*d*.+$\frac{31}{33}$.

(109) If $6\frac{1}{2}$ yards of cloth cost 3*l*. 14*s*. 6*d*., what will 55 English ells cost? *Ans*. 39*l*. 7*s*. $11\frac{3}{4}$*d*.+$\frac{1}{13}$.

(110) How much sterling money can be obtained for 5680 French francs, exchange at 24 fr. 56 cents.?

Ans. 231*l*. 5*s*. $4\frac{3}{4}$*d*.+$\frac{161}{307}$.

(111) Find the least common multiple of 2, 4, 16, 80, 560, 9, 45, 240, 96, 72, and 2240. *Ans*. 20160.

(112) Add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{48}$, $\frac{9}{108}$, $6\frac{5}{6}$, and $\frac{7}{240}$ together.

Ans. $13\frac{253}{360}$.

(113) If $\frac{5}{6}$ of an English ell cost $7\frac{5}{6}$*s*., what will $2\frac{1}{3}$ yards cost? *Ans*. 17*s*. $6\frac{1}{2}$*d*.+$\frac{6}{25}$.

(114) Divide $\frac{2}{3}$ of $\dfrac{96}{\frac{5}{6}}$ by $\dfrac{\frac{1}{2}\text{ of }7}{\dfrac{3\frac{1}{4}}{2}}$ *Ans*. $35\frac{23}{35}$.

(115) Divide ·0000068234 by ·007. *Ans.* 0009747.
(116) Extract the square root of ·000078325. *Ans.* 00885.
(117) Extract the cube root of ·000082741. *Ans.* ·0435+.
(118) Required the square root of ·0095685. *Ans.* ·0978+.
(119) Required the cube root of 29·387. *Ans.* 3·085924.
(120) Required the quotient (by logarithms) of

$$\sqrt[9]{\frac{\dfrac{8\cdot26}{3\frac{1}{2}}}{7\frac{3}{4}}} \text{ by } \sqrt[10]{\frac{\{2\frac{1}{2}-(7\frac{1}{4}-6\frac{3}{4})\}\div6\frac{1}{2}}{\sqrt{9\cdot02}}}$$

Ans. 1·10281.

(121) Multiply $\dfrac{\frac{5}{8}}{\frac{7}{3}}$ by $\sqrt[5]{\dfrac{8\cdot02}{7}}$ *Ans.* 1·499855.

(122) Find the simple interest on 48*l.* 16*s.* 9*d.* for 3¾ years, at 3½ per cent. (by decimals). *Ans.* 6·38429*l.*

(123) Find the simple interest on 596*l.* 18*s.* 6*d.* for 13½ yrs., at 65 per cent. (by logarithms). *Ans.* 5238*l.* 0*s.* 4½*d.*

(124) What are the golden number, epact, and Dominical letter for the year 1869?

Ans. Epact, 17; Golden No., 8; Sunday Letter, C.

(125) Required the day of the month on which Easter falls in the year 1879. *Ans.* April 13.

(126) Required the golden number, epact, and Sunday letter for the year 1865.

Ans. Golden No., 4; Epact, 3; Sunday letter, A.

(127) It is required to convert 337 of the denary scale into a scale whose base shall be 3. *Ans.* 110111.

(128) Convert 100 of the denary scale into a scale whose base shall be 2. *Ans.* 1100100.

(129) Required the number in a base whose scale is 5, answering to 342 of the denary scale. *Ans.* 2332.

(130) Convert 569 of the denary scale into a scale whose base is 2. *Ans.* 1000111001.

(131) A steeple casts a shadow 342 feet long, and a stick 4 feet 2 inches long, when placed upright, casts another of 7 feet 2 inches. What is the height of the steeple? *Ans.* $198\frac{36}{43}$ feet.

(132) The hour and minute hands of a watch are together between 9 and 10; required the time. *Ans.* 9 h. $49\frac{1}{11}$ min.

(133) What is the value of $\frac{13}{15}$ of ½ a moidore?

Ans. 1*l.* 3*s.* 4¾*d*+$\frac{1}{5}$.

(134) Reduce ⅔ of half-a-crown to the fraction of $\frac{1}{5}$ of 7*s.* 9*d.* + ½. *Ans.* $1\frac{51}{749}$.

(135) A man observed a flash of lightning, and 7 seconds afterwards heard the report of the thunder; required the distance of the cloud at first. *Ans.* 1·514 miles.

(136) A ship of war, seeing a merchantman $12\frac{1}{2}$ miles off, gives chase at the rate of 10 miles an hour, whilst the merchantman sails at the rate of $9\frac{1}{4}$ miles per hour; required the distance run before the ship overtakes the merchantman, and the time of chase. *Ans.* Dist. run by ship of war, $166\frac{2}{3}$ miles; time of chase, $16\frac{2}{3}$ hours.

(137) The cylinder of a Bramah press is 134·5 inches superficial measure, the diameter of the pump piston is 1 inch, and it is moved with a power of 100 lbs.; required the power of the press. *Ans.* 17125 lbs.

THE END.

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